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Robustness analysis for closed-loop model reference adaptive attitude control for a satellite launcher

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Abstract
This paper considers a closed loop reference model architecture for an adaptive control system and analyses its robustness in the presence of bounded disturbances. The traditional architecture is modified by an e-modification adaptive law and its bounded stability is proven using concepts of uniform ultimate boundedness. Furthermore, this work proposes a strategy for calculating the size of the bounded set, showing the influence of the CRM architecture in the overall robustness of the system. Finally, the results are applied to a satellite launcher attitude control, illustrating how different controller choices influence the robustness metrics.

1. Introduction

Reference model adaptive control systems are a class of controllers capable of adjusting its own parameters by monitoring the performance on a feedback loop and comparing the output to a reference model. This behaviour makes adaptive systems particularly interesting when dealing with problems with uncertainties on the system dynamics or external disturbances, as it is often true in aeronautics and space applications [1, 2].
However, due to the nonlinear nature of the system, the future use of such controllers in critical systems is still dependent on the development of performance and stability metrics capable of leading to a certifiable control system, and the study of stability and robustness metrics have been a very active field in the past decade [2]. A relevant part of such effort concerns the transient response of such systems.
Even in situations where it is possible to prove the overall system stability, the plant behaviour during the transient may be very different from the reference model [3]. One of the modifications proposed to address these concerns uses the tracking error as a feedback signal to the reference model to maintain fast adaptation while improving transient behaviour [4].
This paper presents a closed loop reference model (CRM) architecture for an adaptive control system and analyses its robustness in the presence of bounded disturbances. The original CRM architecture is modified by an e-modification adaptive law and its bounded stability is proven using concepts of uniform ultimate boundedness (UUB) [5]. Furthermore, this work proposes a strategy for calculating the size of the bounded set based on the bounds of the solution of algebraic matrix equations [6], showing the influence of the CRM architecture gains in the overall robustness of the system. Finally, the results are applied to a satellite launcher attitude control, illustrating how different controller choices influence the robustness metrics for a real critical system.

2. Model reference adaptive control

Model reference adaptive control (MRAC) is a class of adaptive control where the controlled system response is compared to the response of a given reference model, usually assumed linear and time invariant. The goal of the controller is, therefore, for the system response to follow the reference model as close as possible [7].
Figure 1 shows a concept diagram for a MRAC controlled system. As described by [8], one may interpret this configuration as two separate loops: the internal loop mirrors the traditional controller, while the outer loop represents the adaptation loop attempting to drive the error between the reference model and the plant to zero.
If the plant was completely known, finding the control parameters required to match the plant behaviour to the desired reference model would be an algebraic problem. However, since real systems have uncertain parameters, the goal of an adaptive system such as this is to change the controller parameters as the system develops [5].
A well-known issue with this approach is how the adaptive system behaves in the transient period. As mentioned by [3], while it is often possible to prove the stability of model reference adaptive controllers, its performance during the

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transient phase may be widely different than the desired reference. In order to improve such issues, several modifications in the original scheme have been proposed.

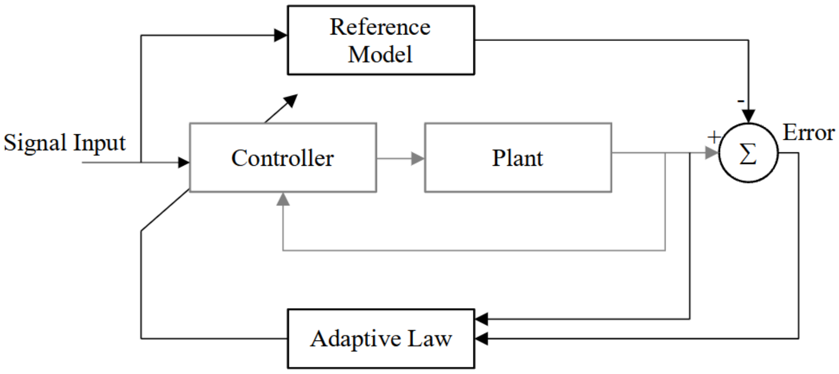

Figure 1: Model reference adaptive control scheme

2.1 Closed-loop model reference adaptive control

Initially proposed by [8], the closed-loop reference model (CRM) is an adaptive control scheme modification that has shown interests results with respect to the transient dynamics of adaptive systems. This configuration, shown in Figure 2, consists of an error feedback loop not only on the adaptive law, but also on the reference model itself.

