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Flight dynamics analysis and trajectory optimization of some gliding phases

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Abstract

As far as the gliding phases are concerned, the specificities of the aircraft flight dynamics and the surrounding aerology must be taken into account seriously, especially since no propulsion is at disposal to cope with possible problems. In this study, several issues are examined. On the one side, pilot induced oscillations of the space shuttle command channel are expertized and on the other side, a performance (range, mean velocity, altitude loss) is maximized for a sailplane in an environment with a given repartition of thermals (or wind). Even if the study is more applied mathematics oriented, the results are interesting for engineers and pilots since it helps to take in-flight decisions and to make pre-flight preparations.

The analysis of some issues coming from nonlinear flight dynamics is accomplished here by means of the bifurcation theory and continuation algorithms. Some nonlinear features are actually due to couplings with pilot reaction here and may imply unexpected events and it is then important to evaluate whether there are dangerous or not. The main topic of this part concerns pilot-aircraft couplings for a configuration with forced oscillations as inputs.[4] The application concerns the space shuttle which meets some issues in its gliding phase.[11] The analysis of the command channel with a rate-limited actuator reveals the presence of jumps (flying qualities cliffs), indeed a little variation of the input pulsation may imply large variation (of amplitude and phase). According to the describing function method (DFM), a continuation algorithm allows to predict the amplitude (and the phase shift) at the entry of the actuator thanks to the resolution of an harmonic balance equation (HBE) which involves the transfer functions of the different parts and the describing function of the nonlinearity (saturation or rate limiting).

Besides another topic is the flight trajectory optimization of a sailplane. When (the mean velocity is) calculated in a stationary environment with only one vertical thermal, it is associated to the so-called MacCready speed, whereas configurations with several thermals raise some questions such as the optimal speeds and the best strategy to adopt (which speed to take or which thermals to target, in which order, etc). Another strategy might be to realize a dolphin-style gliding and keep on flying (in thermals). In this case, optimal control can furnish the best speeds (and altitudes) to follow during a longitudinal flight (in the vertical plane) depending the magnitude distribution of the vertical wind.

Finally for such studies, one hope is surely to help pilots to make wise in-flight decisions and to prepare as well as possible the gliding phases taking into consideration the amount of data and their level of precision at disposal but also to compare different scenarios and to avoid unscheduled off-field landing and more generally to diminish soaring risks. The mathematical analysis, optimization and underlying physics are explained in theoretical and practical terms in order to be valuable for mathematicians and aerospace specialists.

Introduction

Gliding phases are specific in the sense that it is not possible to use thrust. It implies a necessity of carefull handling of the other controls and an obligation to take seriously into account the aerology (and aerodynamics). In this study, two themes are dealt with, the types of aircraft are not the same and the mathematical tools employed neither but all of them present interesting features.

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Concerning the pilot-induced oscillations of the space shuttle during its landing, the trigger is a rate-limited actuator. As a consequence, the classical linear criteria are not sufficient and efficient to assess the susceptibility to these aircraft-pilot couplings successfully. Several methods are at disposal to analyze these phenomena and to attenuate or suppress it. Indeed major accidents of fighters (Gripen, YF22, YF16, etc) occurred for example during the nineties and have forced the aerospace industry and research community to develop methods in order to predict and control the flight dynamics during dangerous phases which demand an important pilot effort like landing, take-off, refueling, etc, thus exacerbating rate limiting effects. Unfortunately the linear theory which was widely employed in the engineering process was not sufficient, therefore some tools and concepts coming from the research world needed to be taken into account and transferred to the industry. Sometimes these last ones might be too conservative in order to be employed successfully for a design process, nevertheless they always present a clear interest in understanding the phenomenon and trying to master it. Describing function method, harmonic balance and bifurcation theory are powerful methodologies, since they allow to diagnose existence, stability of periodical orbits and their changes of characteristics or multiplicity. They are the mathematical core of this study and are extensively used. Nevertheless other methodologies are also relevant and some new ideas are trying to take better into account the transitory characteristics of nonlinear flight dynamics.

As far as the optimization of sailplane flight is concerned, it is always interesting to find a compromise between mathematics and concrete piloting. Indeed on the one side, optimization can furnish precise results but on the other side the pilot must be able to apply them successfully in concrete situations. Especially the pilot does not know perfectly the aerological environment quite often and the in-flight instruments have an inherent delay before displaying the correct measure. Consequently, in order to optimize its flight, the pilot must have an idea of the diverse possible strategies and some knowledge about their mathematical and physical foundations, but also a methodology in order to take quick decisions which may be partly intuitive. For example, when estimating the ascending speed in a thermal, the pilot experience may play a role but also when choosing a strategy amongst several ones. A focus is put in the use of optimal control, since it permits to consider a large variety of flight types, performances to optimize and aerologic situations. Some care must nevertheless be taken in the mathematical formulation in order to facilitate the calculation process.

