

Modelling of plasma discharge for aerospace applications: Streamer to arc transition

Eve Pachoud^{1,†}, Fabien Tholin¹, François Pechereau¹, Anne Bourdon³

¹ DPHY, ONERA, Université Paris-Saclay, F-91123 Palaiseau, France, surname.name@onera.fr

² LPP, UMR CNRS 7648, Ecole Polytechnique, France, anne.bourdon@lpp.polytechnique.fr

Abstract

Streamer and electric arc discharges play a very important role in many aerospace applications. The formers are related to many safety issues during lightning stroke to aircraft [1] and streamer discharges are involved in the triggering phase of electric arcs [4]. MHD (Magneto-Hydro-Dynamics) models have been successfully used to simulate electric arcs [7], but they cannot model non-neutral streamer discharges [8] that are usually modelled by non-neutral multi-fluid drift diffusion approaches [9]. However, streamer models require timesteps and cell sizes decreasing dramatically as the plasma transits to an arc. The goal of this work is to propose an innovative numerical scheme, based on recent discretization techniques called AP (Asymptotic Preserving) that could make it possible to unify streamer and arc models consistently [10-11].

1. Introduction

Streamer discharges and electrical arcs play a significant role in many aerospace applications. First, streamers are a type of discharge characterized by the propagation of a fast-moving ionization front (10^6 m/s) called the “head of the streamer” in the literature, that is connected to an electrode by a filamentary plasma channel with a diameter of a few hundreds of microns in atmospheric pressure air. Streamers can be observed in many applications involving discharges at high pressure and they play a crucial role in the triggering of lightning stroke to aircraft. During a thunderstorm, the atmospheric electric field is of the order 10^5 V/m, which is much lower than the breakdown field in air: ~ 3 MV/m. Then, the triggering of a lightning stroke requires the field to be greatly enhanced by point effect in the vicinity of sharp edges. In these locations, ionization takes place and streamer discharges may propagate along field lines. If enough electrostatic energy is available, these streamers may transit to a leader discharge regime able to propagate on large distances (km), and to give birth to a lightning arc. Given that an aircraft is struck by lightning, on average, once or twice a year, it is important to understand and predict these phenomena in order to design aircraft structures whose materials are resistant to the consequences of lightning. This issue has become increasingly topical with the use of composite materials and the quest for lightweight materials, unlike the first generation of aircraft made from aluminum alloys [1]. Streamer discharges may also appear as a result of insulation defect in high voltage equipment and result in the so called Partial Discharges (PD) [4]. They are known to occur in the insulators of high-voltage power lines but can increasingly be found in onboard electronic systems which are subjected to ever higher voltages [12]. In some aerospace applications streamer discharges are generated on purpose as for example for plasma assisted combustion technologies or flow control. In the field of plasma-assisted combustion (PAC), the plasma is used to help the ignition of the combustion reactions by generating reactive species such as radicals. They control the ignition of electric arcs that can help the combustion by thermal effect and by improving the mixing between the fuel and the combustive thanks to instabilities created in the gas flow. Streamer-to-arc transition is then particularly important for PAC to maintain the flame in particularly difficult situations such as supersonic combustion in scramjets [6].

The lack of model in the literature able to deal both with streamer and arc discharge plasma regimes is a serious limitation to the understanding and progress in these different fields of research. The main reason for this is the very different physics involved in these plasma regimes: in electric arcs, the plasma is quasi-neutral, possibly magnetized, and at local thermal equilibrium (LTE), which is well suited to resistive MHD (MagnetoHydroDynamics) simulations. In this approach, the mono-fluid Euler equations are solved with basically two MHD source terms: The Laplace forces in the momentum conservation equation, and the Joule effect in the energy conservation equation. Sometimes, more complete models involve replacing the Euler equations by Navier Stokes equations to take account of viscous effects and some radiative models may be necessary [7]. Streamers on the other hand are non-neutral due to the presence of

space-charges, and highly out of thermal and chemical equilibrium, making MHD approaches inappropriate. To model streamer discharges, non-neutral multi-fluid drift diffusion models have proven successful [9]. It consists of solving the conservation equations for charged particles and stating that all velocities follow the drift-diffusion approximation. These equations are then coupled to the Poisson's equation to solve the field enhancement due to space charges. Depending on the gas, different kinetic models are used to compute the chemical source terms due to ionization and other inelastic collisional processes. However, these drift-diffusion models require timesteps and cell sizes decreasing dramatically as the plasma becomes more and more conductive, making the transition to an arc regime with explicit approaches out of reach.