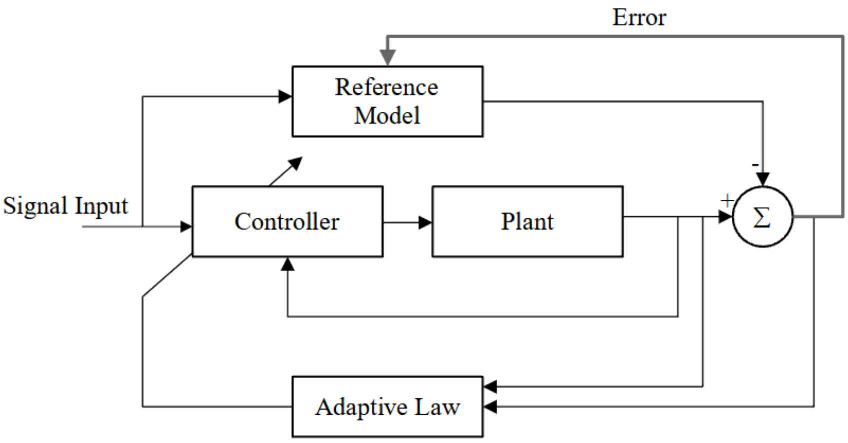

Figure 2: Closed-loop model reference adaptive control scheme

The stability and overall transient properties of this configuration have been explored in several different works [4, 5, 9, 10,11].

Consider a plant modelled by Equation (1), where x_p is the n-dimensional state (fully available), u is the control signal, A is an unknown matrix, B is the control matrix, fully known and Λ is a positive defined diagonal matrix responsible for modelling potential control failures. It is assumed that the pair (A, B Λ) is controllable [5].

$$\dot{x}_p = Ax_p + B\Lambda u \tag{1}$$

The behaviour desired for the plant under a limited signal input r is modelled by a reference model represented by Equation (2). The state error is multiplied by a matrix gain L.

$$\dot{x}_m = A_{ref}x_m + B_{ref}r - L(x_p - x_m) \tag{2}$$

The controller follows a feed-forward structure given by Equation (3). $\widehat{K}_x^T$ and $\widehat{K}_r^T$ are the estimated controller gains and it is assumed that exists ideal gains K_x^T and K_r^T so that $A + B\Lambda K_x^T = A_{ref}$ and $B\Lambda K_r^T = B_{ref}$.

$$u = \widehat{K}_x^T x_p + \widehat{K}_r^T r \tag{3}$$

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3. Robustness analysis in the presence of bounded disturbances

In this paper, we are interested in studying the robustness properties of such system in the presence of bounded disturbances. Equation (4) represents the plant in this case, where ξ represents a bounded external disturbance where $|\xi(t)|_{L_\infty} \leq \xi_{max} \geq 0$.

$$\dot{x}_p = Ax_p + B\Lambda u + \xi \tag{4}$$

Considering the same reference model as Equation (2), the state error dynamics is given by Equation (5), where $\Delta K_x = \hat{K}_x - K_x$ and $\Delta K_r = \hat{K}_r - K_r$.

$$\dot{e} = \left(A_{ref} + L\right)e + B\Lambda\left[\Delta K_x^T x_p + \Delta K_r^T r\right] + \xi \tag{5}$$

We shall consider a quadratic Lyapunov function candidate with adaptation gains such as that $\Gamma_x = \Gamma_x^T > 0$ and $\Gamma_r = \Gamma_r^T > 0$ given by Equation (6) [5].

$$V(e, \Delta K_x, \Delta K_r) = e^T P e + tr(\Lambda \Delta K_x^T \Gamma_x^{-1} \Delta K_x) + tr(\Lambda \Delta K_r^T \Gamma_r^{-1} \Delta K_r) \tag{6}$$

Here, $P=P^T>0$ solves the Riccati equation $(A_{ref}+L)^T P + P(A_{ref}+L) + Q = 0$ for a given $Q=Q^T>0$. The temporal derivative of the Lyapunov function candidate may be calculated by the expression in Equation (7).

$$\dot{V} = \frac{\partial V}{\partial e}\dot{e} + \frac{\partial V}{\partial \Delta K_x}\Delta\dot{K}_x + \frac{\partial V}{\partial \Delta K_r}\Delta\dot{K}_r \tag{7}$$