Even if both themes are different, they each present huge interest and some common mathematical methodologies and flight experiences can contribute to improve the pilot ability and the engineering design process.

1. Pilot induced oscillations of the space shuttle

The space shuttle met pilot induced oscillations during its approach and landing tests. Nasa technical reports furnishes well documented details on the command channel as well as possible improvements in its technical memoranda.[11,12] In order to evaluate the susceptibility to such aircraft - pilot couplings, several methodologies exist. A direct attempt consists in applying forced oscillations with various amplitudes and pulsations.

1.1 Forced oscillations

In order to evaluate the susceptibility of the space shuttle command channel, forced oscillations may be applied as pilot stick inputs like described in figure 1. Depending on the reaction of the aircraft to these direct solicitations, it is already possible to have an idea of the potential issues and to their hazardousness.

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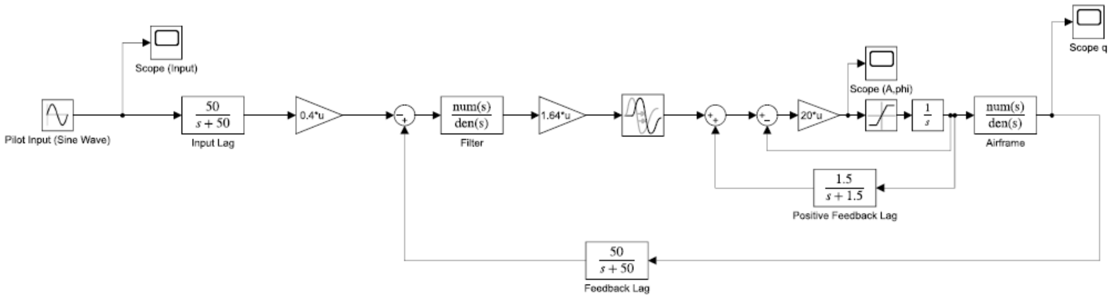

Figure 1: Command channel of the space shuttle.

Since the source of pilot induced oscillations is a rate limited actuator, a nonlinear analysis is required (linear considerations cannot catch such behaviours). The describing function method is here employed, thus we try to find solutions to equation 1 thanks to a pseudo-arclength continuation.[2,8]

$$Ae^{j\phi} = \frac{Filter(j\omega)}{1 - N(A,\omega)Actuator(j\omega)Airframe(j\omega)Feedback(j\omega)Filter(j\omega)}Feedforward(j\omega)Input \tag{1}$$

The results (for the worst case) with a time delay $T = 0.4\ s$ and an input amplitude of 15° are shown in figure 2.

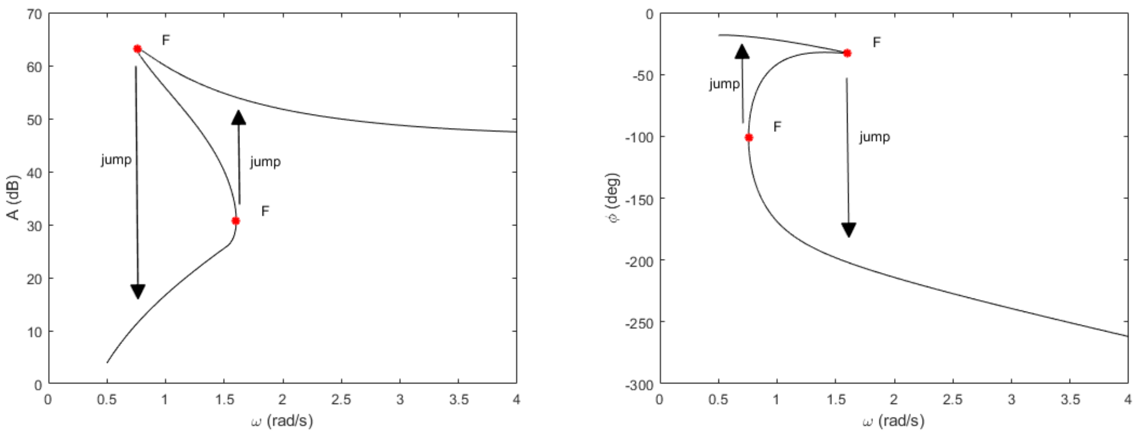

Figure 2: Amplitude A and phase ϕ at actuator entry in function of the pulsation ω of the pilot inputs according to DFM. Diagnosis of jumps due to a rate limited actuator in the space shuttle command channel.

Some bifurcations of periodical orbits are diagnosed and are responsible for sudden jumps. From the flight dynamics viewpoint, these last ones may take the pilot by surprise and lead to an accident. Another issue is that once the branch with higher amplitudes is reached by a little change of input frequency, a huge decrease in the frequency of the inputs is necessary so as to jump back to the branch with lower amplitudes. This hysteresis phenomenon may render the situation quite difficult to tackle.

