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Safe Attitude Controller Design for Multicopter via High-order Control Barrier Function

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Abstract

A safe controller is designed for multicopter attitude control using control barrier function. Two safety constraints on the maximum angular speed and maximum total thrust direction deviation from a given desired direction are considered. High-order control barrier function method is utilized to deal with the two safety constraints. The resulting safety filter is minimally invasive with respect to a given nominal attitude controller, and the control input is obtained by solving a linearly constrained quadratic optimization problem in real time. The performance of the proposed method is demonstrated by numerical simulation.

1. Introduction

Ensuring safety is of significant importance in many physical systems. Typically, safety constraints are defined as state constraints, which aim to prevent the state of interest from escaping certain predefined safe sets. Nagumo's theorem provides a tool to achieve the safe conrtol by guaranteeing the set invariance.[5] However, it is hard to form a stabilizing control law which ensures safety by Nagumo's theorem. For example, Kim et al. used Nagumo's theorem to show the invariance of field-of-view limits for a stabilizing impact angle control law.[11,12] Because the stability and safety should be simultaneously considered, it is difficult to design controllers and further improve some performance. To deal with this issue, Ames et al. proposed a control barrier function (CBF) approach with a modern interpretation of Nagumo's theorem.[2] In CBF method, the control input is modified by a safety filter in a minimally invasive way along with a nominal (stabilizing) controller.

Several studies have been conducted to design the safe control of multicopter, such as model predictive control (MPC),[3] barrier Lyapunov function (BLF),[16] and CBF with a cascade controller design.[10] These existing studies mostly consider the safety of multicopter with respect to position and velocity. From the fact that the multicopter dynamics can be viewed as a cascade system with fast inner- and relatively slow outer-loops of rotational (attitude) and translational dynamics, the safety fulfillment in rotational dynamics should be guaranteed in real time. On the other hand, conventional CBF approach may not be applicable for the cases that the designed CBF has relative degree more than one. To treat high relative degree cases, a high-order CBF (HOCBF) method was proposed,[21] with a specific form of exponential CBF (ECBF).[18]

In this study, two safety constraints in rotational dynamics are considered for the case of i) rotating not too fast, and ii) restricting the attitude of multicopter. The two suggested safety constraints can be imposed by higher-level autonomy or mission requirements including visual odometry and terrain navigation using mounted cameras. To guarantee the two constraints, a safety filter is designed by adopting appropriate CBFs with relative degrees of one and two, respectively, considering the control allocation of multicopter. The proposed safety filter is demonstrated by numerical simulation with multicopter quaternion controller[7] as a nominal attitude controller. Figure 1 illustrates the attitude of multicopter without and with the designed safety filter (Section 4). Note that this study is an extension of the previous work[13] by incorporating a control allocation inside the safe controller.

The rest of this paper is organized as follows. Section 2 briefly summarizes the conventional and high-order control barrier function methods for the safe control synthesis, and the multicopter dynamics is described including the inner-loop system of rotational dynamics. In Section 3, two safety constraints in rotational dynamics considering maximum angular speed and maximum total thrust direction deviation are introduced. For the safeties, an attitude safety filter is designed by utilizing the conventional and high-order control barrier functions. The proposed safety filter provides a safe control input in real time, which is minimally invasive with respect to a given nominal attitude