Where,

$$\begin{aligned} \frac{\partial V}{\partial e}\dot{e} &= 2e^T P \dot{e} \\ \frac{\partial V}{\partial \Delta K_x}\Delta\dot{K}_x &= 2tr\left(\Lambda \Delta K_x \Gamma_x^{-1} \dot{\hat{K}}_x\right) \\ \frac{\partial V}{\partial \Delta K_r}\Delta\dot{K}_r &= 2tr\left(\Lambda \Delta K_r^T \Gamma_r^{-1} \dot{\hat{K}}_r\right) \end{aligned} \tag{8}$$

Therefore, considering the error dynamics in Equation (5),

$$\dot{V} = -e^T Q e + 2e^T P\xi + 2tr\left(\Lambda\left[\Delta K_x{}^T x_p e^T P B + \Delta K_x^T \Gamma_x^{-1}\dot{\hat{K}}_x\right]\right) + 2tr\left(\Lambda\left[\Delta K_r{}^T r e^T P B + \Delta K_r^T \Gamma_r^{-1}\dot{\hat{K}}_r\right]\right) \tag{9}$$

The presence of the $2e^T P\xi$ term here means that traditional adaptation laws are not enough to guarantee stability under external bounded disturbances [5, 12].

3.1 e-modification

In order to ensure the system stability under these circumstances, several modifications have been proposed in literature. This paper will consider an adaptation law with the e-modification [13], shown in Equation (10). Here, $\sigma > 0$ is a scalar constant.

$$\begin{aligned} \dot{\hat{K}}_x &= -\Gamma_x\left(x e^T P B + \sigma||e^T P B||\widehat{K_x}\right) \\ \dot{\hat{K}}_r &= -\Gamma_r\left(r e^T P B + \sigma||e^T P B||\widehat{K_r}\right) \end{aligned} \tag{10}$$

Therefore, Equation (9) becomes

$$\dot{V} = -e^T Q e + 2e^T P\xi - 2\sigma||e^T P B||tr\left(\Lambda \Delta K_x^T \widehat{K_x}\right) - 2\sigma||e^T P B||tr\left(\Lambda \Delta K_r^T \widehat{K_r}\right) \tag{11}$$

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Since $\Delta K_x=\widehat{K}_x-K_x$ and $\Delta K_r=\widehat{K}_r-K_r$, the derivative of the Lyapunov function is given by Equation (12).

$$\dot{V} = -e^TQe + 2e^TP\xi - 2\sigma||e^TPB||tr(\Lambda\Delta K_x^T\Delta K_x) - 2\sigma||e^TPB||tr(\Lambda\Delta K_x^T K_x) - 2\sigma||e^TPB||tr(\Lambda\Delta K_r^T\Delta K_r) - 2\sigma||e^TPB||tr(\Lambda\Delta K_r^T K_r) \quad \textbf{(12)}$$

Considering the definition of trace, we may write Equations (13) and (14), where $||\,.\,||_F$ represents the Frobenius norm [5].

$$tr(\Lambda\Delta K_x^T\Delta K_x) \equiv \sum_{i=1}^{n}\sum_{j=1}^{m}\Delta K_{x_{ij}}^2\,\Lambda_{jj} \geq ||\Delta K_x||_F^2\Lambda_{min} \quad \textbf{(13)}$$

$$tr(\Lambda\Delta K_r^T\Delta K_r) \equiv \sum_{j=1}^{m}\Delta K_{r_{jj}}^2\,\Lambda_{jj} \geq ||\Delta K_r||_F^2\Lambda_{min} \quad \textbf{(14)}$$

Finally, we may use the Schwarz inequality and the trace cyclic property on the other elements.

$$\begin{aligned} tr(\Lambda\Delta K_x^T K_x) &\leq ||\Delta K_x^T K_x||_F||\Lambda||_F \leq ||\Delta K_x||_F||K_x||_F||\Lambda||_F \\ tr(\Lambda\Delta K_r^T K_r) &\leq ||\Delta K_r^T K_r||_F||\Lambda||_F \leq ||\Delta K_r||_F||K_r||_F||\Lambda||_F \end{aligned} \quad \textbf{(15)}$$