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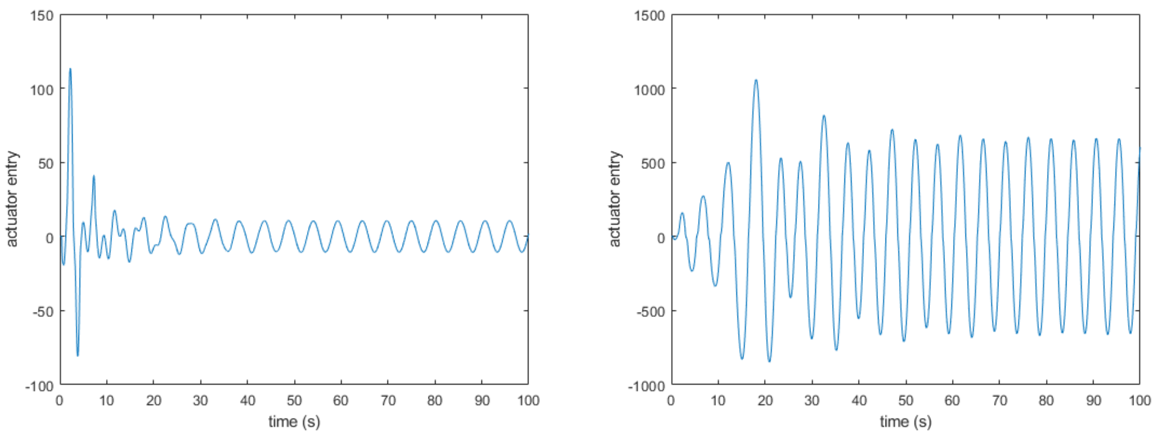

Figure 3: Time simulations at actuator entry for pilot inputs of amplitude 15° and of pulsation $\omega = 1.2\ rad/s$ (left) and $\omega = 1.3\ rad/s$ (right). Stabilization to the branch of low amplitude or to the one of high amplitude.

In figure 3, the time simulations realized for different input pulsations ($\omega = 1.2\ rad/s$ and $\omega = 1.3\ rad/s$) are converging towards the limit cycle of low amplitude or high amplitude. If this jump occurs, the pilot may be astonished. But the time simulations show that once a second limit cycle exists, the switch to the high amplitudes can happen anytime. This may create a dangerous situation and a linear analysis is far from predicting such an unpleasant occurrence.

Other methodologies may also be employed in order to analyze the behaviour. Some fresh ideas are brought by[4] with (equivalent) nonlinear bode diagrams. Besides continuation of limit cycles is a classical attempt to use bifurcation theory.[2] Nevertheless since fold bifurcations of limit cycles are the only critical elements, employing a sinusoid with a sweeping frequency (increasing and decreasing) as input seems to be enough in order to catch the two branches of periodical orbits and to have an idea on the underlying dynamics and especially the fold bifurcation occurrences.

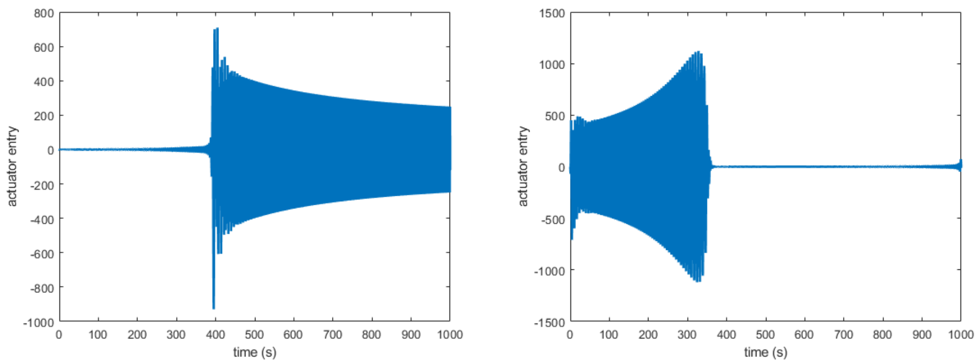

Figure 4: Time simulations at actuator entry with an increasing (left) and decreasing (right) pulsation ω of the pilot inputs. Diagnosis of jumps in the space shuttle command channel response.

On the time simulations of the figure 4, the amplitudes at the entry of the actuator saturation are suddenly jumping at frequencies whose values correspond approximately to the ones coming from the analysis with the describing function method. The inputs are sinusoids with a sweeping frequency from 0.2 rad/s to 2 rad/s (increasing and decreasing on 1000 s).

After accomplishing an analysis and diagnosis of pilot induced oscillations, we will try next to attenuate or suppressing this issues thanks to anti-windup techniques.

1.2 Anti-windup

When trying to cope with PIO problems due to rate limiting, the most direct way consists in adapting the sensitivity of the stick and thus to adapt the shaping function. A first proposal is made at the end of the technical memorandum[11]

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and another one is exposed in the report.[12] Moreover, it is also possible to add a feedback block so as to re-use the PID tuning know how.