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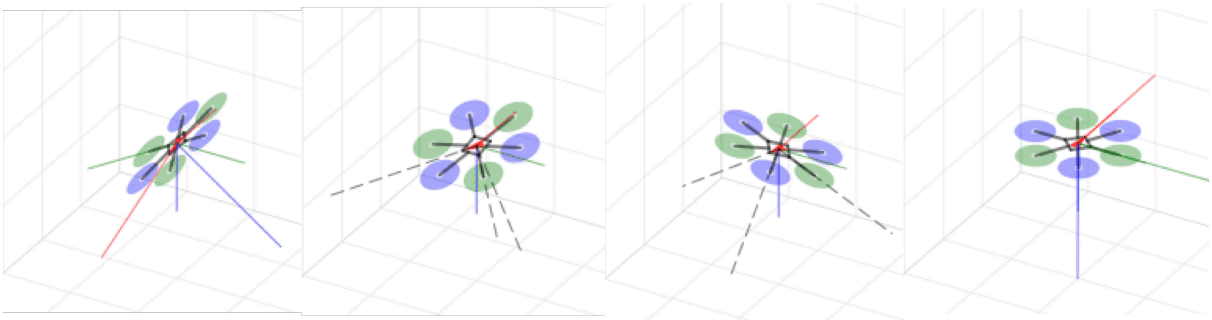
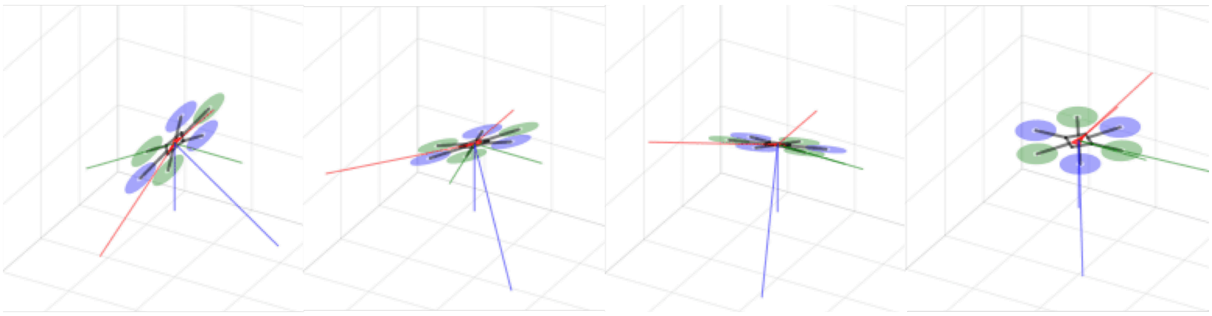


(a) Without safety filter



(b) With safety filter

Figure 1: Snapshots of the multicopter attitude (a) without and (b) with the proposed safety filters considering two safety constraints: i) maximum angular speed and ii) maximum total thrust direction deviation. Top and bottom figures of each column correspond to the same time stamp (left to right correspond to initial to final times). Axes become dashed lines when the multicopter rotates too fast. Axes become black when the angle between multicopter's z-axis and gravity direction is too large. The proposed method does not violate any safety constraints, while the controller without the proposed safety filter does.

controller. In Section 4, numerical simulation is conducted to demonstrate the performance of the proposed safe controller. Section 5 concludes this study.

2. Preliminaries

$\mathcal{K}_\infty$ denotes the set of all strictly increasing continuous functions $\alpha : [0, \infty) \to [0, \infty)$ such that $\alpha(0) = 0$, and $\mathcal{K}_\infty^e$ denotes the extended class $\mathcal{K}_\infty$, i.e., $\alpha \in \mathcal{K}_\infty$ and $\lim_{r \to -\infty} \alpha(r) = -\infty$ if $\alpha \in \mathcal{K}_\infty^e$.

Consider an input-affine control system as follows,

$$\dot{x}(t) = f(x(t)) + g(x(t))u(t), \tag{1}$$

where $x(t) \in X$ and $u(t) \in U$ denote the state and input at time $t \in \mathbb{R}_{\geq 0}$, respectively, with state and input spaces, $X \subset \mathbb{R}^n$ and $U \subset \mathbb{R}^m$, respectively. The functions $f : \mathbb{R}^n \to \mathbb{R}^n$ and $g : \mathbb{R}^n \to \mathbb{R}^{n \times m}$ are supposed to be locally Lipschitz continuous. Given locally Lipschitz continuous controller $k : \mathbb{R}^n \to \mathbb{R}^m$, the closed-loop dynamics can be written as

$$\dot{x}(t) = f(x(t)) + g(x(t))k(x(t)). \tag{2}$$

Note for given initial condition $x_0 \in X$ that there exists a maximal time interval $I(x_0) = [0, t_{\max}(x_0))$ such that the unique solution to the closed-loop dynamics (2) exists for all $t \in I(x_0)$.[9]

2.1 Conventional control barrier function

Safe controllers can be synthesized with control barrier functions as follows.