Thus,

$$\begin{aligned} \dot{V} \leq & -\lambda_{min}(Q)||e||^2 + 2||e||\lambda_{max}(P)\xi_{max} - 2\sigma||e^TPB||||\Delta K_x||_F^2\Lambda_{min} \\ & + 2\sigma||e^TPB||||\Delta K_x||_F||K_x||_F||\Lambda||_F - 2\sigma||e^TPB||||\Delta K_r||_F^2\Lambda_{min} \\ & + 2\sigma||e^TPB||||\Delta K_r||_F||K_r||_F||\Lambda||_F \end{aligned} \quad \textbf{(16)}$$

Using a similar technique as [5], we may complete the squares, so that Equation (17) represents the derivative inequality.

$$\begin{aligned} \dot{V} \leq & -\lambda_{min}(Q)\left[||e|| - \frac{\lambda_{max}(P)\xi_{max}}{\lambda_{min}(Q)}\right]^2 + \frac{\lambda_{max}^2(P)\xi_{max}^2}{\lambda_{min}(Q)} - 2\sigma||e^TPB||\Lambda_{min}\left[||\Delta K_x||_F - \frac{||K_x||_F||\Lambda||_F}{2\Lambda_{min}}\right]^2 \\ & + \sigma||e^TPB||\frac{||K_x||_F^2||\Lambda||_F^2}{2\Lambda_{min}} - 2\sigma||e^TPB||\Lambda_{min}\left[||\Delta K_r||_F - \frac{||K_r||_F||\Lambda||_F}{2\Lambda_{min}}\right]^2 \\ & + \sigma||e^TPB||\frac{||K_r||_F^2||\Lambda||_F^2}{2\Lambda_{min}}. \end{aligned} \quad \textbf{(17)}$$

Therefore, $\dot{V}(e,\Delta K_x,\Delta K_r) < 0$ if at least one out of Equations (18), (19) or (20) is true.

$$\lambda_{min}(Q)\left[||e|| - \frac{\lambda_{max}(P)\xi_{max}}{\lambda_{min}(Q)}\right]^2 > \frac{\lambda_{max}^2(P)\xi_{max}^2}{\lambda_{min}(Q)} + \sigma||e^TPB||\frac{||K_x||_F^2||\Lambda||_F^2}{2\Lambda_{min}} + \sigma||e^TPB||\frac{||K_r||_F^2||\Lambda||_F^2}{2\Lambda_{min}} \quad \textbf{(18)}$$

$$\begin{aligned} & 2\sigma||e^TPB||\Lambda_{min}\left[||\Delta K_x||_F - \frac{||K_x||_F||\Lambda||_F}{2\Lambda_{min}}\right]^2 \\ & \qquad > \frac{\lambda_{max}^2(P)\xi_{max}^2}{\lambda_{min}(Q)} + \sigma||e^TPB||\frac{||K_x||_F^2||\Lambda||_F^2}{2\Lambda_{min}} + \sigma||e^TPB||\frac{||K_r||_F^2||\Lambda||_F^2}{2\Lambda_{min}} \end{aligned} \quad \textbf{(19)}$$

$$\begin{aligned} & 2\sigma||e^TPB||\Lambda_{min}\left[||\Delta K_r||_F - \frac{||K_r||_F||\Lambda||_F}{2\Lambda_{min}}\right]^2 \\ & \qquad > \frac{\lambda_{max}^2(P)\xi_{max}^2}{\lambda_{min}(Q)} + \sigma||e^TPB||\frac{||K_x||_F^2||\Lambda||_F^2}{2\Lambda_{min}} + \sigma||e^TPB||\frac{||K_r||_F^2||\Lambda||_F^2}{2\Lambda_{min}} \end{aligned} \quad \textbf{(20)}$$

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These expressions represent limiting conditions, so that $\dot{V} < 0$ outside a compact set $\Omega \subset (R^n \times R^{n\times m} \times R^{m\times m})$, closed and bounded, where c_1, c_2 and c_3 are defined by Equations (22), (23) and (24).

