

# Oscillating Boundary Layer in Hybrid Rocket Combustion

Jina Kim\*, Sang Hun Kang\*\* and Changjin Lee\*

\* Konkuk University, 120, Neungdong-ro, Gwangjin-gu, Seoul 05029, Korea

\*\* Sejong University, 209, Neungdong-ro, Gwangjin-gu, Seoul 05006, Korea

## Abstract

The resonance of thermal lags of solid fuel with heat transfer oscillations in the boundary layer flow is the primary mechanism of exciting low frequency instability (LFI) in hybrid rocket combustion. Recent studies reported that swirl injection seems to stabilize combustion by suppressing boundary layer oscillations. This study focused on the experimental investigation of how the boundary layer is perturbed and it leads to the LFI. Also the appearance of temporal stabilization of combustion between two consecutive LFIs was investigated in terms of boundary layer oscillation. Special attentions were also made to monitor the overall behavior of oscillating boundary layer and the occurrence of the LFI as swirl intensity increased. Fluctuating boundary layer was successfully captured by visualized images and POD (Proper Orthogonal Decomposition) analysis. In the results, oscillating boundary layer became stabilized as the swirl intensity increases. And the coupling strength between high frequency  $p'$ ,  $q'$  diminished and periodical amplification of RI (Rayleigh Index) with similar frequency band of thermal lag was also decayed. Results confirmed that oscillating axial boundary layer triggered by periodic coupling of high frequency  $p'$ ,  $q'$  is the primary mechanism to excite thermal resonance with thermal lag characteristics of solid fuel. Numerical simulation also confirmed that transitioning to a positive coupling of high frequency  $p'$ ,  $q'$  is the mechanism responsible for exciting boundary layer oscillation.

## 1. Introduction

Combustion in hybrid rocket displays very interesting nonlinear phenomena such as low frequency instability (LFI), where frequency peaks of pressure are located far less than 100 Hz. Many studies confirmed that the LFI is a manifestation of physical resonance between thermal lag characteristics of solid fuels and the oscillatory heat transfer in response to external disturbances in the boundary layer [1]. Especially, Goutham et al. [2] suggested a mathematical model of LFI occurrence which describes oscillating boundary layer flow by vertical fluctuations of evaporative fuel flow.

Recent studies reported that two different frequency bands are observed at the LFI occurrence; a strong pressure oscillation of 20 Hz band and a weak pressure oscillation of 500 Hz band with small amplitudes [4]. As already known, the thermal delay of solid fuel is primarily responsible for generating the pressure oscillations of low frequency band. However, the pressure oscillations ( $p'$ ) of 500 Hz band are appeared as small vortices are introduced into the post chamber.

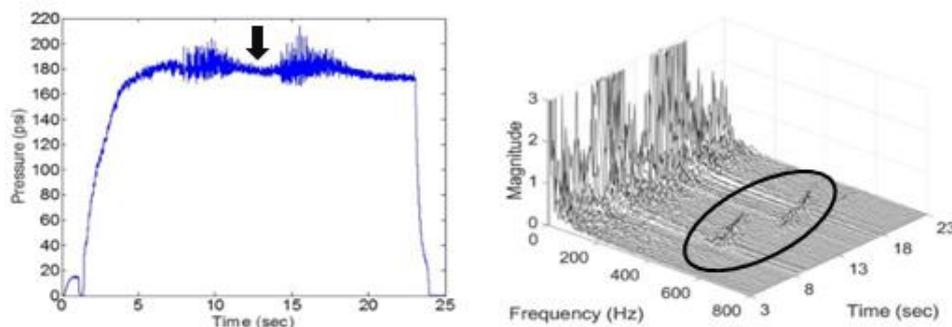


Figure 1: Trajectory of combustion pressure and frequency spectrum with the LFI

Kim et al. [5] studied that the mutual interference between the fuel evaporative flow and the axial oxidizer flow is the responsible mechanism of producing small vortices. Interesting features are also observed in the fluctuations of combustion reaction by using PMT (photomultiplier tube) device. Analysis revealed that the frequency peaks in combustion reaction coincide with those of pressure oscillations of about 500 Hz. In addition, they observed a positive coupling between pressure oscillation ( $p'$ ) and heat release oscillation ( $q'$ ) of 500 Hz band when the LFI occurred. Thus, they presumed that the establishment of a positive coupling between  $p'$  and  $q'$  could be a precondition of the LFI occurrence. However, they did not offer any possible due processes how the coupling does induce an oscillatory heat transfer flux to the solid fuel.

Figure 1 shows a typical example of the trajectory of combustion pressure and the spectrum analysis when the LFI occurs. One can notice that combustion becomes temporally stabilized after the first appearance of the LFI. It is worth noting that frequency jumps of combustion pressures ( $p'$ ) of 500Hz band are found at the same point of transition of the LFI. Therefore, it seems that unveiled physical processes may act to suppress or delay the LFI occurrence during the frequency jumps of high frequency  $p'$ .