Definition 1 (Control barrier function[2,20]) *Let $C \subset \mathbb{R}^n$ be the 0-superlevel set of a continuously differentiable function $h : \mathbb{R}^n \to \mathbb{R}$ with $\frac{\partial h}{\partial x}(x) \neq 0$ when $h(x) = 0$. The function h is a control barrier function for (1) on C if there exists $\alpha \in \mathcal{K}_\infty^e$ such that for all $x \in \mathbb{R}^n$, $\sup_{u \in \mathbb{R}^m} \dot{h}(x, u) > -\alpha(h(x))$.*

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If h is a CBF for (1), then the set $K_{\mathrm{CBF}}(x) = \{u \in \mathbb{R}^m | \dot{h}(x,u) \geq -\alpha(h(x))\}$ is non-empty for all x,[2,20] and any locally Lipschitz continuous controller k renders C a safe set, i.e., C is forward invariant.[2,20]

2.2 High-order control barrier function

When h's relative degree r is more than one and h is r-times continuously differentiable, the conventional CBF approach introduced in Section 2.1 is not applicable. For the high-order cases, high-order control barrier function (HOCBF) provides a tool to synthesize a safe controller. One can recursively define functions h_i with $h_0 := h$ as follows,

$$\begin{aligned} h_i(x) &:= \dot{h}_{i-1}(x) + \alpha_i(h_{i-1}(x)), \forall i \in \{1, \ldots, r-1\}, \\ h_r(x,u) &:= \dot{h}_{r-1}(x,u) + \alpha_r(h_{r-1}(x)) \geq 0, \end{aligned} \tag{3}$$

with $\alpha_i \in \mathcal{K}_\infty^e$ for $i \in \{1, \ldots, r\}$. If $\sup_{u \in \mathbb{R}^m} h_r(x,u) \geq 0$ for all $x \in \cap_{i=0}^{r-1} C_i := \cap_{i=0}^{r-1} \{x \in \mathbb{R}^n | h_i(x) \geq 0\}$, then h is said to be a HOCBF. The input u satisfying (3) implies the forward invariance of C_0 if the initial condition x_0 is in $\cap_{i=0}^{r-1} C_i$.[21] Note that the exponential CBF (ECBF) is an example of HOCBF.[18,21]

2.3 Multicopter dynamics

The multicopter equations of motion can be represented as[15]

$$\dot{p} = v, \tag{4}$$

$$\dot{v} = ge_3 - \frac{1}{m} T R e_3, \tag{5}$$

$$\dot{R} = R\omega^\times, \tag{6}$$

$$\dot{\omega} = J^{-1}(M - \omega \times J\omega), \tag{7}$$

where $p \in \mathbb{R}^3$ and $v \in \mathbb{R}^3$ denote the position and velocity in the inertial coordinate, respectively, $R \in \mathrm{SO}(3)$ is the rotation matrix representing the attitude of the multicopter, $\omega \in \mathbb{R}^3$ is the angular velocity in body-fixed coordinate, $T \in \mathbb{R}$ and $M \in \mathbb{R}^3$ are the total thrust and torque applied to the center of gravity, respectively, $g \in \mathbb{R}_{>0}$ is the gravitational acceleration, $e_3 := [0,0,1]^\intercal$ is the unit vector pointing to the ground, $m \in \mathbb{R}_{>0}$ is the mass of the multicopter, and $J \in \mathbb{R}^{3\times3}$ is the moment of inertia. $(\cdot)^\times : \mathbb{R}^3 \to \mathrm{SO}(3)$ denotes the *hat map*.[15] The virtual input is defined as