The aim of this work is to propose a model containing both the physics of streamers and the physics of electric arcs and an innovative numerical scheme that can be stable under all the limit physical cases that are called asymptotic regimes in literature [14,15,16]. For example, the scheme has been built in the goal of remaining stable in the highly collisional regimes that corresponds to the drift-diffusion approximation (streamer regime). But it had to be built in such way as to retain its stability properties in the quasi-neutral regimes (electric arc regime). Such schemes that conserves some stability properties through asymptotic regimes are called Asymptotic-Preserving (AP) schemes [14,15,16]. First approach relies on starting from Euler-Poisson equations which are then simplified with the quasi-neutral hypothesis and solved in subdomains of computation where the plasma is thermal and quasi-neutral. In the other subdomains the complete set of equations without any quasi-neutrality assumptions is solved if non-neutral effects are expected. These methods require the computational domain to be partitioned into subdomains and each of them to be treated with the appropriate physics, but this partitioning presents many numerical difficulties [17,18,19,20]. Other class of approaches tend to keep the all set of Euler-Poisson equations without simplifying assumptions by the use of a fully time implicit integration [21,22,23,24,25,26,27]. Some of these methods exhibit good AP properties for the Euler-Poisson system but a few was applied to the modeling of streamer discharges [16]. The scheme developed in this work is an extension of the isotherm plasma model of [31] to multi-species and highly collisional plasmas with temperature dependence (energy conservation equation added). It is a semi-implicit AP scheme based on the decoupling of fast (acoustic) and slow (advection) phenomena.

The paper is structured as follows : In section 2, the model of equations for the streamers and arcs' physics is detailed. In section 3, some details about the choices of the collisional model and chemistry are presented and discussed. In section 4, a brief description of the numerical strategy used for solving the equations is proposed in order to get an AP scheme. In section 5, some test cases for the AP scheme are shown and the section 6 concludes.

2. The 1D model of equations

The multifluid collisional Euler-Poisson model was chosen :

$$\partial_t n_k + \partial_x (n_k u_k) = S_k \quad (1)$$

$$m_k \partial_t (n_k u_k) + \partial_x (n_k m_k u_k^2 + p_k) = -s_k q_e n_k \partial_x \Phi - n_k \mu_k \nu_{kn} (u_k - u_n) \quad (2)$$

$$m_k \partial_t (n_k E_k) + \partial_x (n_k m_k E_k u_k + p_k u_k) = -s_k q_e n_k \partial_x \Phi u_k + \frac{1}{2} n_k m_k \nu_{kn} (u_k - u_n)^2 - \frac{3}{2} n_k \kappa_{kn} \nu_{kn} k_B (T_k - T_n) \quad (3)$$

$$\partial_{xx} \Phi = \sum_k -\frac{s_k n_k q_e}{\epsilon_0} \quad (4)$$

For each charged species k (k can be equal to e for electron, p for positive ions and m for negative ions) and for neutrals (k equal to n), the compressible Euler equations are solved. n_k, u_k, E_k, m_k stand respectively for the density, the velocity, the total energy and the mass of the species k . Each kind of particles is considered as a fluid assumed to follow the perfect gas law. Hence, the pressure p_k and the temperature T_k are linked by the relation (5) where k_B the Boltzmann constant:

$$p_k = n_k k_B T_k \quad (5)$$

In equations (2-3) Φ is the electric potential and so, in 1D, $-\partial_x \Phi$ is the electric field. In the RHS (Right Hand Side), the electrostatic terms which result from the interaction between the electric field and the charged particles are involved in the momentum equation (2) as Coulomb forces and in the energy equation (3) as Joule effect. q_e is the elementary electric charge. The other source terms describe the collisions between charged particles and neutrals. ν_{kn}

refers to the collision's frequencies between the charged particles k and the neutrals n in (2). μ_{kn} is the reduced mass and is equal to $m_k m_n / (m_k + m_n)$ and κ_{kn} is the energy coefficient transfer expressed by $2\mu_{kn}/(m_k + m_n)$. In the momentum equation, the collision terms represent the transferred momentum associated to the collision. In the energy equation, the collision terms are expressed into two different parts. The first part represents the directed kinetic energy transfer and the second part represents the transfer of undirected thermal energy. These expressions can be found in the lecture of Jean-Luc Raimbault and Roch Smets and are based on the hard sphere approximation [43]. Collisions between charged particles are not considered here. Indeed, in the Coulombian interactions' approximation, the collision frequency ν_{kl} between two charged particles k and l follows the relation $\nu_{kl} \propto 1/v_{kl}^3$ where v_{kl} is the relative velocity between the particles, which leads to frequencies negligible compared to collisions' frequencies of charged particles with neutrals [43]. More precisely, for streamer conditions at atmospheric pressure, due to the very low ionization degree the electron-ion collision frequency is around 5 orders of magnitude lower than the electron-neutral collision frequency. Models for the calculation of these frequencies ν_{kl} are discussed in section 3. In (1) S_k is a source term that relates to the chemical interaction between species. Thus, during a chemical interaction between two species, one of them may lose matter or gain matter and/or the interaction may create a third species. Therefore, S_k can be a production term if it is positive or a loss term if it is negative. By conservation of matter in a closed system these terms must verify the relation $\sum_k S_k = 0$. The model chosen to compute these terms is discussed in part 3.

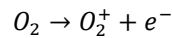
These Euler equations (1-3) are coupled to the Poisson equation (4). Indeed, the electrostatic approximation is stated as it is used to do in the literature of streamer modeling. The influence of the magnetic field and the Lorentz forces has been neglected as a first approximation in this approach since the current level in streamer discharges is generally very small in the mA range. In future works, the Lorentz force will be added in the RHS of equations (1-3) to take into account the magnetic forces and extend this model to the high current arc discharges. This model is therefore a non-neutral Eulerian multi-fluid model that solves the space charges. It is consistent with the physics of streamers and could even capture phenomena that cannot be represented using the classic drift-diffusion model (the inertia of charged species is considered). The physics of low-intensity current arc is also contained in this model and Laplace force terms could always be added later.