For example, when adding a PID block before the lead-lag filter, we may hope to change sufficiently the dynamics by a fine tuning so as to modify the reaction to strong solicitations in the stick. In the next configuration a PID under the form $PID(s) = k_p + \frac{k_i}{s} + k_d \frac{s}{T_f s+1}$ is selected. A comparison is made between the nominal configuration and the one with this modification for forced oscillations of amplitude 5 deg as input (and a time delay of $0.5s$) and is illustrated in the figures 5.

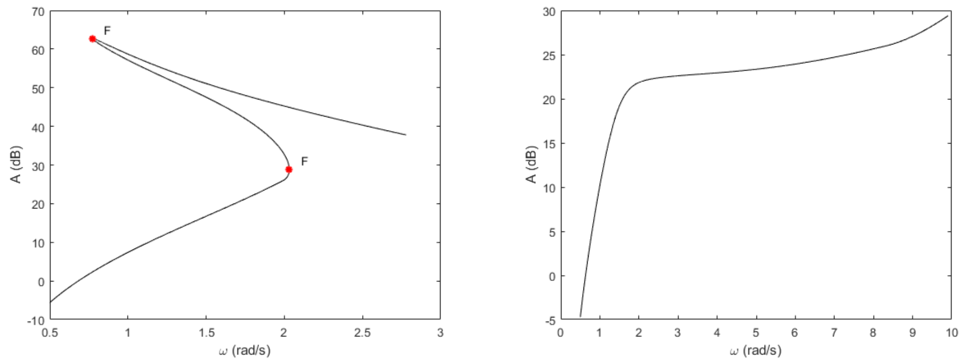

Figure 5: Influence of a PID in a command channel without (left) or with it (right). Presence or absence of jumps for the space shuttle command channel with forced inputs as shown by the diagrams presenting amplitude A (of the periodical orbit) in function of pulsation ω.

A direct anti-windup strategy consists in measuring the difference of the input and the output of the rate-limited element, than to filter the signal with a low-pass filter and to apply the little correction at the entry like described in the modified command channel 6. The effect can be direct and are illustrated in the time simulations 7.

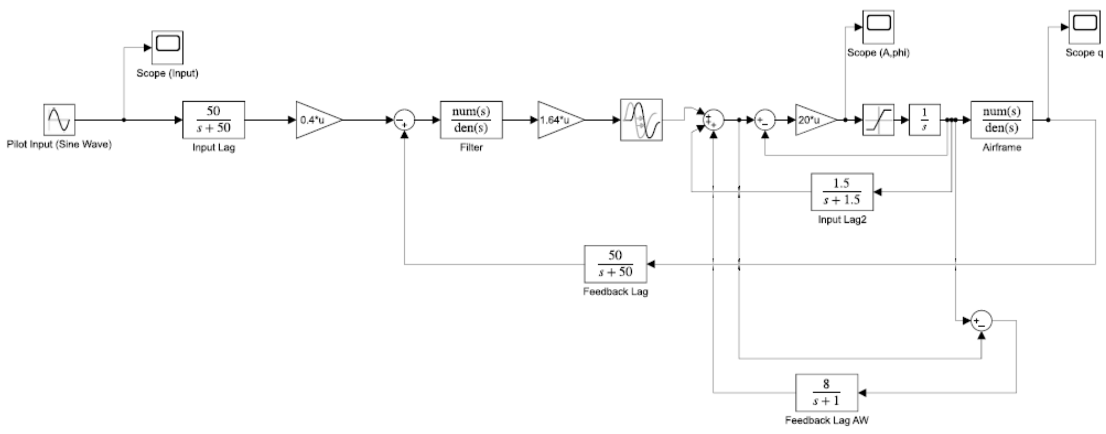

Figure 6: Low-pass filter correction applied to a command channel and taking into account the diffence between the input and the output of the rate-limited actuator.

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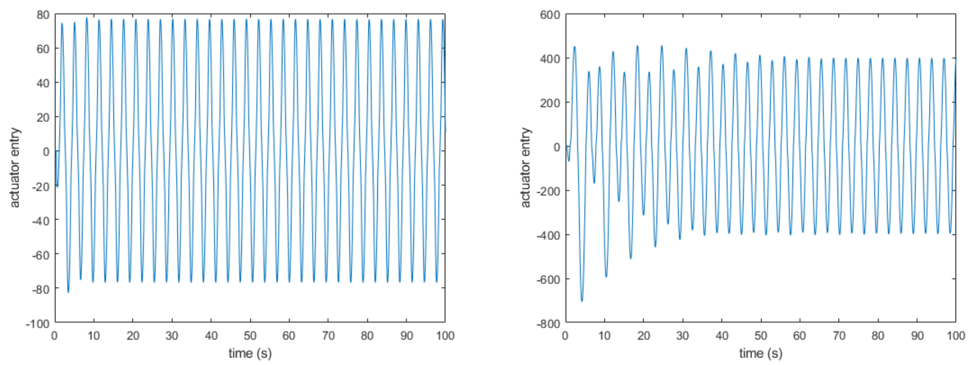

Figure 7: Influence of a low-pass filter correction. Time simulations for an input of amplitude 15° and pulsation $\omega = 2\ rad/s$. Disappearance of jumps with the feedback correction (left) in comparison with the original command channel (right).