$$\Omega = \{(e, \Delta K_x, \Delta K_r) : (||e|| \leq c_1) \wedge (||\Delta K_x||_F \leq c_2) \wedge (||\Delta K_r||_F \leq c_3)\} \quad \textbf{(21)}$$

$$\frac{\lambda_{max}(P)\xi_{max}}{\lambda_{min}(Q)} + \sqrt{\frac{\lambda_{max}^2(P)\xi_{max}^2}{\lambda_{min}^2(Q)} + \frac{\sigma||e^T PB||}{\lambda_{min}(Q)}\left[\frac{||K_x||_F^2||\Lambda||_F^2}{2\Lambda_{min}} + \frac{||K_r||_F^2||\Lambda||_F^2}{2\Lambda_{min}}\right]} \equiv c_1 \quad \textbf{(22)}$$

$$\frac{||K_x||_F||\Lambda||_F}{2\Lambda_{min}} + \sqrt{\frac{\lambda_{max}^2(P)\xi_{max}^2}{2\sigma||e^T PB||\Lambda_{min}\lambda_{min}(Q)} + \frac{1}{2\Lambda_{min}}\left[\frac{||K_x||_F^2||\Lambda||_F^2}{2\Lambda_{min}} + \frac{||K_r||_F^2||\Lambda||_F^2}{2\Lambda_{min}}\right]} \equiv c_2 \quad \textbf{(23)}$$

$$\frac{||K_r||_F||\Lambda||_F}{2\Lambda_{min}} + \sqrt{\frac{\lambda_{max}^2(P)\xi_{max}^2}{2\sigma||e^T PB||\Lambda_{min}\lambda_{min}(Q)} + \frac{1}{2\Lambda_{min}}\left[\frac{||K_x||_F^2||\Lambda||_F^2}{2\Lambda_{min}} + \frac{||K_r||_F^2||\Lambda||_F^2}{2\Lambda_{min}}\right]} \equiv c_3 \quad \textbf{(24)}$$

Equation (21) shows the ultimate uniform boundedness of all signals in the closed-loop dynamics.

3.2 Closed-loop reference model influence

Equations (21) – (24) show that the compact sets defining the ultimate uniform boundedness of all signals are influenced by the eigenvalues of the P and Q matrices.

Considering the same Q for both cases, the main difference between both cases will be in the algebraic equation solved for P. Equation (25) shows the equation for the traditional, "open-loop", reference model, while Equation (26) presents the equation used for the CRM case. Here, the subscript O is used to differentiate the open-loop reference model.

$$A_{ref}^T P_o + P_o \mathrm{A_{ref}} = -Q \quad \textbf{(25)}$$

$$\left(A_{ref} + L\right)^T P + P\left(A_{ref} + L\right) = -Q \quad \textbf{(26)}$$

According to [6, 12], for a given Q, there is a single symmetric definite positive solution for each of these equations. Therefore, if $\alpha_{o_{max}} \geq \alpha_{o_2} \geq \ldots \geq \alpha_{o_n} > 0$ are the eigenvalues for P_o, there is a lower bound on the maximum eigenvalue of P_o, described by Equation (27).

$$\lambda_{max}(P_o) = \alpha_{o_{max}} \leq \frac{1}{-\lambda_{max}\{(A_{ref} + A_{ref}^T)Q^{-1}\}} \quad \textbf{(27)}$$

For the closed-loop reference model, we have an equivalent expression, given by Equation (28).

$$\lambda_{max}(P) \leq \frac{1}{-\lambda_{max}\{\left[A_{ref} + L + \left(A_{ref} + L\right)^T\right] Q^{-1}\}} \quad \textbf{(28)}$$

Comparing Equations (27) and (28), we see that the closed loop reference model also directly affects the size of the ultimate uniform boundary as long as $\lambda_{max}(P) < \lambda_{max}(P_o)$. That is, if the largest eigenvalue of the matrix $[A_{ref}+L+(A_{ref}+L)^T]Q^{-1}$ is smaller than the largest eigenvalue of the matrix $(A_{ref}+A_{ref}^T)Q^{-1}$ for a given Q. This result is valid as long as the denominator in Equations (27) and (28) is positive.

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4. Closed-loop model reference adaptive control for satellite launcher

As a way to illustrate the control system studied in this paper, let us consider a thrust vector controlled (TVC) symmetric satellite launcher. The following assumptions are made:

1. The axial velocity u (t) is considered an independent parameter, since it is assumed the vehicle flies under a designed trajectory.
2. The vehicle has independent roll control, so that its roll velocity is null.
3. The velocity u (t) is much larger than the wind velocities vv_z and vv_y, so that $\alpha \approx (w-vv_z)/u$ and $\beta \approx (v-vv_y)/u$.
4. The yaw angle ψ is small, so that $\sin(\psi) \approx \psi$ and $\cos(\psi) \approx 1$.
5. Euler angle rotations follow the sequence (θ,ψ,ϕ).