3. Collisions and chemistry model

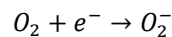
In this part, the model used to solve the chemistry reactions between species is described. This is followed by a discussion about collision frequencies.

3.1. Chemistry of charged species

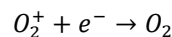
Positive and negative ions and electrons can interact through ionization, attachment, recombination and photoionization reactions. Ionization reactions refer to the collision of an electron to a neutral species that produces a positive ion and an electron. For example, the ionization reaction of the dioxygen molecule can be written:



The main loss term for electrons is two-body and three body attachment reactions: an electron interacts with a neutral atom or molecule to produce a negative ion. Eventually, attached electrons may be detached in high field regions:



The other loss term for electrons is electron-ion recombination. Recombination also exists between positive and negative ions. For example, electrons may recombine with dioxygen positive ions:



The photoionization reaction is a process in which a positive ion is formed from the interaction of a photon with an atom. As a first step, photoionization has been neglected in this study. Moreover, it has been shown that it only has a limited influence if the pre-ionization level is high enough, above 10^9 cm^{-3} at 1000 K for example as explained in [38]. All these reactions can be found in details for further information in [43]. Differential equations that corresponds to the kinetic of these reactions can be written as (6-8):

$$\partial_t n_e = n_e(\alpha - \eta)|V_e| - n_e n_p \beta \quad (6)$$

$$\partial_t n_p = n_e(\alpha|V_e| - n_p \beta) - n_m n_p \beta \quad (7)$$

$$\partial_t n_m = n_e \eta |V_e| - n_p n_m \beta \quad (8)$$

With V_e the electron drift velocity and α , η , β the ionization, attachment and recombination coefficients. These coefficients can be deduced from the calculations carried out by Morrow and Lowke [40]. Indeed, they fitted experimental data available in literature by formulating the local field hypothesis stating that the coefficients depend on the reduced electric field E/N where E is the electric field and N the neutral density [39,40]. In [39,41], the local field assumption is discussed. Hence, this assumption is considered valid as long as the electric field remains constant over a length defined as the equilibrium distance. Let us mention that $\alpha - \eta$ is known as the effective ionization coefficient. Indeed, when it is greater than 0, it means that electrons produced by an ionization reaction will not necessary be implied in an attachment reaction. In the air, $\alpha - \eta > 0$ above 36 kV.cm^{-1} . Once the chemical coefficients are obtained, the system (6-8) can be solved using an ODE solver. In this study, the RADAU5 ODE solver has been used.

3.2. Collision frequencies

In the multi-fluid Eulerian system (1-3), collision frequencies have to be specified to compute the RHS. A first approximation to study simple test cases is to consider constant frequencies, with typical values that can be encountered in streamer discharges at atmospheric pressure. For example, a constant collision frequency of 10^{14} s^{-1} has been considered in many test-cases for the electron-neutral elastic collisions. For more realistic air plasma discharge simulations, collision frequencies consistent with the chemical coefficients of Morrow and Lowke [40] have been derived. Based on the expression given for the mobility of the different species, it is possible to derive the momentum collision frequency thanks to the Drude-Lorentz formula:

$$\nu_{mn} = -\frac{q_e}{m_m \mu_m}, \quad \nu_{pn} = \frac{q_e}{m_p \mu_p}, \quad \nu_{en} = -\frac{q_e}{m_e \mu_e}$$

4. Fluid solver development

The Euler equations are hyperbolic systems that can be solved numerically by finite volume methods of the Godunov type. These involve discretizing the spatial domain into control volumes over which the physical variables are averaged. At each interface, the discontinuity of the variables can be seen as a Riemann problem for which exact and approximate solvers are available. However, in the context of simulating the streamer-arc transition, significant numerical difficulties can be encountered because the physics contained in the model is multi-scale. Space charges in a collisional plasma tends to recombine on a characteristic timescale called the dielectric relaxation time, or Maxwell's time τ , given by equation (9) where sigma is the electric conductivity and epsilon ϵ_0 the vacuum permittivity:

$$\tau = \frac{\epsilon_0}{\sigma} \quad (9)$$

The typical size of space charges is given by the Debye length λ (10):

$$\lambda = \sqrt{\frac{\epsilon_0 k_B T_e}{n_e q^2}} \quad (10)$$

To describe properly the non-neutral dynamics of a plasma, it is then required to consider timesteps lower than τ and cell size lower than λ . However, as the plasma becomes more and more conductive (higher sigma, higher n_e), those numerical constraints become prohibitive. For example, during the arc transition, the medium is quasi-neutral and highly conductive (conductivity increases from 1 S/m to $10^4 - 10^5 \text{ S/m}$) making Maxwell's time step collapse and the calculation of the arc transition prohibitive. Similarly, the Debye length decreases drastically in the quasi-neutral plasma of the electric arc (it goes from $1-10 \text{ }\mu\text{m}$ to $10^{-4} \text{ }\mu\text{m}$), thus constraining the cell size. It is then crucial not to keep a classical non-neutral plasma model to describe the quasi-neutral arc regime. One particular class of numerical scheme caught our attention: The Asymptotic Preserving schemes. In what follows, a definition of asymptotic regimes and asymptotic schemes is provided. This is followed by an explanation of the basic principles of the scheme developed for the simulations.