By adapting equation 1 so as to employ the describing function method and a continuation algorithm, it is possible to determine the (first harmonic) characteristics of the periodical orbits at the entry of the actuator saturation in function of the input pulsation for this specific case with a sole feedback loop.

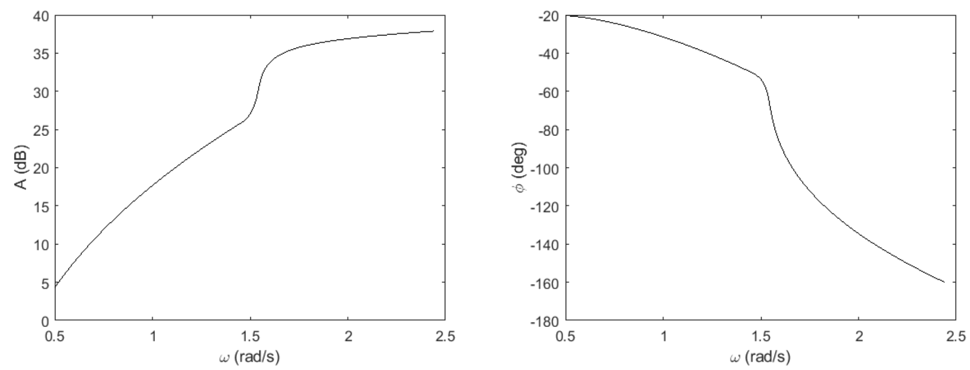

Figure 8: Diagrams presenting amplitude A and phase ϕ (towards forced oscillations inputs) of the periodical orbit at actuator entry in function of pulsation ω for the pitch attitude control system with a low-pass filter correction according to DFM.

This last simple anti-windup technique has bad reputation for the high frequencies and is thus often extended with a bypass.[10] But here it seems to offer sufficient effects to tackle this issue.

The interest of these nonlinear analysis tools are clearly shown, since they permit to know in advance whether a configuration might be hazardous or not toward PIO or PIO-like. Once the methodology and the numerical tools are adapted and implemented, it is possible to compute systematically the different control systems, the limitations (numerical values or types of command channels) being known in advance by experience.

The first topic of this study dealt with the pilot induced oscillations of the space shuttle. An analysis was accomplished and strategies to cope with it were evoked. The next part of this study is now devoted to soaring in an environment with a specific aerology.

2. Sailplane flight in an environment with vertical wind

Several strategies are explored in order to obtain the best performances for a sailplane facing an environment with some thermals during a longitudinal flight. The classical method is first described for a unique thermal before trying to optimize the flight in more general wind configurations.

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2.1 MacCready speed-to-fly

When competing in the presence of an (ascending) thermal, it is possible to choose an optimal airspeed, the so-called MacCready speed as exposed in textbooks[3,9] on sailplane. It aims at achieving cross-country flight as quickly as possible. During the first phase of the flight in calm air, the glider flies with a faster airspeed and may thus loose some excessive altitude but in the thermal, it is possible to return to the initial altitude in a reasonable amount of time. This permits to develop an optimal strategy in the context of a competition.

Graphically the speed-to-fly can be read thanks to a tangent line to the speed polar curve crossing the point $(0, V_{Z_1})$ where V_{Z_1} is the estimated vertical speed of the glider in the thermal.

As mathematical foundation to this theory with simplified hypotheses (no time lost in stabilizing the glider in the thermal for example), the demonstration leads to the equality of the ratio

$$\frac{V_{mean}}{V_a} = \frac{V_{Z_1}}{V_{Z_1} - V_Z\left(V_a\right)} \tag{2}$$

where $V_{mean} = \frac{D}{T_{total}}$ is the mean speed required to perform the overall flight that is to say the distance (to the thermal) divided by the total amount of time T_{total} which englobes a gliding phase in still air and an ascending phase in the thermal. Browsing all the points of the speed polar $V_Z\left(V_a\right)$ allows the optimisation of V_{mean} and leads to the best airspeed $V_a = V_{speed-to-fly}$.