Meanwhile, many studies reported that the application of swirl oxidizer injection contributes to stabilize hybrid rocket combustion as well. Bellomo et al. [6] reported that swirl injection can significantly reduce the amplitude of the combustion pressure oscillations. Messineo et al. [7] also reported that small-sized vortex generation and their shedding are reduced when swirl injection is applied. Also, they claimed that the reduction in vortex generation affects to decrease the amplitude of the pressure fluctuations of 500 Hz band and results in the combustion stabilization. Although swirl injection was found very beneficial for combustion stabilization, detailed physics are not comprehensively understood. In this regard, Kim et al. [8] reported interesting results on the due process leading to the LFI in their experimental study with swirl injections. As the swirl intensity increased, they observed combustion became stabilized and the phase between  $p'$  and  $q'$  of 500Hz band was gradually changed from a positive coupling to a negative one. Furthermore, they observed the formation of 15~18 peaks in Rayleigh index per second when a positive coupling was established. Here Rayleigh index is defined as  $RI = \oint p'q' dt$ . Interestingly, the number of RI peaks per second is about 15~18 that coincides with the frequency characteristic of thermal lag of solid fuel. Therefore, it is not surprising to assume that the formation of RI peaks may act to perturb boundary layer fluctuations leading to heat transfer fluctuations to solid fuel. Nonetheless, no studies have been done to investigate which physical processes are responsible for perturbing boundary layer flow.

Therefore, in this study, two different cases are investigated to monitor the response of boundary layer to the formation of RI peaks when the LFI occurs. First case is the temporal appearance of stable combustion between two consecutive frequency jumps of combustion pressure as shown in Fig. 1. The other one is transitional behavior of unstable combustion into the stabilization as swirl intensity increases. Special attentions are made on understanding the physical links between the formation of RI peaks and boundary layer fluctuations. To this end, a series of experimental test is designed. And, combustion pressure and intensity are measured to analyze the interaction of the combustion pressure and the chemical reaction, and their effects on the LFI is also assessed. The light intensity emitting from the combustion chamber is measured by PMT to monitor the oscillatory behavior of the combustion reaction. Also, numerical calculations for non-reactive flows were performed using DNS method to examine the change in flow structure caused by the interaction of wall blowing and swirl injection. The calculation results were also used to investigate the qualitative effects on combustion instability.

## 2. Experimental setup

A series of experimental tests was designed with gaseous oxygen (GOx) and PMMA in this study. Chamber lengths of main, pre and post-chamber were fixed as 400 mm, 45 mm and 75 mm respectively, whereas a fuel diameter was 50 mm in all test cases. Pressure was measured at the pre-chamber. Table 1 summarizes test conditions of all test cases. Test 1 is the reference case where the LFI occurred.

Test	Swirl angle (degree)	Remark
Test 1 (Baseline)	0(Axial)	LFI
Test 2	4	↓ Weak ↓
Test 3	5	
Test 4	6	
Test 5	7	
Test 6	8	
Test 7	9	Stable
Test 8	10	

Table 1. Summary of test conditions of all cases

Based on the results of previous study [8], swirl angle was varied from 4 degree in Test 2 to 10 degree to increase swirl intensity. Figure 2 shows configurations of experimental set up with pre- and post-chamber.

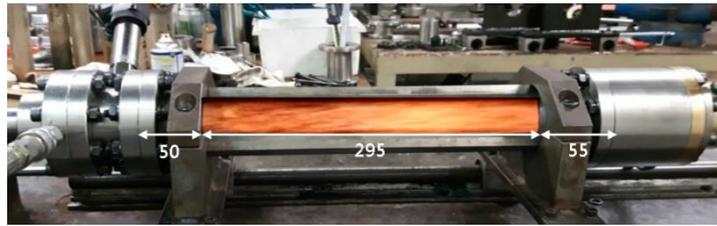


Figure 2: Configuration of main chamber for visualization

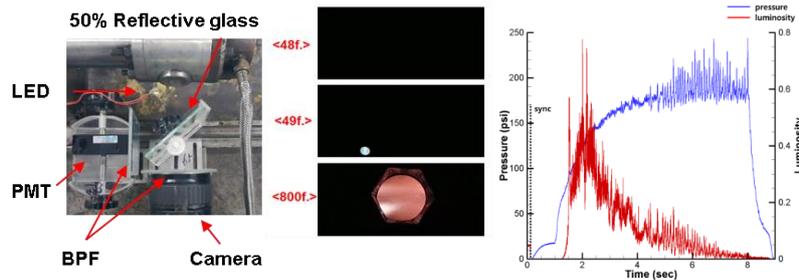


Figure 3: Synchronization setup for pressure and flame luminosity measurement [11]