4.1. Asymptotic regimes and asymptotic scheme

Proposition – definition :

A system of equations F^{p_1, \dots, p_N} depending on N parameters p_1, \dots, p_N such that

$$\forall p_i, p_i \rightarrow 0 \Rightarrow F^{p_1, \dots, 0, \dots, p_N}$$

is said to be associated with a so-called AP scheme if and only if the scheme is unconditionally stable and uniformly consistent at the passage with respect to the parameters p_i .

The limits $p_i \rightarrow 0$ define asymptotic regimes and the parameters p_i are called asymptotic parameters.

Three asymptotic parameters determining three asymptotic regimes have been identified in our electrons system after normalization of the equations. First, the ratio between the mass of the electron and the mass of the ion denoted ε can become very small in massless electron regimes and lead to numerical stability problems. Streamers can be an illustration of this asymptotic regime. Second, the Debye length λ that is the characteristic length of a space charge in a plasma can become very small and tends to 0 as the plasma becomes quasi-neutral and conductor. Finally, the collision time between electrons and neutrals ν_{en}^{-1} (inverse of the collision frequency) can be very small in collisional plasma. This highly collisional asymptotic regime is supposed to be achieved in the drift-diffusion approximation which is the main model used in the community of the streamer modeling. The goal of our approach is to develop an AP scheme that preserve the stability properties described in the proposition above when passing through these three different asymptotic regimes. In the next subsection, the process used to get such a scheme is described.

4.2. Principles used in our approach

For ionic species and neutrals, classical volume finite schemes have been used in our approach to solve their dynamics (HLL or Rusanov flow calculations were tested). However, due to the low inertia of electrons compared to the ionic species and neutrals' one, two important problems well-known in numerical resolutions may appear problematic. Firstly, as the inertia of the electrons is low, their thermal velocity can be high and then the electrons in the simulations often evolve under supersonic conditions (defined by a fluid velocity higher than the acoustic velocity). This tends to eliminate the slow dynamics phenomena and create accuracy problems in some areas of the subdomain for some simulation times for which electrons are in subsonic conditions (defined by a fluid velocity lower than the acoustic velocity). Hence, accuracy problems may arise for Low-Mach regimes. This Low-Mach problem is all the more acute for electrons, as their thermal speed is high and they are therefore more sensitive to numerical fluctuations. To overcome this accuracy problem, a Low-Mach correction was used to attenuate the numerical diffusion artificially introduced by the numerical flux reconstruction term. This term depends on the relative velocity of the fluid with respect to a reference velocity (chosen as the acoustic velocity and/or the thermal velocity of the electrons). [28,29,30]. Secondly, in Low-Mach regimes, fast dynamic phenomena related to the acoustic properties do not play an important role in fluid dynamics. However, classical explicit methods require the use of a small timestep due to the fast dynamics contained in the equation as a stability condition. A strategy of decoupling acoustic and advective phenomena has been described by [37] and has been used for the electrons system. By applying a decomposition operator, acoustic phenomena can be decoupled and solved numerically independently and implicitly from the advective phenomena. In this way, a first acoustic step is performed and an intermediate state is obtained and used for the initial conditions of the other phenomena. However, by doing so, some divergency terms appear in the electron acoustic equation at the cost of conservativity properties.

To summarize, the scheme developed in this work consists of solving acoustic phenomena implicitly in a first step for the electron system. During this step, it is assumed that ions and neutrals have sufficient inertia for their conservative variables to remain unchanged. This approximation is justified by the fact that the ions and neutrals move more slowly than the electrons on the acoustic scale. An intermediate state is obtained and used as the initial condition for the advective system of the electrons and for the ionic and neutral systems. In the integration of the ion dynamics, there may be a problem of imbalance in the integration of the source terms for low-temperature plasmas. As the plasma potential scales with the electron temperature the electrostatic force on the ions can be much larger than the ions pressure gradient. Artificial numerical oscillations can be arisen. [32,33,34,35,36] propose an upwind discretization to overcome this problem and this method have been used for the calculation of source terms of ions and neutrals systems. The basic principle of this scheme has been set out by [31] for the isothermal Euler-Poisson system without energy equations and without collisions. Therefore, the scheme developed in this work is an extension of the isotherm plasma model of [31] to multi-species and highly collisional plasmas with temperature dependence. For this purpose, collisions with neutral particles have been added as well as the energy conservation equation. The schematic view of the algorithm during one time-step Δt is depicted on figure (1).