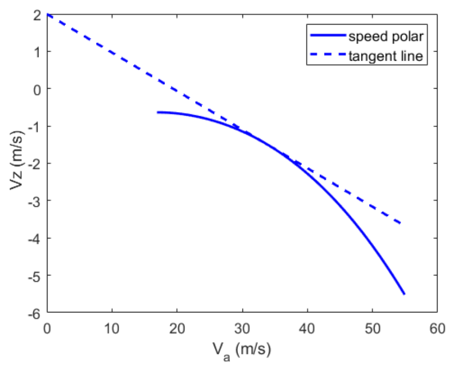

Figure 9: Speed polar of the Slingsby Dart T51 (calculated) and tangent line for the determination of the MacCready speed-to-fly.

The figure 9 plots a speed polar calculated (with data taken in flight dynamics literature[1]) and its tangent line passing through $(0, V_{Z_1})$. The ascending speed in the thermal is here equal to $V_{Z_1} = 2m/s$ and the speed-to-fly is calculated to be $V_{speed-to-fly} = 33.7m/s$.

After presenting and employing a standard way to tackle soaring with thermals, a more mathematical approach is employed and uses optimal control theory. This will be the next evoked subject.

2.2 Optimal Control

Soaring in a environment with a certain wind distribution can be treated thanks to specific mathematical tools. The model and some results exploiting optimal control theory can be found in[6,7] and are valuable since writing the equations and formulating the optimization problem correctly is important in order to be able to employ successfully such tools. For optimal control calculations, the code ICLOCS2[5] is here employed. Aircraft data are here the ones of the Slingsby Dart T51 (taken in[1]). But identification of the speed polar in the flight manual may be possible for other gliders. A polynomial model of reduced order (2 for the drag polar and 3 for the speed polar) may be sufficient even if at the highest speed, the error may sometimes begin to be significant.

The equations 6 describe the aircraft motion in the aerodynamic frame $\mathcal{R}_b$ and include the vertical wind distribution W_h.

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$$\frac{dh}{dt} = V_a \sin\gamma + W_h \tag{3}$$
$$\frac{dV_a}{dt} = -\frac{\rho V_a^2 C_X S}{2m} - \left(V_a \cos\gamma \frac{dW_h}{dX}\frac{dX}{dt} + g\right)\sin\gamma \tag{4}$$
$$\frac{d\gamma}{dt} = \frac{\rho V_a C_Z S}{2m} - \left[\cos\gamma \frac{dW_h}{dX}\frac{dX}{dt} + \frac{g}{V_a}\right]\cos\gamma \tag{5}$$
$$\tag{6}$$

Concretely, re-writing all the time derivatives with distance derivatives seems to simplify greatly the computation process. That's why we will use the following equations for the numerical resolution, exploiting the fact that $\frac{dx}{dt} = V\cos\gamma$.

$$\frac{dh}{dx} = \tan\gamma + \frac{W_h(x)}{V_a\cos\gamma} \tag{7}$$
$$\frac{dV_a}{dx} = \left(-\frac{\rho V_a^2 C_X S}{2m} - g\sin\gamma\right)/\left(V_a\cos\gamma\right) - \frac{dW_h(x)}{dx}\sin\gamma \tag{8}$$
$$\frac{d\gamma}{dx} = \left(\left(\frac{\rho V_a C_Z S}{2m}\right) - \frac{g}{V_a}\cos\gamma\right)/\left(V_a\cos\gamma\right) - \left(\cos\gamma\frac{dW_h(x)}{dx}\right)/V_a \tag{9}$$

When the objective consists in flying as far as possible, the performance to optimize (altitude loss in fact) can be re-written

$$J = \int_0^{t_f} (W_h + V\sin\gamma)\,dt = \int_0^{x_f}\left(\frac{W_h + V\sin\gamma}{V\cos\gamma}\right)dx \tag{10}$$

A calculation is performed in the case of two vertical thermals of different ascending airspeeds. Here the ascending speeds of the two thermals are equal to $2m/s$ and $3m/s$ (respectively between 20 km and 25 km and between 40 km and 45 km). The speed to adopt by the glider is plotted in figure 10 and also the corresponding altitude during the flight. As constraint, the lift aerodynamic coefficient cannot exceed its maximum value $C_{Z_{max}}$.