Figure 3 shows the test setup and configurations for visualizing the combustion. One side of main chamber was modified to visualize the combustion flow inside the chamber. A special glass with a reflectance of 50% was installed in front of the window, and the light generated by the combustion reaction was divided into two directions. The divided light was photographed using Casio Ex-1 camera and photomultiplier tube (PMT) H10722 manufactured by Hamamatsu. Camera and PMT have the same shooting area, and are synchronized. Event-based synchronization technique, which uses LED as trigger is used to synchronize pressure and high-speed image data. Visualization tests were conducted in complete darkroom in order to minimize the noise in the PMT. In addition, a band pass filter (BPF) for 430 nm wavelengths was attached to the front of the camera and the PMT to clearly capture the combustion reaction. The shooting speed of the camera is 1200 fps, and this speed can measure changes with frequency characteristics up to 600 Hz. Details regarding validation of post-chamber configuration, extraction of the light intensity from the visualization images, and the image post-processing are consulted from reference [9].

### 3. Test Results

#### 3.1 LFI occurrence with an axial injection

Figure 4 displays FFT results of chamber pressure and heat release fluctuations with the LFI when axial oxidizer injection was used. Note that stable and unstable combustion are marked with different region number to compare their characteristics; the stable (Region I, 13-14sec) and unstable (Region II, 16-17sec). This case is the baseline case and the basic features of coupling status between  $p'$  and  $q'$ , RI index behavior and boundary layer oscillation are closely monitored for comparing to other cases.

In order to capture the approximate behavior of main and boundary layer flow, POD (Proper Orthogonal Decomposition) was applied even though the analysis results are not quite accurate compared to other flow visualization technics such as PIV method. Details of the method how to extract flow information from the captured images are well addressed in the previous studies [10]. Since visualized images contain intrinsically 2-D information of the flow, POD analysis is only able to provide overall behavior of axial motion of combustion flow. Also, since the swirl angle is at most 10 degree or less, POD method is assumed to be capable of sufficiently capturing dominant axial flow motion even in the swirl injection. Note that there are two typical axial flow motions in the combustion chamber; main combustion flow and boundary layer flow. Since POD analysis assumes that flow is 2 dimensional, the boundary layers are found both at the top and bottom of the visualization domain. Thus, POD is expected to be able to provide valuable information of boundary layer behavior in relation to the transition of coupling status between  $p'$  and  $q'$ .

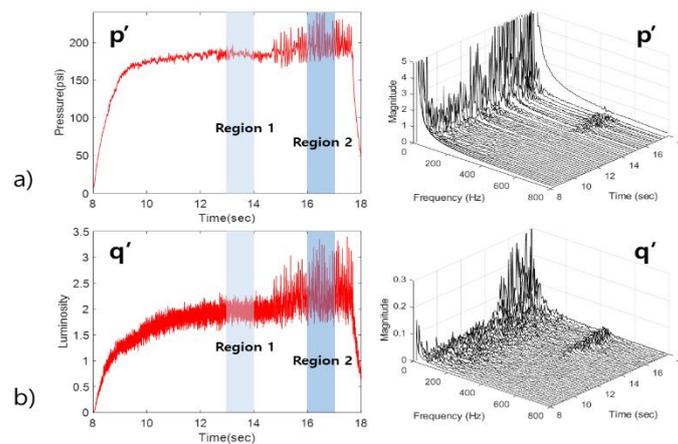


Figure 4: Time traces and FFT results of (a)  $p'$  and (b)  $q'$  in Test 1 (baseline)

Figure 5 shows the energy distribution of first 10 modes obtained by POD analysis of Test 1 during the period of LFI occurrence (Region II). As shown, mode 1 has the largest energy because mode 1 represents the main axial flow in the chamber. From the physical aspect, the main axial flow is considered to have the largest energy and the next one may be the boundary layer flow. Energy contents of modes higher than 3 are so small compared to those of mode 1 and 2 that physical significance of these modes may not be quite big. And these modes may simply represent some complicated flow behaviors inevitably appeared due to the 2D flow analysis. Spatial distributions of energy of mode 1 and 2 are compared in Figure 6. As mentioned, mode 1 represents mean flow in axial direction. And most of flow energy are nearly concentrated near the wall surface that mode 2 is the flow behavior near the wall surface, which means boundary layer flow.