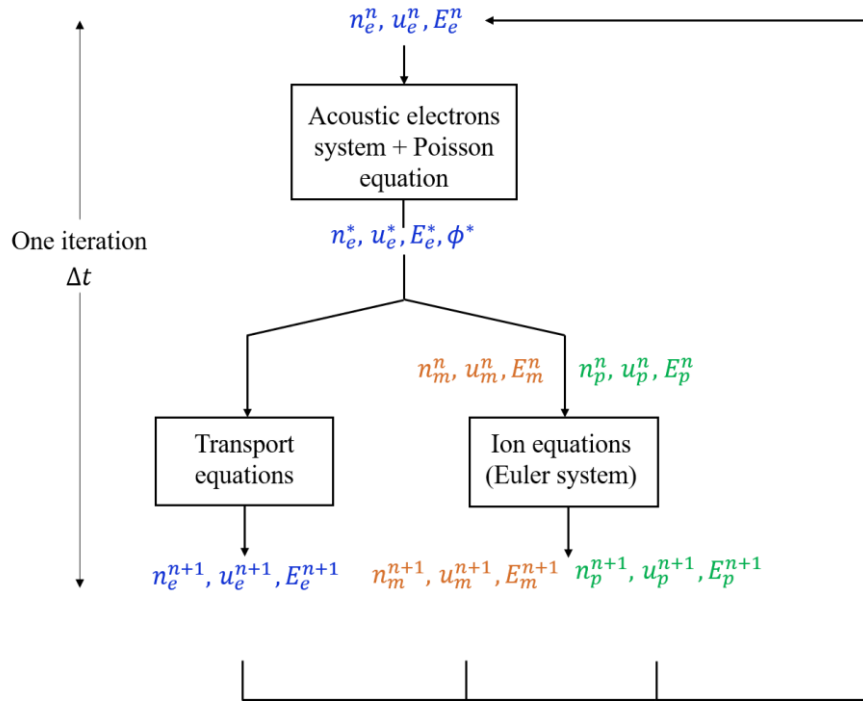


Fig. 1. Schematic view of the algorithm developed for one temporal iteration Δt .

5. Test cases and first steps for a streamer modeling

5.1. Collisional plasma in a uniform electric field

In the first test case, a uniform plasma subjected to a uniform electric field was simulated as well as electron-neutral and ion-neutral collisions with fixed frequencies that remain constant throughout the simulation. Initially, the electron and ion densities are uniformly set over the domain, and Neumann boundary conditions are applied to all physical variables of the Eulerian systems. For the Poisson equation, Dirichlet boundary conditions (BC) have been imposed on the left and right BC with $\Phi = 0$ V on the left and $\Phi = E \cdot l$ V where E is the applied field and l the length of the domain. In figure (2), the dimensionless electron velocity at the middle of the simulation domain is plotted as a function of time. At the beginning of the simulation, an acceleration proportional to the electric field is observed for the electrons. This corresponds to a ballistic phase 1, during which electrons are accelerated by the Coulomb forces. After a sufficiently long time compared to the electron-neutral collision time, a plateau appears in the electron velocity which becomes proportional to the electric field as collisions slow down the acceleration of the electrons. The plasma enters a collisional phase and follows Ohm's law, referred to as the Ohmic or resistive regime 3. Between the two regimes, a ballistic-resistive transition phase 2 is observed. Two important facts should be noted. When we let the electron-neutral collision time tend to zero in the electron equation (reaching so a collisional regime), an electron velocity expressed as combination of a drift term and a diffusion term is gotten (11).

$$\bar{v}_e = \frac{\nu_{en}^{-1} \bar{T}_e \partial_x \bar{n}_e}{\varepsilon + 1} + \varepsilon^{-1} \nu_{en}^{-1} \partial_x \phi \quad (11)$$

Thus, the asymptotic regime where the electron-neutral collision time tends to zero corresponds to a drift-diffusion regime. It has been found that the expected velocity resulting from this approximation is exactly equal to what is obtained in the collisional phase 3, thus confirming the accuracy of the numerical resolution. Secondly, when the simulation timestep is larger than the electron-neutral collision time, the simulation remains stable, and the value of the velocity obtained is directly the one that is expected in the resistive phase, so the accuracy is maintained. This first simple test case highlights the AP (Asymptotic-Preserving) nature of the scheme from the ballistic regime to the collisional, drift diffusion regime.