Under these, the system attitude dynamics in pitch and yaw are independent. Assuming slow parametric variations with respect to the process time constants, the longitudinal motion of a satellite launcher may be modelled as a transfer function shown in Equation (30) [15], where β_z is the deflection of the thrust vector angle and θ is the deflection angle.

$$\frac{\theta(s)}{\beta_z(s)} = \frac{-M_\beta s + M_\alpha Z_\beta/u - M_\beta Z_\alpha/u}{s^3 + \left(\frac{Z_\alpha}{u} + M_q\right)s^2 + \left(\frac{M_q Z_\alpha}{u} - M_\alpha\right)s + g\frac{M_\alpha}{u}} \tag{29}$$

Equation (25) represents the different efficiency coefficients used in Equation (29), where T is the thrust, m is the vehicle mass, I is the moment of inertia, l_c is the control lever, l_a is the static margin, Q is the dynamic pressure, S is the reference area, C_{n_α} is the normal force coefficient and x_e is the distance between the centre of gravity and the nozzle.

$$\begin{aligned} M_\beta &= \frac{T l_c}{I} \\ M_\alpha &= \frac{C_{n_\alpha} Q S l_a}{I} \\ Z_\beta &= \frac{T}{m} \\ Z_\alpha &= \frac{C_{n_\alpha} Q S}{m} \\ M_q &= \frac{\dot{I}}{I} + \frac{C_{M_q} Q S l r^2}{2Iu} + \frac{\dot{m} x_e^2}{I} \end{aligned} \tag{30}$$

If the velocity u is very large and $M\alpha \gg M_q$, the system may be represented by the following transfer function.

$$\frac{\theta(s)}{\beta_z(s)} = \frac{-M_\beta}{s^2 - M_\alpha} \tag{29}$$

Therefore, assuming external bounded disturbances as defined in the previous sections, the plant dynamics may be written as [16,17]

$$\dot{x}_p = \begin{bmatrix} 0 & 1 \\ M_\alpha & 0 \end{bmatrix} x_p + \begin{bmatrix} 0 \\ -M_\beta \end{bmatrix} u + \xi \tag{29}$$

For this paper, we will compare the different reference models using the vehicle modelled in [17] and considering frozen poles at t=55s. The reference model will be the one described by Equation (2) and designed so that the reference model converges to a second order model in the traditional form with ω_n=4 and ζ = 0.7 as the feedback error goes to zero.

Figure 3 shows the response of this system to a step command of 5 degrees with both model reference adaptive control configurations with the same $Q = 100I_2$, where I_2 represents the identity matrix. Here, ORM represents the open loop, traditional, reference model and CRM represents the closed loop reference model. In both cases, $\sigma_x = \sigma_r = 0.1$, $\Gamma_r =$

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1000, $\Gamma_x = 1000 I_2$ and the initial conditions were set to zero on the reference model and [-1deg 1 deg/s] for the plant state. In the closed loop reference model, the controller feedback gain was defined as $L =$ -$0.8 I_2$.