$$\nu := [T, M^\intercal]^\intercal \in \mathbb{R}^4. \tag{8}$$

The virtual input ν is realized by the rotor thrust vector $u \in U \subset \mathbb{R}^m$ as follows,[14]

$$\nu = Bu, \tag{9}$$

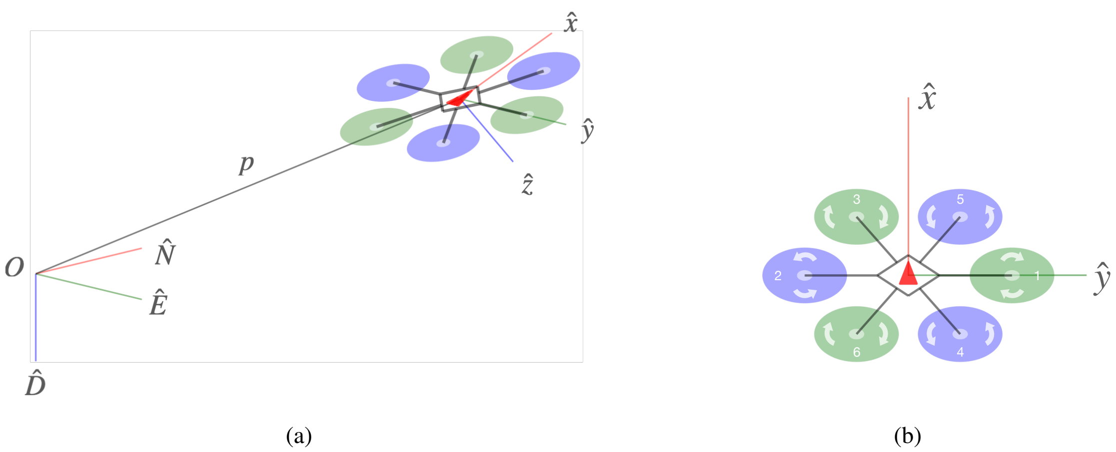

Figure 2: Illustration of multicopter. (a) Multicopter with six rotors (hexacopter-x) and frames, (b) top view

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where $B \in \mathbb{R}^{4\times m}$ is the control effectiveness matrix, which depends on the configuration of the multicopter. For example, the control effectiveness matrix B of the hexacopter with hexa-X configuration is written as[19]

$$B = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ -l & l & \frac{1}{2}l & -\frac{1}{2}l & -\frac{1}{2}l & \frac{1}{2}l \\ 0 & 0 & \frac{\sqrt{3}}{2}l & -\frac{\sqrt{3}}{2}l & \frac{\sqrt{3}}{2}l & -\frac{\sqrt{3}}{2}l \\ -k_M & k_M & -k_M & k_M & k_M & -k_M \end{bmatrix}. \tag{10}$$

Note that the position p and velocity v can be controlled by the rotation matrix R and the total thrust T. Therefore, the rotational dynamics can be seperated from the full dynamics of multicopter as (6) and (7) with control allocation in (9). For the rotational dynamics, the state vector is redefined as $x = [\mathrm{vec}(R)^\intercal, \omega^\intercal]^\intercal \in \mathbb{R}^{12}$ where $\mathrm{vec}(\cdot)$ a mapping that vectorizes a given matrix. The illustration of a hexacopter, a multicopter with six rotors, is depicted in Figure 2.

3. Main results

3.1 Safety in rotational dynamics

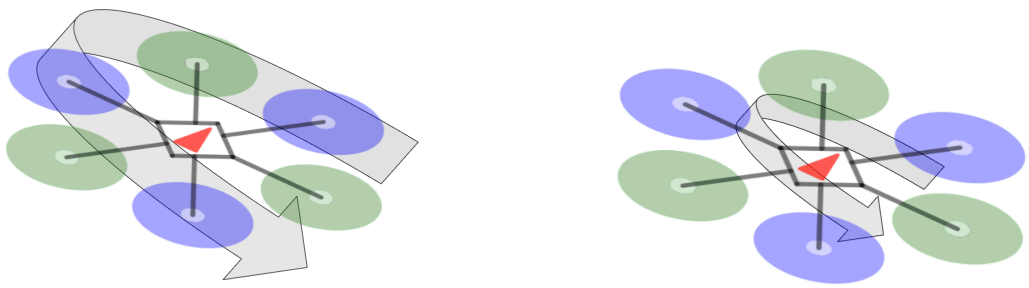
In this section, two safety constraints with respect to angular speed and total thrust direction are considered. Each safety constraint is guaranteed by CBF methods.