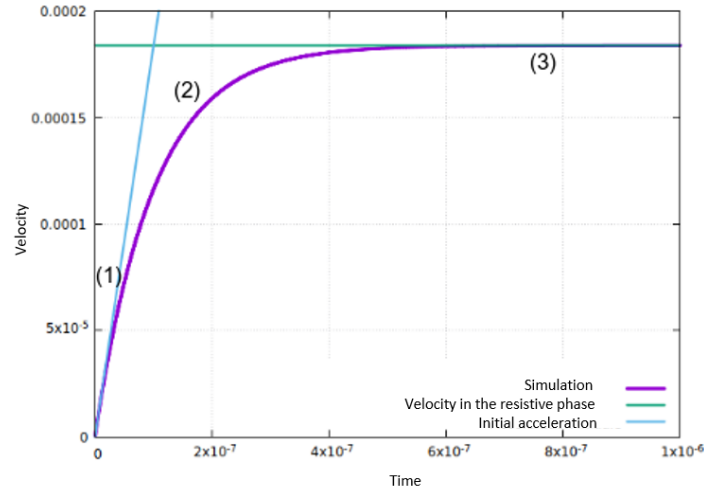
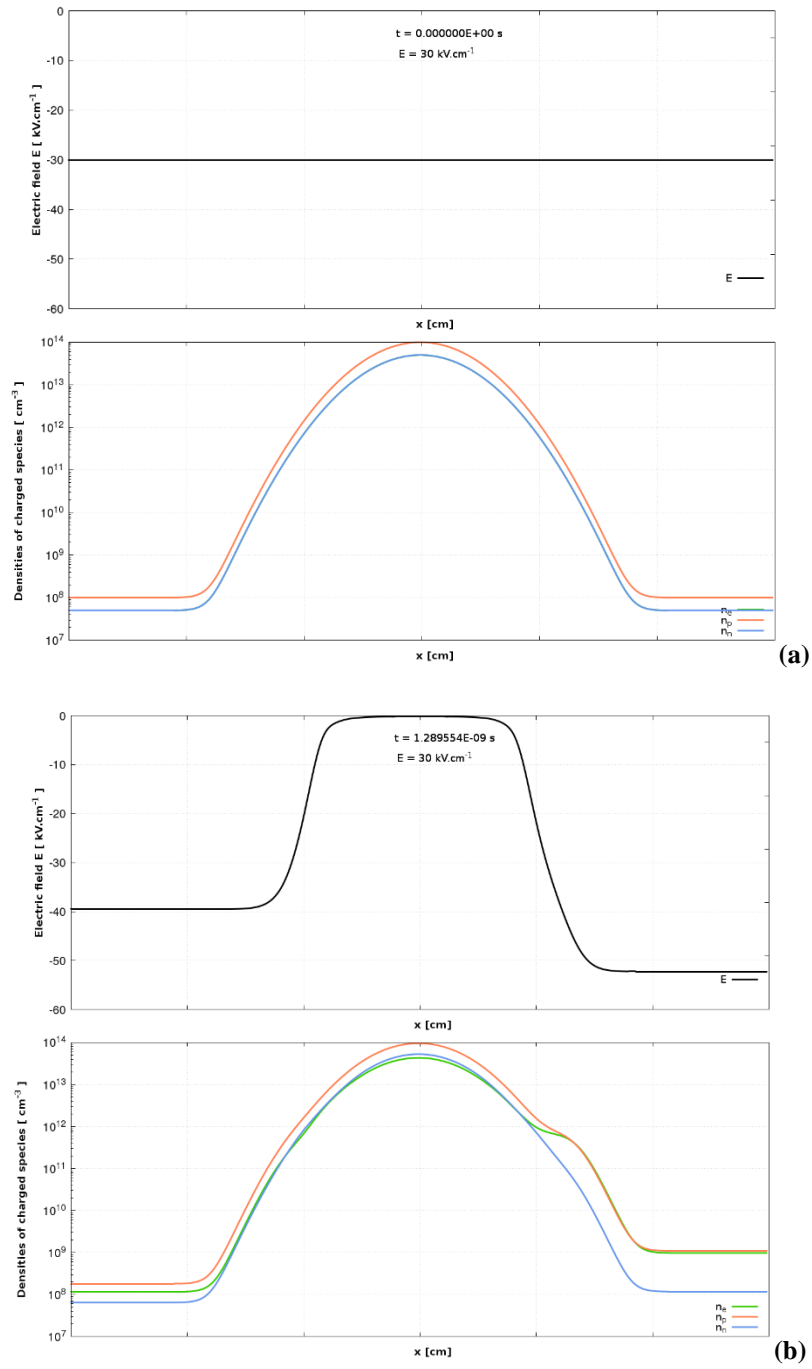
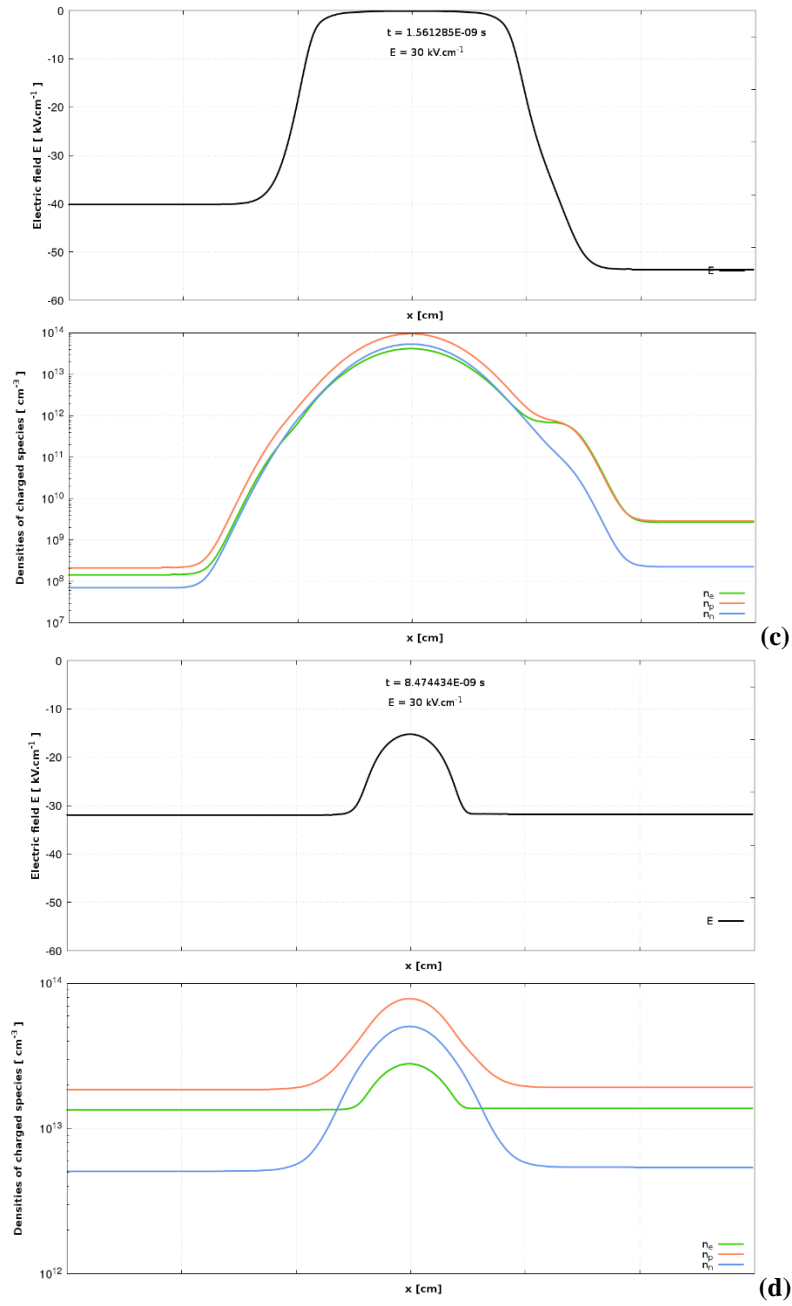