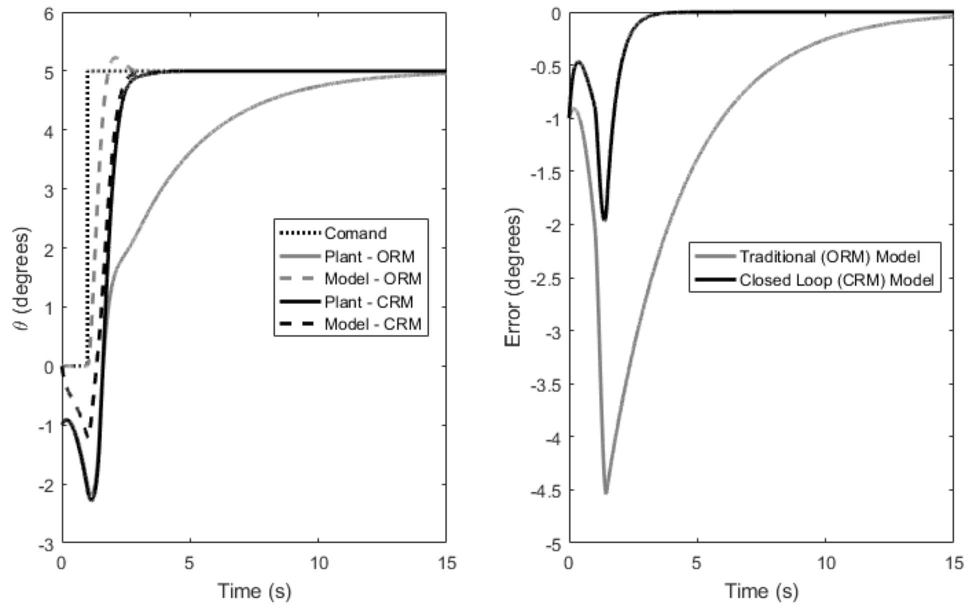

Figure 3: System response comparison to a 5 degree step input

Figure 3 shows how the closed loop reference model, under the same design parameter Q, responds faster to the step control signal.

Figure 4 shows the same comparison under a bounded disturbance. In this case, the external disturbance was modelled as a pulse impulse of 3 degrees, applied at t=2s.

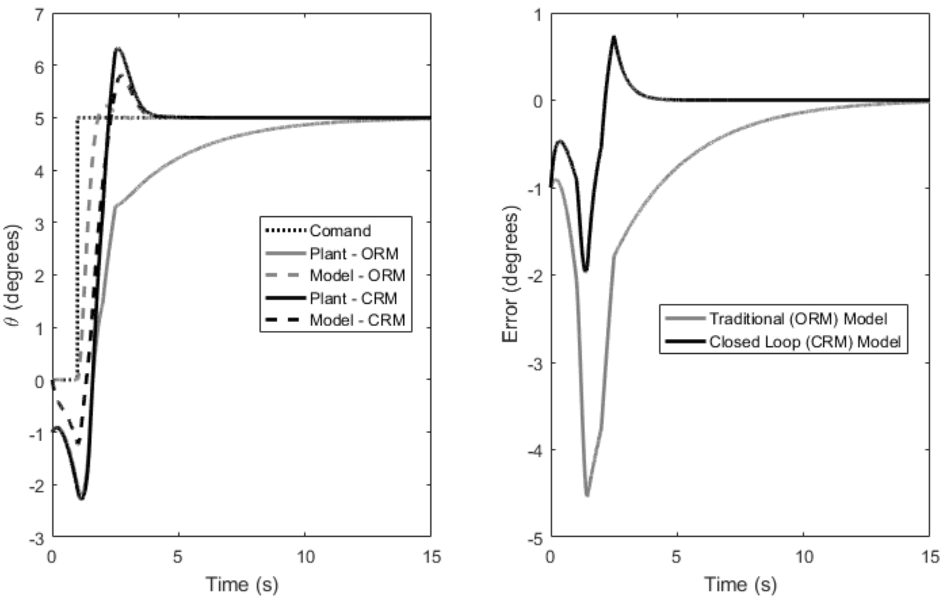

Figure 3: System response comparison to a 5 degree step input and external bounded disturbance

As seen, since the closed loop system was already close to the reference input when the disturbance occurred, there is a considerable peak, but the system is able to recover. Interestingly, it is possible to see when comparing Figures 3 and 4 that, as expected, the reference model adapted to the larger error, changing the reference the controller was following for a brief moment and softening the control effort.

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5. Conclusions

This paper considered a closed loop reference model for a model reference adaptive controller with e-modification. The architecture's robustness with respect to external bounded disturbances was shown and stability was proven with respect to uniform ultimate boundedness (UUB) arguments, presenting boundaries for all signals in the system. The limits presented on the compact set were considered and a way to calculate relevant variables on the bounds was shown for a class of systems. Finally, the results of the CRM architecture were demonstrated considering a satellite launcher. The study showed how, for the same design parameters, the closed loop reference model provided a faster response to a step input. In the presence of the modelled disturbances, the simulations also demonstrated how the feedback loop influences the reference model when adapting to undesired external perturbations.
Future works should expand on the analysis with respect to time variant systems, as well as considering how a time invariant closed loop reference model could be integrated to a slow varying system such as a satellite launcher.1

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