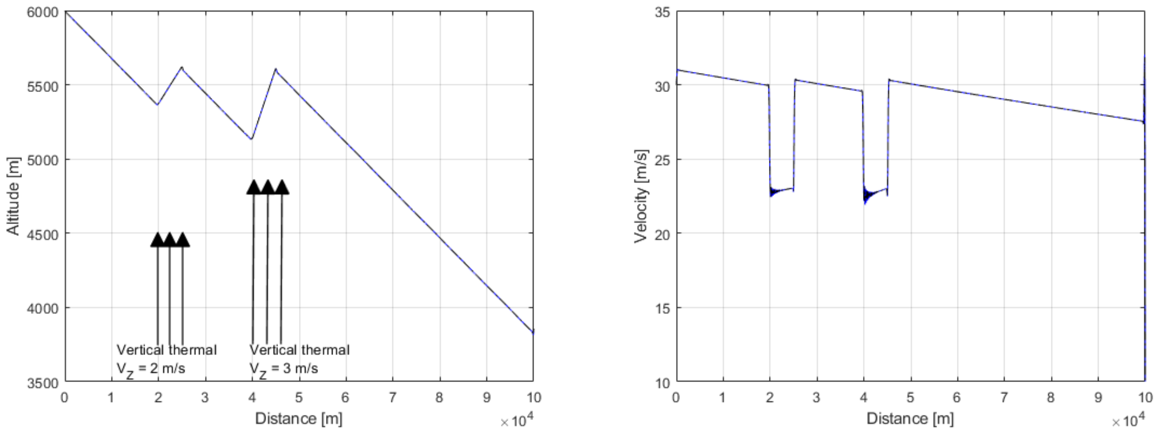

Figure 10: Best speeds to adopt in order to minimize the altitude loss of a glider in a calm environment with two stationary ascending thermals. Results obtained thanks to optimal control of dolphin soaring.

Another performance to optimize may be the time needed so as to cover a certain range. The mathematical objective is here the following one.

$$J = \int_0^{t_f} dt = \int_0^{x_f} \frac{1}{V\cos\gamma}dx \tag{11}$$

As constraints, the lift aerodynamic coefficient cannot exceed its maximum value $C_{Z_{max}}$ and the certified maximum airspeed implies the existence of a minimal value $C_{Z_{min}}$. With the same aerology as before, calculations based on optimal control are accomplished for two ranges of 50 km (with a final altitude of 5000 m) and 100 km (with a final

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altitude of 1000 m). According to figures 11, the velocities to adopt proves to be different.

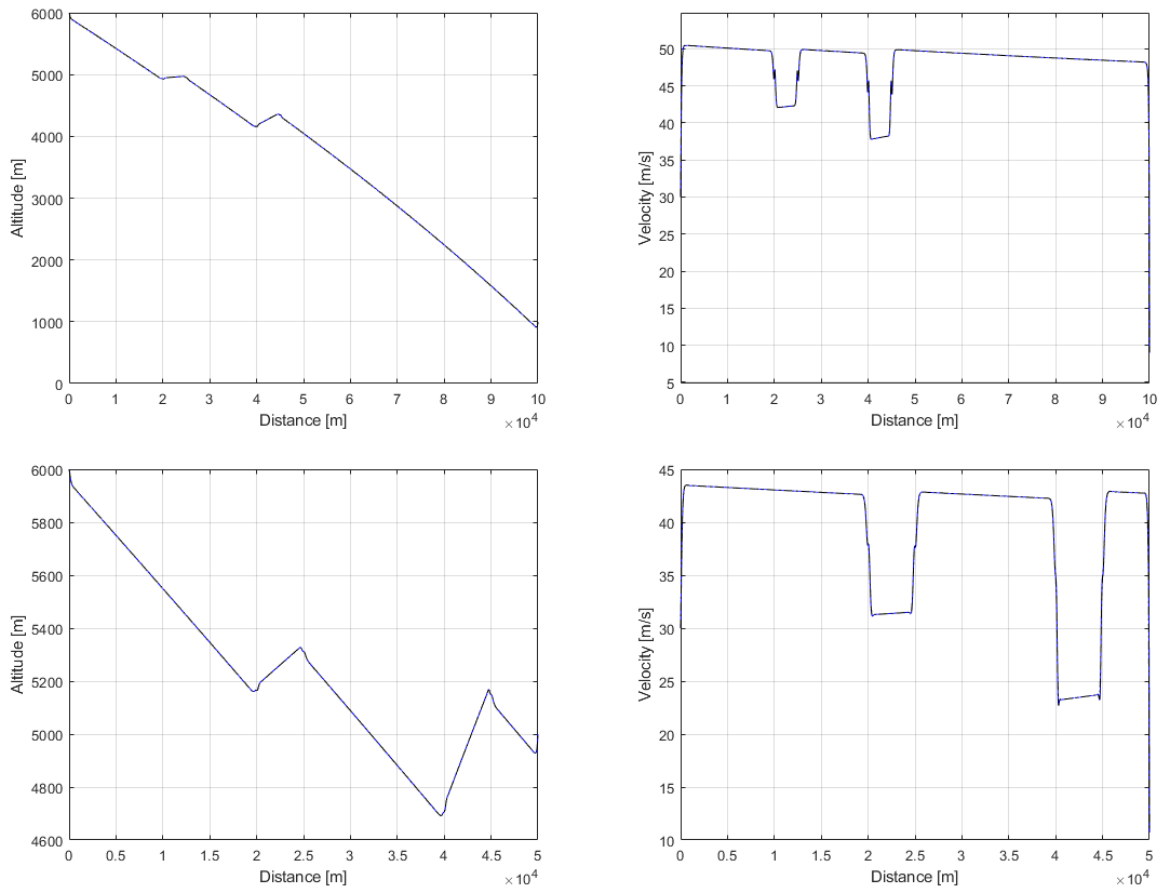

Figure 11: Best speeds to adopt in order to minimize the time needed to cover a certain range for a glider in a calm environment with two stationary ascending thermals. Results obtained thanks to optimal control of dolphin soaring for two ranges of 50 km (bottom) and 100 km (top).