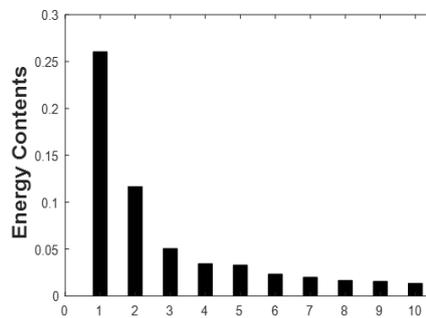


Figure 5: Energy distribution of higher 10 mode in Region 2

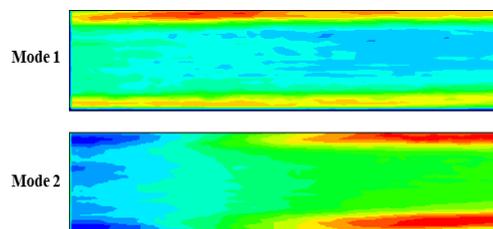


Figure 6: Spatial energy distribution in Mode 1 and Mode 2

Since POD analysis provides not only spatial distribution (mode) but also temporal behavior (time coefficient) of each flow mode, temporal behavior of each flow mode can be obtained by FFT analysis of time coefficient. In this regard, interesting features are found in the comparison of frequency spectrum of mode 1 with that of combustion pressure. Figure 7 compares frequency spectrum of combustion pressure with time coefficient of mode 1. As mentioned there are two distinct frequency peaks in combustion pressure when the LFI occurs; low frequency peaks about 15~18Hz with large amplitudes and high frequency peaks about 450~500Hz with very small amplitudes. FFT analysis of time coefficient of mode 1 suggests very interesting results regarding similarity of frequency peaks both in low and high frequency range.

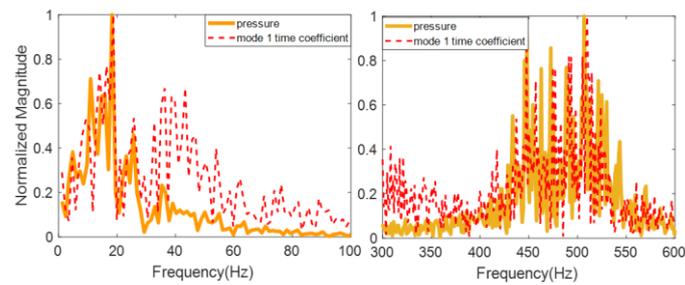


Figure 7: Comparison of peak frequencies of pressure and time coefficient of mode 1 in region II; low frequency range (0~100Hz) and high frequency range (300~600Hz)

As shown in Fig. 7, frequency peaks of mode 1 are mostly found at the location similar to those of combustion pressure, but additional peaks near 40Hz also appear. Therefore, the fact that the peak frequency characteristics of two oscillations are quite similar indicates that the mode 1 flow represents the axial main flow containing most of the energy of the combustion flow.

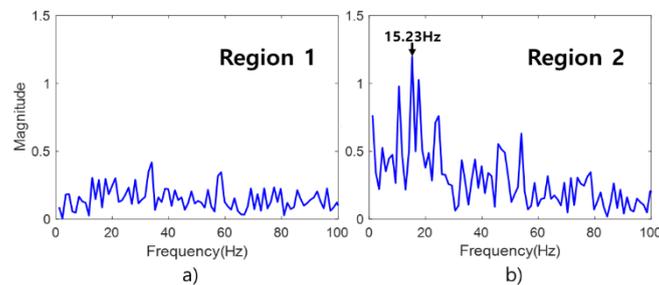


Figure 8: FFT results of time coefficient of Mode 2 in Region 1(a) and Region 2(b) of baseline case

Figure 8 compares the FFT results of time coefficient of mode 2 in region I with those in region II. In region I, no typical frequency peaks are observed in mode 2. However, strong peaks appear near at frequency of 15~18Hz in region II (unstable combustion). Interestingly, the locations of the frequency peaks are at the same frequency of thermal lag characteristic of solid fuel. In this regard, Karabeyoglu et al. [1] confirmed that the LFI is the manifestation of the resonance of thermal lag oscillations with the heat transfer oscillations caused by the boundary layer perturbations. Therefore, the appearance of strong frequency peaks near 15~18Hz in mode 2 is the evidence that the boundary layer is oscillating. And oscillating boundary layer induces the oscillation of heat transfer to the solid fuel, in turn, and leads to the resonance with thermal lag. Although many studies [1-3] already suggested that the oscillation of boundary layer is directly related with the development of the LFI, studies on the major causes of oscillating boundary layer have not yet been elucidated. Nonetheless, recent study suggests a plausible scenario in which a positive coupling of high frequency  $p'$  and  $q'$  causes RI peaks to oscillate and ultimately triggers the LFI [11].