3.1.1 Safety 1: Maximum angular speed

The CBF for angualr speed safety is given as $h_\omega(x) = \overline{\omega}^2 - \omega^\intercal\omega$ with maximum angular speed $\overline{\omega} \in \mathbb{R}_{>0}$. For example, $h_\omega(x) \geq 0 \iff \|\omega\| \leq \overline{\omega}$. The angular speed safety can be guaranteed by the conventional CBF method because the relative degree of h_ω is one,

$$\dot{h}_\omega(x, M) + \alpha_\omega(h_\omega(x)) = -2\omega^\intercal J^{-1}\left(M - \omega \times J\omega\right) + \alpha_\omega\left(\overline{\omega}^2 - \omega^\intercal\omega\right) \geq 0, \tag{11}$$

with $\alpha_\omega \in \mathcal{K}_\infty^e$. Figure 3 illustrates the angular speed safety considered in this study.



(a) Safety violated

(b) Safety guaranteed

Figure 3: An illustration of angular speed safety

3.1.2 Safety 2: Maximum deviation of total thrust direction

The CBF for total thrust direction safety is given as $h_{0,z_B}(x) = z_B^\intercal z_{B_d} - \cos(\overline{\theta}_{z_B})$ with maximum deviation $\theta_{z_B} \in (0, \pi]$, the actual and reference z-axis directions $z_B \in \mathbb{S}^2$ and $z_{B_d} \in \mathbb{S}^2$, where $\mathbb{S}^2$ denotes the set of unit vector in $\mathbb{R}^3$. For example, $h_{0,z_B}(x) \geq 0 \iff \theta_{z_B} \leq \overline{\theta}_{z_B}$ where $\theta_{z_B} \in [0, \pi]$ is the angle between unit vectors z_B and z_{B_d}. The total thrust direction safety is guaranteed by the high-order CBF method as the relative degree of h_{0,z_B} is two,

$$\begin{aligned} h_{1,z_B}(x) &:= \dot{h}_{0,z_B}(x) + \alpha_{1,z_B}(h_{0,z_B}(x)) = z_{B_d}^\intercal R\omega^\times e_3 + \alpha_{1,z_B}\left(z_B^\intercal z_{B_d} - \cos(\overline{\theta}_{z_B})\right), \\ h_{2,z_B}(x, M) &:= \dot{h}_{1,z_B}(x) + \alpha_{2,z_B}(h_{1,z_B}(x)) = z_{B_d}^\intercal\left(R(\omega^\times)^2 e_3 + R(J^{-1}(M - \omega \times J\omega))^\times e_3\right) + \left.\frac{d\alpha_{1,z_B}}{dh_{0,z_B}}\right|_x z_{B_d}^\intercal R\omega^\times e_3 \geq 0, \end{aligned} \tag{12}$$

with $\alpha_{1,z_B}, \alpha_{2,z_B} \in \mathcal{K}_\infty^e$. Figure 4 illustrates the total thrust direction safety considered in this study.