After addressing the classical issue of minimizing the time and the altitude loss when flying in an aerology containing some fixed thermals, it is also possible to examine other situations which may also happen.

2.3 A more general vertical wind profile

The distribution of the wind profile for which the gliding is optimized can be more general. The practical knowledge of the aerology comes most of times from an estimation of the ascending speed in the forecoming thermals by observing the other gliders or by collecting information on them. It can also be the experience of the pilot who has an idea of the properties of a thermal under a cloud for example. Here another situation is examined. The wind profile may be estimated by other means (like computation). The mathematical frame remains described by equations 6 and 10 but the wind profile W_h is more general (it depends just on the position).

For example, in the case of a sinusoidal wind profile of periodic length 5000 m and of amplitude 2 m/s, the optimal trajectory is computed. The figures 12 present the airspeed to follow and the altitude in function of the distance covered.

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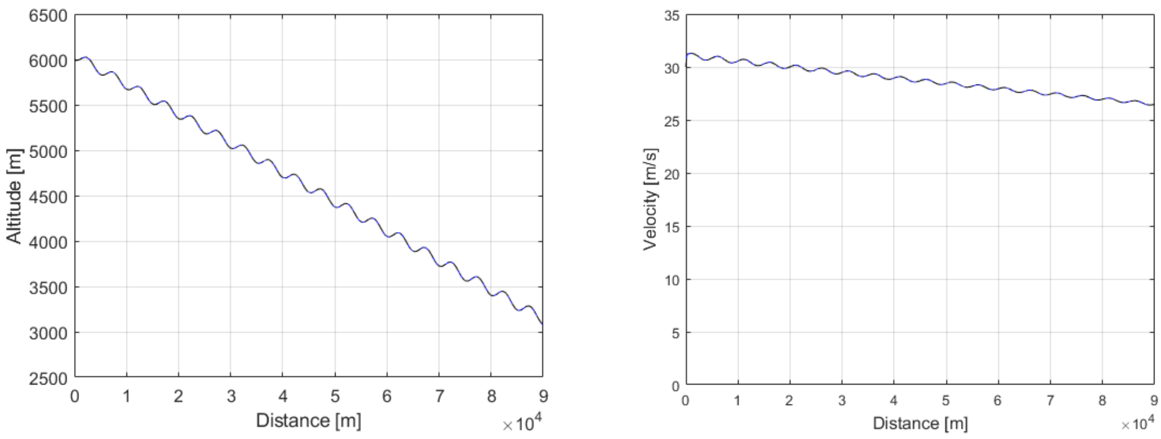

Figure 12: Best speeds to adopt in order to minimize the altitude loss of a glider in an environment with sinusoidal ascending wind. Results obtained thanks to optimal control of dolphin soaring.

The indications and concrete optimal values are interesting, but such an optimal trajectory is maybe more difficult to follow as far as sailplane piloting is concerned.

3. Conclusion

In this study of gliding phases, two main topics were dealt with. An analysis of the pilot induced oscillations of the space shuttle due to rate limiting was first accomplished thanks to the describing function method and the associated curves were computed by means of a continuation algorithm. It revealed to be a pertinent method to diagnose the hazardousness of a configuration before and after potential modifications of the command channel. The practical experience consisted in applying direct oscillatory inputs and observing (or not) some jumps corresponding to handling qualities cliffs. Several anti-windup strategies were tested in order to cope with this issue. The other topic focused on the trajectory optimization of a sailplane with a distribution of vertical wind. After describing the mathematical model employed, some practical configurations, situations were considered with isolated thermals or with a periodic wind distribution. The loss of altitude was optimized but other performances such as the time needed could be sought as well.

In both parts, mathematical treatments allowed to determine valuable results and pieces of information which can help pilots to know how to react adequately or in an optimal manner. In the design process, engineers can also apply these numerical tools to fix in advance potential issues or to estimate optimal solutions.

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Nomenclature

α	angle-of-attack (AoA)
δ_e	elevator angle (pitch control)
γ	flight-path angle
ρ	air density
θ	pitch angle
c	chord
g	Earth's gravitational acceleration
h	altitude
m	mass
q	pitch rate
I_{YY}	pitching moment of inertia
C_X	aerodynamic coefficient of drag
C_Z	aerodynamic coefficient of lift
Cm	aerodynamic coefficient of pitching moment
R_X	drag force
R_Z	lift force
$\mathcal{R}_a$	aerodynamic frame
$\mathcal{R}_b$	body-fixed frame
$\mathcal{R}_0$	Earth frame
S	(main wing) reference area
V_a	airspeed
W_h	vertical wind distribution

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