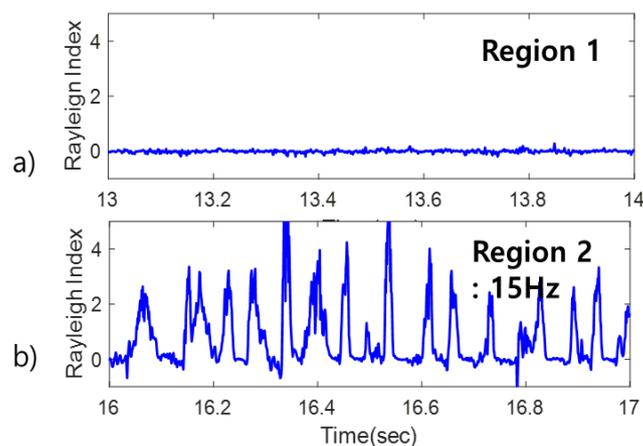


Figure 9: Rayleigh Index in (a) Region I, (b) Region II

Figure 9 shows the comparison of RI peaks in region I and II. Here, RI peaks are the consequence of coupling process

between high frequency  $p'$  and  $q'$ . Thus, it is not surprising that there is no coupling between  $p'$  and  $q'$  in region I because a stable combustion dominates. However, periodic formation of RI peaks with 15~18 Hz is observed in region II. At the same time, the boundary layer approximated by mode 2 in region II is also oscillating with the same frequency. Therefore, it can be assumed that establishing a positive coupling of  $p'$  and  $q'$  excites periodic formation of RI peaks with 15~18Hz and boundary layer starts to oscillate in the axial direction as the consequence of the excitation. In this sense, oscillatory heat transfer to the fuel surface is the final physical process leading to the LFI occurrence. It is worth noting that the change in combustion temperature due to O/F variation is the primary cause of shifting the coupling status between  $p'$  and  $q'$  during the combustion. Reference [11] describes the physical processes responsible for the temperature variation in more detail.

### 3.2 Transition of Oscillating Boundary Layer with Swirl Injection

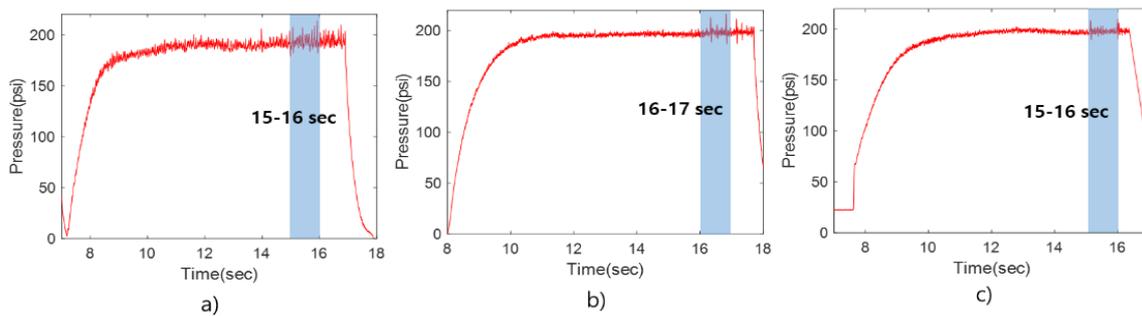


Figure 10: Trajectories of combustion pressure in (a) Test 2 (b) Test 4 and (c) Test 5

Generating swirl motion in a pipe flow is generally one of the favorable methods to enhance the heat transfer to the surface. And the swirl injection was widely used to increase the regression rate in hybrid rocket combustion because swirl motion increases convective heat transfer to the fuel surface. Also, many studies reported swirl injection is very effective method to stabilize combustion in hybrid rocket combustion. Another advantage of swirl injection is the combustion stabilization. Kim et al. [8] claimed that the swirl injection with an appropriate intensity substantially reduces the generation of high frequency  $p'$  by modifying the interaction of boundary layer flow with evaporative fuel flow. In this regard, Obata et al. [12] reported very interesting behavior of diffusion flame using the combustion visualization. They observed that the location of diffusion flame moves toward to the fuel surface as swirl intensity increases. Therefore, the increase in swirl intensity not only enhances convective heat transfer but also modifies the flow structure by increasing the interactions with boundary layer flow. And the reduction of the small vortices generation can be a result of the augmented interaction of swirl flow in the boundary layer, which is the main source of high frequency  $p'$ . This study is the continuing effort to expand understanding the role of swirl injection on the combustion stabilization and a follow-up study of a previous study [8].