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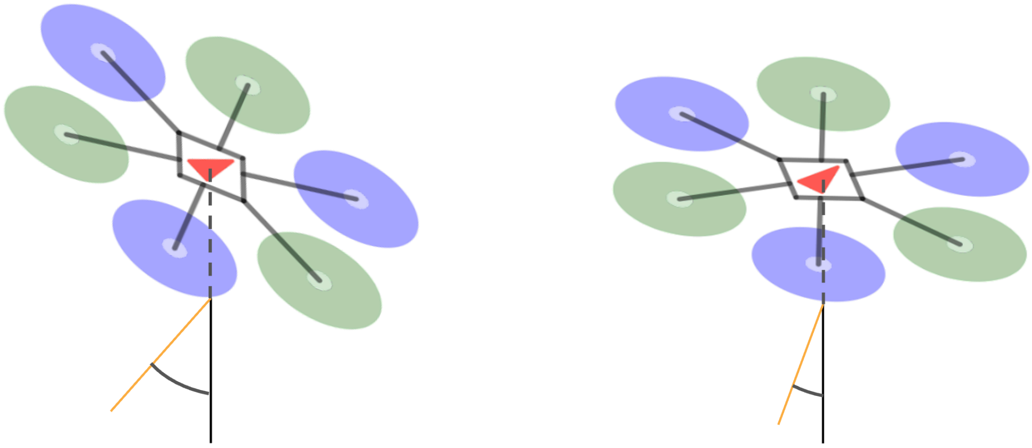

(a) Safety violated

(b) Safety guaranteed

Figure 4: An illustration of total thrust direction safety

3.2 Safety filter in rotational dynamics

Suppose that a nominal attitude controller $(x, \theta_{\text{cmd}}) \mapsto k_{\text{nom}}(x, \theta_{\text{cmd}})$ is given where $\theta_{\text{cmd}} \in \mathbb{R}^p$ denotes the attitude command generated by a higher-level controller, e.g., a position tracking controller. The proposed safe controller is designed as the following optimization-based safety filter,

$$
\begin{aligned}
k(x) = \ & \underset{u \in U}{\arg\min} \ \|u - k_{\text{nom}}(x, \theta_{\text{cmd}})\|^2 \\
& \text{s.t. } (9), (11), (12),
\end{aligned}
\tag{13}
$$

for given state x and attitude command θ_{cmd}. That is, the proposed safety filter is minimally invasive with respect to the given nominal control input while guaranteeing the safeties with the highest priority. Note that the resulting optimization problem in (13) is a linearly constrained quadratic problem, and therefore the safe control input can be obtained reliably by state-of-the-art convex optimization solvers in real time.[6,17] Figure 5 shows the block diagram of the overall control system with the proposed safety filter.

Remark 1 *Each safety constraint suggested in this study can be imposed seperately by imposing the corresponding inequality constraints in* (13)*. That is, if one may want to impose only the total thrust direction safety, the proposed safety filter in* (13) *can be modified by removing the other constraint* (11)*. One can impose more constraints on the proposed safety filter in consideration of more safety constraints.*

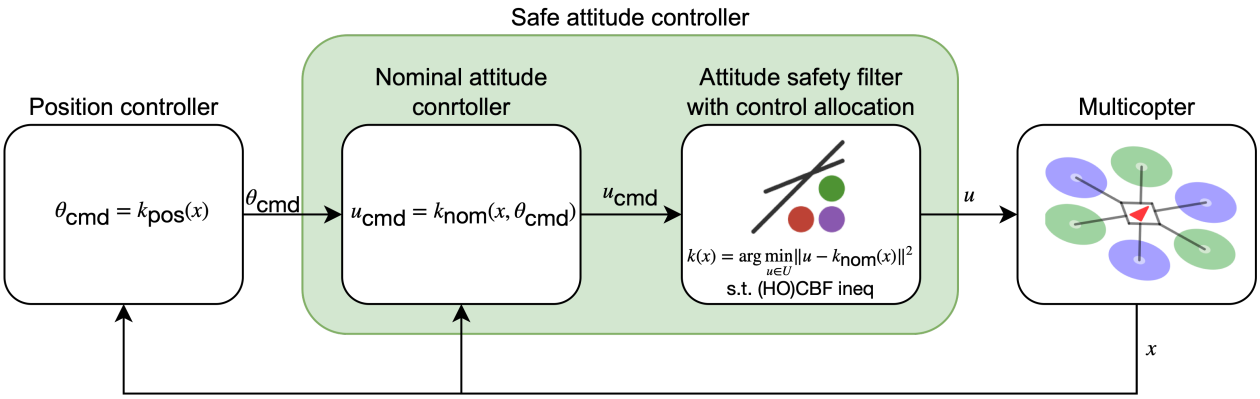

Figure 5: Block diagram of the control system with the proposed attitude safety filter