Fig. 2. Electron velocity in the middle of the domain over time during the simulation of a uniform plasma subjected to a uniform electric field.

5.2. Plasma seed subjected to a uniform electric field

In this test case, a Gaussian quasi-neutral plasma seed with $n_e = 10^{14} \text{ cm}^{-3}$ is placed in the center of a 3 cm domain with a 10^8 cm^{-3} preionization level and a uniform background field of 30 kV/cm. The electron temperature is set to 10 000 K while the temperature of other species is set to 300 K. As for the simulation timestep, only the plasma oscillation time remains a constraint throughout the simulation and spatially, the Debye length is under-resolved by 1 to 2 orders of magnitude. The fluid dynamics resolution is coupled with the ionization, recombination, and attachment reactions system, with corresponding frequencies obtained from Morrow's coefficients as explained in section 3.2. In figure 3, the electric field is plotted on the top graph and the electron densities (in green), negative ion densities (in blue), and positive ion densities (in orange) are plotted on the bottom graph. These data are shown at 6 moments during the simulation: the initial instant (a), $t=1.29 \text{ ns}$ (b), $t=1.56 \text{ ns}$ (c), $t=8.47 \text{ ns}$ (d), $t=37.48 \text{ ns}$ and (e) $t=48.00 \text{ ns}$. At first (a-b-c), positive and negative charges are created on the left and right sides of the domain, respectively and the medium is perfectly neutralized at the center of the domain. Thus, we can see that the electric field is exactly zero at the center of the domain, due to the screening effect of the plasma seed. In the absence of chemical reactions, this state would remain stationary until the end of the simulation. However, in this case, we have ionization reactions, and by looking at the electron densities, the ionization of the medium can be observed on the sides of the domain and then throughout the entire domain (c-d). Eventually, the plasma transition to a quasi-neutral regime in which species densities become uniform and neutralize each other, and the densities of charged species continue to increase until the end of the simulation (e-f). The electric field becomes uniform and equal to the background field in this regime (30 kV.cm^{-1}). What is important to note in this simulation is that it remains stable even when the collision time is not a constraint for the temporal resolution. This highlights the concept of asymptotic stability in a collisional medium observed in the previous test case. The simulation remains also stable throughout the simulation, including the transition to the quasi-neutral arc regime, even when the Maxwell timestep and the Debye length drop dramatically, far below the space and time resolutions. This highlights the asymptotic stability during the transition to the quasi-neutral asymptotic regime. These simulations are encouraging in the context of our goal to model a streamer and its transition to an arc within the same simulation.