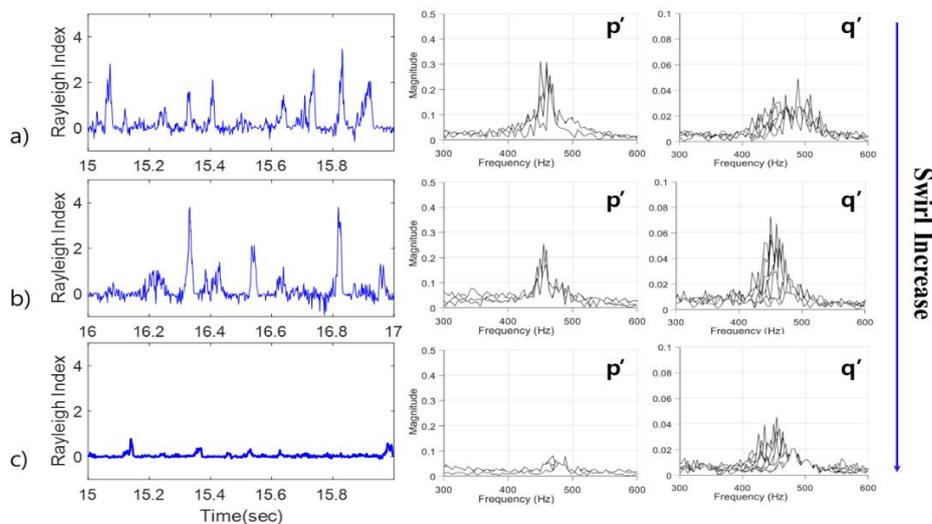


Figure 11: Rayleigh Index and frequency peaks of  $p'$  and  $q'$  in (a) Test 2 (b) Test 4 and (c) Test 5

Figure 10 shows the comparison of combustion pressure traces of Test 2, 4 and 5 as swirl intensity increases. Here

swirl intensity was controlled through the injection angle from 4 to 7 degree. As expected, the increase in swirl intensity brings about the stabilization of unstable combustion. And the critical swirl angle is about 6 degree, above which swirl injection contributes to stabilize the unstable combustion with the LFI. However, as seen in Test 2, swirl injection is not able to suppress the LFI, if the swirl intensity is not sufficiently enough to change the flow structures.

Figure 11 displays the overall behavior of periodic amplification of RI peaks and distinct frequencies of  $p'$  and  $q'$  as swirl intensity increases. As mentioned, the appearance of periodic RI peaks is the result of a positive coupling of high frequency  $p'$  and  $q'$ . In Test 2, RI peaks with large amplitudes are identified and distinctive frequency peaks of  $p'$  and  $q'$  appear. Thus, Test 2 is the case where a positive coupling dominates even with swirl injection because swirl intensity is not sufficiently enough to change flow interactions in the boundary layer. However, Test 5 shows a typical example where swirl injection surely modifies flow interactions in the boundary layer. Firstly, RI peaks are negligibly small that there are no coupling behaviors of  $p'$  and  $q'$ . Also, the amplitudes of  $p'$  are negligibly small because of the active interaction with swirl motion in the boundary layer.

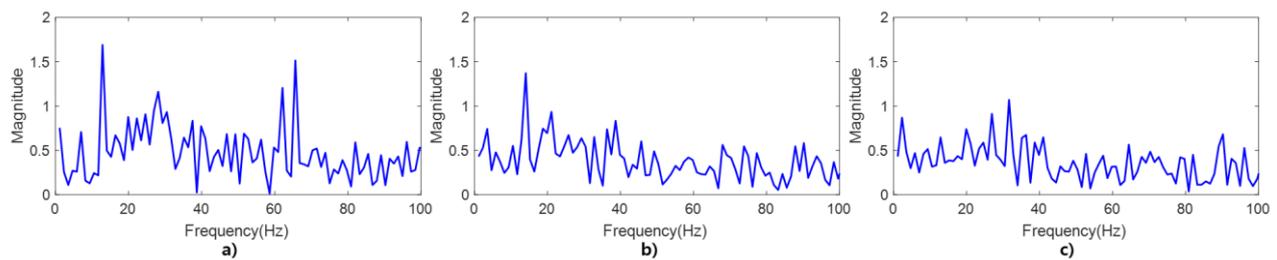


Figure 12: FFT of time coefficients of Mode 2 in (a) Test 2 (b) Test 4 and (c) Test 5

Figure 12 compares FFT results for the time coefficient of mode 2. In Test 2, typical low frequency peaks are found near at 15Hz, which are approximately the same frequency characteristics of thermal lag of solid fuel. As swirl intensity increases, the amplitudes of low frequency peaks of time coefficient of mode 2 are gradually reduced and no typical peaks are found in Test 5, where swirl intensity is strong enough so that swirl injection affects to reduce  $p'$  generation and to shift to a negative coupling of  $p'$  and  $q'$ . Thus, boundary layer is neither affected nor oscillated because RI peaks are not appeared in Test 5. Thus, the resonance of thermal lag with heat transfer oscillation is not possible and the occurrence of LFI is completely suppressed. In summary, the appearance of oscillating boundary layer is a necessary precondition for the initiation of the LFI because boundary layer oscillation provides oscillatory heat transfer to the fuel surface and consequently leads to the resonance with a thermal lag characteristic. This can be a new scenario about the LFI occurrence.