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4. Numerical Simulation

4.1 Simulation settings

A hexacopter model is considered in this study,[14] which modifies a quadcopter model[15] for the hexacopter with hexa-X configuration.[19] The input space is set as $U = [0, 0.6371\text{mg}]^6$, which is modified for hexa-X configuration.[1,19] Quadcopter quaternion controller is modified for the hexacopter and used as a nominal attitude controller.[7]

Conventional and exponential CBFs are used for the safety constraints with respect to the angular speed and total thrust direction safeties. The class $\mathcal{K}_\infty^e$ functions for the CBFs are set as $\alpha_\omega(h) = 5h$, $\alpha_{1,z_B}(h) = 10h$, and $\alpha_{2,z_B}(h) = 10h$. The control input obtained from the proposed safety filter is applied in the same way as a zero-order-hold (ZOH) control for every 5ms. The simulation is written in Julia,[4] and ECOS is used as the convex optimization solver.[8]

The initial state is set as follows; initial angular velocity $\omega_0 = [30, 30, -15]^\intercal$ deg/s, and the initial Euler angles $(\phi, \theta, \psi) = (30, -30, -180)$ deg (roll, pitch, yaw, respectively). The parameters for the safety filter are set as $\overline{\omega} = 50$ deg/s and $\overline{\theta}_{z_B} = 45$ deg. The desired attitude corresponds to the zero Euler angles.

4.2 Simulation results

The nominal controllers with and without the proposed safety filter are compared. The control input of the nominal controller is saturated to be in the input space U if it is out of U. Figure 1 shows the snapshots of the simulation result, and Figure 6 shows the responses of state and input variables. As shown in Figure 6, the attitude controller with the proposed safety filter does not violate any safety constraints of maximum angular speed and maximum total thrust direction deviation. On the other hand, the nominal attitude controller without the safety filter violates all safety constraints. Note that the computation time for the safety filter is approximately 1ms on a laptop (MacBook Air M1, 2020). The simulation result supports that the proposed safety filter guarantees the safety constraints with respect to the maximum angular speed and maximum total thrust direction deviation in real time.

4.3 Limitations and outlooks

Typically, the multicopter control system is designed as a cascade system with inner- and outer-loops of the rotational and translational dynamics. The proposed safety filter for the rotational dynamics may violate the objective of the outer-loop controller.[10] One may employ the cascade design with a risk of violating the outer-loop performance or incorporate the outer-loop dynamics in the safety filter.

5. Conclusion

A safety filter was proposed for the attitude control of multicopter based on high-order control barrier functions. Considering the rotational dynamics of multicopter, two safety constraints were considered; maximum angular speed and maximum total thrust direction deviation. With the control barrier functions, the resulting safety filter forms a linearly constrained quadratic optimization problem, which can be solved reilably and effectively by state-of-the-art convex optimization solvers. Numerical simulation result showed that the proposed attitude safety filter can effectively ensure the safeties in rotational dynamics by solving a convex optimization problem in real time.

Future works include working on safety constraints considering translational dynamics and corresponding safe controller. Also, fault-tolerant safety filter for multicopter is included as future work.