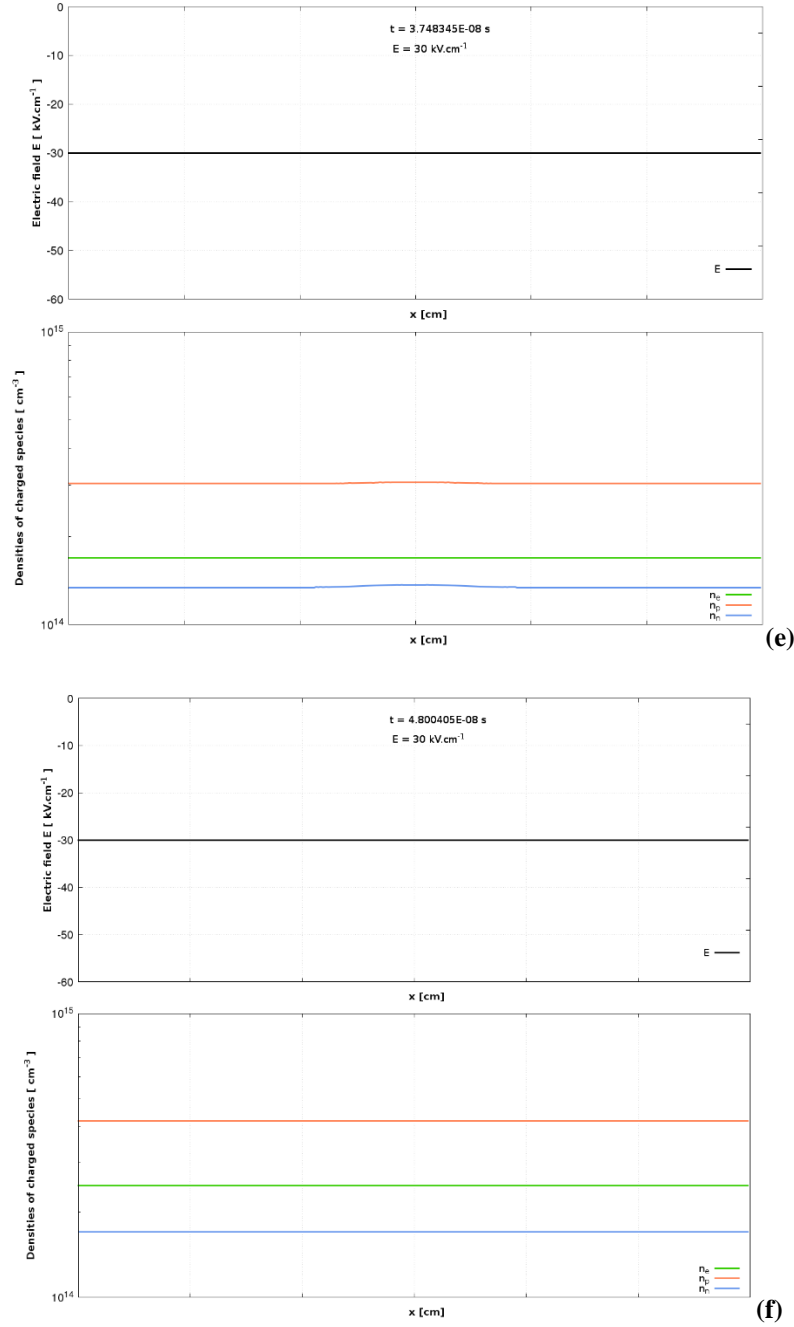


Fig. 3. Evolution over time of a plasma seed subjected to a uniform electric field of 30 kV.cm⁻¹. On the upper plot, the electric field is plotted. On the bottom one, the electron density (in green), the negative ion density (blue) and the positive ion density (orange) are plotted. (a) : initial time, (b) corresponds to $t=1.29$ ns, (c) to $t=1.56$ ns, (d) to $t=8.47$ ns, (e) to $t=37.48$ ns, (f) to $t=48.00$ ns.

6. Conclusion

A 1D numerical solver has been developed with the future aim of simulating the streamer discharge and its transition to the arc regime. This numerical solver solves the multi-fluid Euler-Poisson equations with a finite volume discretization and has the particularity of exhibiting asymptotic stability properties at the transition to physical asymptotic regimes, making it an Asymptotic Preserving scheme (AP). This AP property is very useful and important in the simulation of a streamer and its transition to the arc, given that the plasma passes from an asymptotic regime characterised by a highly collisional medium ($\nu_{en}^{-1} \rightarrow 0$) to an asymptotic regime characterised by a quasi-neutral

medium ($\lambda \rightarrow 0$). Simplified test cases have demonstrated the asymptotic stability properties of the scheme. In particular, the test case of a plasma seed in a homogeneous electric-field subject to ionization, attachment and recombination reactions, shows that the stability is preserved during the transition to a quasi-neutral arc regime. This model is currently extended in 2D, and in particular axisymmetric 2D. This will make it possible to model the plasma seed with the space-charge electric-field decreasing with distance. This effect could in principle makes it possible to model a realistic streamer discharge and its transition to an arc. However, many assumptions and simplifications performed in this work may be discussed. Particularly, the electrostatic assumption formulated in the model could be questioned. Furthermore, the absence of a magnetic field can be called into question, especially during the transition to an arc, as the electric current rises. It would be interesting to incorporate its contribution in the source terms through Lorentz forces and to make it evolve over time using either the full set of Maxwell's equations or a quasi-static approach. Finally, Coulombian collisions between charged species have been neglected in our model, while they are mandatory when the ionization degree becomes important. Additionally, the local field assumption used in the context of Morrow's chemistry could be questioned in favor of a chemistry that depends on electron temperature. All these points are still open and will be addressed in future works.

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