### 3.2 Temporal Stabilization between Two LFIs

As seen in Fig. 1, the combustion in Test 1 became temporally stabilized between two consecutive pops up of the LFIs. It is worth noting that a frequency jump of  $p'$  is also found nearly at the same location of temporal stabilization. Therefore, unveiled physical processes act to suppress or delay the LFI occurrence during the frequency jump of high frequency  $p'$ . Summarizing the discussions in the previous section and the current analysis on the temporal combustion stabilization, the appearance of RI peaks with large amplitudes and the oscillation of boundary layer are presumed to be a prerequisite to activate the LFI. In this regard, the analysis with similar approaches is made to monitor overall behavior of boundary layer oscillations and RI peaks while the combustion becomes temporally stable.

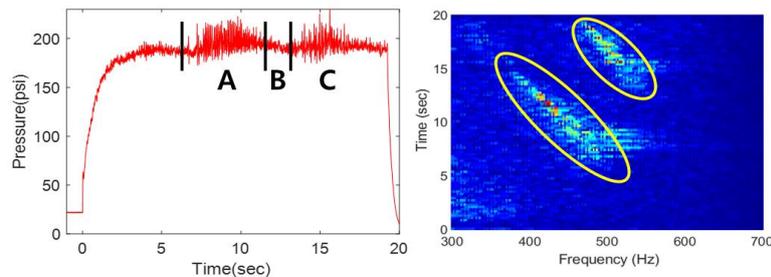


Figure 13: Trajectory of combustion pressure and spectrum analysis in high frequency domain

The most interesting feature in the hybrid rocket combustion is the frequency jump in  $p'$  with 500Hz band. Figure 13 is an enlarged version of pressure trace with two consecutive instabilities and spectrum analysis of  $p'$ . As mentioned, there is a temporal stable region in combustion between two pops up of the LFI. In the figure, letter 'A' and 'C' represent the domain with the LFI and 'B' is a temporally stable region. A frequency jump of  $p'$  is started nearly at the same moment of temporal relief of instability.

Note that frequency jumps are usually observed in a cavity flow. Rossiter [13] claimed that the occurrence of frequency jumps is the result of complicate flow interactions between vortex shedding flow and feedback disturbances in a cavity. If the vortex shedding flow collides with the end of the cavity wall, additional feedback disturbances are excited and suddenly affect to increase the frequency of shedding flow characteristic. Messineo et al. [14] successfully simulated a frequency jump in hybrid rocket combustion by using Rossiter 's mechanism. Wang et al. [15] also did a numerical simulation to describe how a frequency jump is created through the interactions with shedding shear layer in a cavity flow. Their simulation reveals that a frequency jump is the combined result of creating additional large vortex flow in the cavity and the sudden acceleration of shear layer flow.

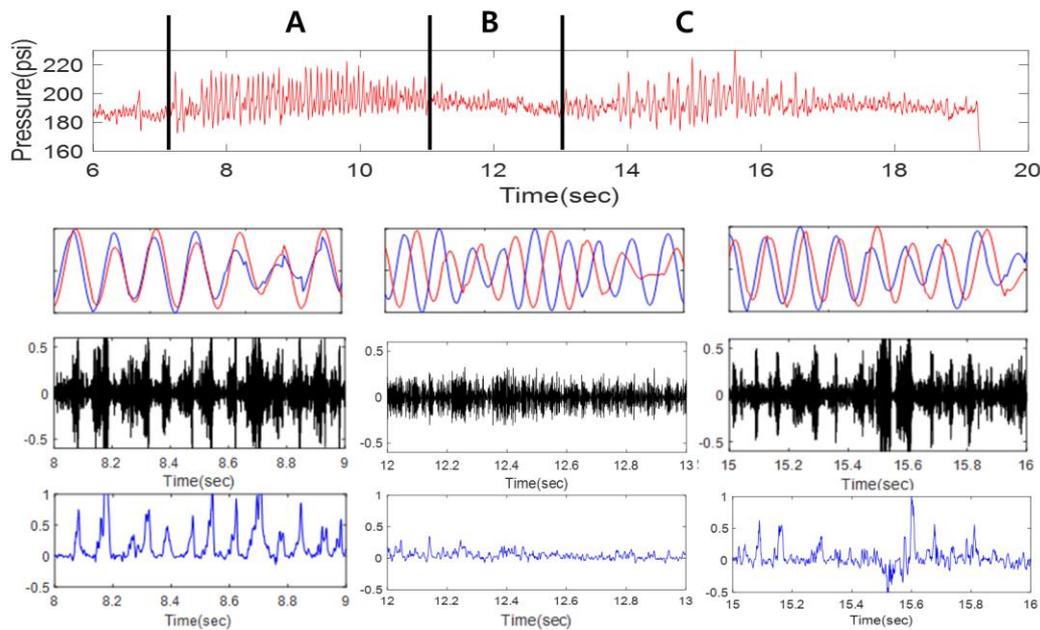


Figure 14. Comparison of three major parameters including, coupling status of  $p'$  and  $q'$ , trace of high frequency  $p'$ , and Rayleigh index