6. Acknowledgments

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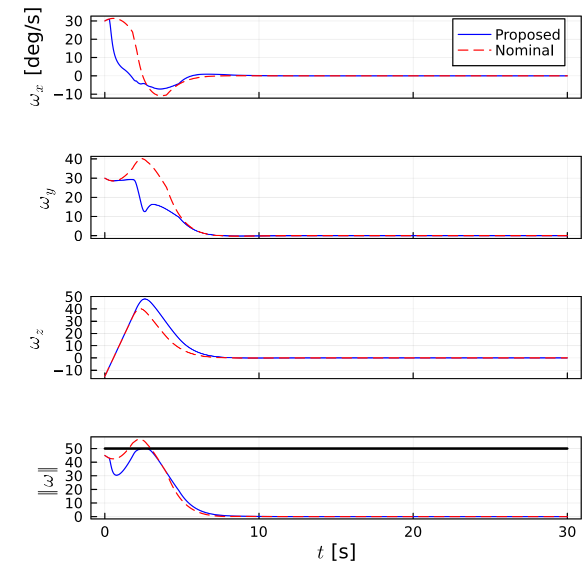
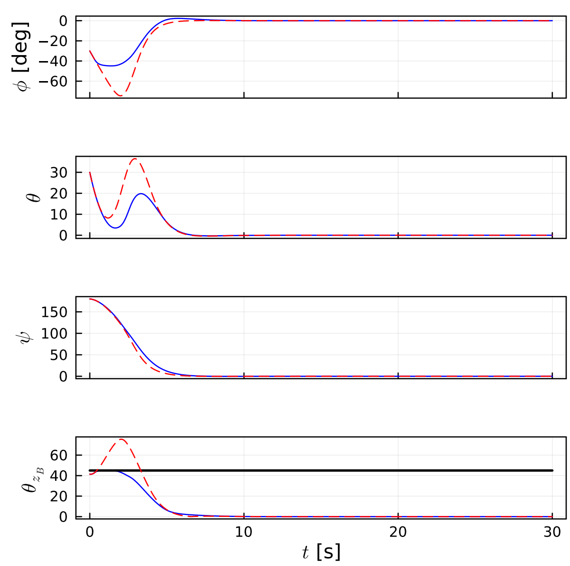
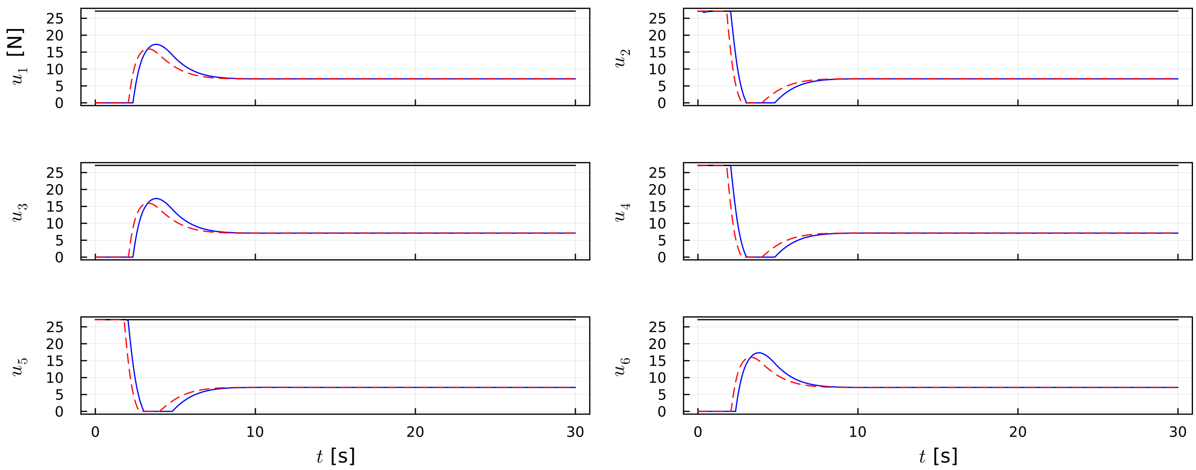


(a) The angular velocity and its two-norm of hexacopter



(b) Euler angles and total thrust direction deviation



(c) Rotor thrusts

Figure 6: Simulation result. Blue and red lines denote the responses with and without the proposed safety filter, respectively. The safe controller does not violate any safety constraints suggested in this study considering input constraint, while the nominal controller violates all the safety constraints.

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