Figure 14 shows the overall behavior of three major parameters including, coupling status of  $p'$  and  $q'$ , magnified trace of  $p'$ , and Rayleigh index in the region of A, B, and C. In region of A, typical features of the LFI are found with the formation of RI peaks, the appearance of well-developed amplitudes of  $p'$ , and the positive coupling between  $p'$  and  $q'$ . However, RI peaks becomes negligibly small and the coupling status changes to a negative as the phase difference increases in B. These changes are presumed as the due processes leading to stabilize the combustion. After the temporal achievement of stable combustion in region B, all measures representing flow characteristic return to its initial conditions associated with the LFI occurrence in region A. In other words, RI peaks regain its large amplitudes, well-defined  $p'$  reappear, and a positive coupling resumes between  $p'$  and  $q'$ .

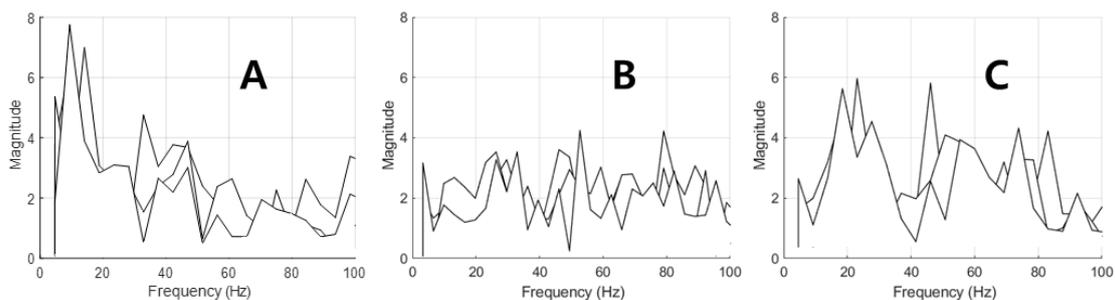


Figure 15. Frequency peaks of time coefficient of Mode 2 in region A, B and C

Another interesting feature can be found in the transition of oscillating boundary layer. Figure 15 compares the transition of peak frequencies of boundary layer flow (Mode 2) in region of A, B and C. Again, note that Mode 2 approximates boundary layer flow in POD analysis. As seen in the figure, peak frequencies migrate from low values of about 16~18Hz, which is the same frequency as thermal lag characteristic, to higher ones of about 60~65Hz in region B. This is the transitional key process responsible for the temporal achievement of stable combustion because oscillating frequencies of boundary layer in region B do not match with those of thermal lag of solid fuel. Because of the mismatch of frequency characteristic, the resonance of heat transfer to the fuel surface is not able to occur at all. However, oscillating boundary layer gains its initial frequency characteristic of about 16~18Hz as new  $p'$  restarts after a frequency jump in region C. Therefore, the appearance of oscillation boundary layer with a frequency characteristic of the thermal lag of solid fuel is a prerequisite for the LFI occurrence. If the oscillation of boundary layer could be suppressed or the oscillating peak frequency would be surely changed, the control of the LFI occurrence is no longer theoretical one.

#### 4. Conclusion

Experimental study was conducted for understanding the physical relationship between the LFI occurrence and the oscillation of boundary layer in hybrid rocket combustion. Baseline case (Test 1) was closely investigated in terms of Rayleigh index, coupling status between high frequency  $p'$  and  $q'$ , and frequency peaks in Mode 2 oscillation. Results of baseline case shows that frequency peaks of oscillating boundary layer are approximately the same as thermal lag characteristics of solid fuel. Since establishing a positive coupling between  $p'$  and  $q'$  excites periodic formation of RI peaks with 15~18Hz and boundary layer starts to oscillate in the axial direction as the consequence of the excitation.

In the experiments with swirl injection, special attentions are also made to monitor transitional behavior of oscillating boundary layer using Rayleigh index, and the coupling status between high frequency  $p'$  and  $q'$ . Results show that the swirl injection with an appropriate intensity substantially reduces the generation of high frequency  $p'$ . If swirl intensity is not sufficiently strong enough, results show that a positive coupling still dominates even with swirl injection and RI peaks are also distinguishable.

However, if swirl intensity is strong enough so that swirl injection affects to reduce  $p'$  generation and to shift to a negative coupling of  $p'$  and  $q'$ , boundary layer is neither affected nor oscillated because external excitations by RI peaks are not appeared. Thus, the resonance of thermal lag with heat transfer oscillation is not possible and the occurrence of LFI is completely suppressed. Thus, the resonance of thermal lag with heat transfer oscillation is not possible and the occurrence of LFI is completely suppressed. In this sense, the appearance of oscillating boundary layer is a necessary precondition for the initiation of the LFI because boundary layer oscillation provides oscillatory heat transfer to the fuel surface and consequently leads to the resonance with a thermal lag characteristic.

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