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An Indirect Estimation Approach of Control Parameters for Underactuated Spacecraft Orbital Pursuit-evasion Game

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Abstract

This article presents an incomplete information game strategy for underactuated spacecraft pursuit-evasion problem in the absence of along-track control. Based on the underactuated dynamics, the linear constraint of underactuated players subject to the uncontrollable state variable is discussed. By using differential game theory, the underactuated saddle-point strategy pair is derived. Note that the underactuated players can also complete the three-dimensional orbital pursuit-evasion game. An indirect estimation approach is developed to calculate the control parameters of the evader to derive an incomplete information game strategy. Numerical simulations also verify the validity of the proposed underactuated game strategy.

1. Introduction

Recently, the spacecraft pursuit-evasion game has been widely used in space applications such as on-orbit servicing, avoiding runaway satellites or space debris, etc.[1–4] Traditional trajectory tracking control approaches are only suitable for one-sided optimization problems,[5,6] while the pursuit-evasion game is modeled as a two-sided optimization problem.[7,8] The differential game proposed by Isaacs[9] is an effective method to solve the pursuit-evasion game, which is employed in scenarios such as spacecraft orbital pursuit-evasion game,[1] missile interception,[10] and multi-drone cluster control.[8]

Whether in circular or elliptic orbits, the orbital pursuit-evasion problem between two spacecraft is conventionally modeled as a zero-sum differential game in which both sides try to find optimal control sequences for minimizing the objective function. Linear quadratic differential games are similar to linear quadratic regulators in that the objective function is associated with interception time, relative distance, and game cost indicators.[11–13] Moreover, solving the differential game is also a challenge.[14] One way is to transform the two-sided optimization problem into a two-point boundary-value problem to yield an accurate saddle-point solution.[15] However, this class of methods can only be used for linear dynamics. Numerical optimization algorithms are applicable to solve nonlinear games, but the optimality of the solutions cannot be guaranteed.[16] Besides the two-player game, there may be more than one player in the game. Ye et al.[17] transformed the three-spacecraft game into the two-spacecraft game with two phases so that the pursuer spacecraft can intercept the evader without being counter-intercepted by the defender spacecraft.

Indeed, the incomplete information game aligns more with the real scenario. In this sense, it is impossible for the pursuer to access the evader's motion state and control parameters due to the non-cooperative nature of the game.[18,19] In practical applications, it has been proven to be a feasible approach to estimate the motion state of the opponent by the Kalman filter.[18,19] For the control parameters, Ye et al.[18] introduced a multiple model adaptive approach to find the control parameter and game strategy of the evader spacecraft with a group of filters running in parallel. Also, Li et al.[19] took the weight matrix as an augmented state vector and then estimated the true value of the control parameter by an extended Kalman filter. It is clear that the closer the initial value of the estimated control parameter is to the true value, the better the convergence of the filter.

Notably, the outlined game strategies are derived from the fully-actuated dynamical equation. To the best of our knowledge, seldom work has been reported on the underactuated spacecraft game. In spacecraft rendezvous and

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formation flying applications, the relative orbital control between spacecraft has been investigated in the absence of radial or along-track thrust.[20–27]

For the case without along-track control, Godard et al.[20] stated that there is an uncontrollable eigenvalue in the linearized dynamical equation. Huang et al.[24–26] further derived that the uncontrollable state variable is equivalent to the stability condition of natural formations. In the literature,[27] we gave a new underactuated control scheme to obtain a smaller system stable reconfiguration accuracy than the previous results. Therefore, the underactuated spacecraft pursuit-evasion game is also an interesting issue.

Inspired by the preceding discussion, the enhancements and contributions of this paper are twofold.

(i) Different from the fully-actuated game strategy, the differential game theory is applied to derive the underactuated spacecraft game strategy in the absence of along-track thrust to achieve the three-dimensional orbital pursuit-evasion game.

(ii) An indirect estimation approach is proposed to calculate the opponent's control parameter by estimating the control acceleration, based on which the incomplete information game strategy is derived.

The remainder of the paper is structured as follows. Section 2 analyzes the controllability and feasibility of the underactuated pursuit-evasion game. Section 3 derives the underactuated game strategies with complete and incomplete information. Section 4 presents the simulations, and the remarking conclusions are given in Section 5.

2. Dynamics and problem statement

2.1 Game dynamics

For the short-range orbital pursuit-evasion game, the relative dynamics is typically constructed within the local-vertical-local-horizontal frame. In Fig. 1, a leader flies in a circular orbit surrounded by a pursuer and an evader. In the O_Lxyz coordinate, O_Lx-axis is along the vector O_CO_P, O_Lz-axis is along the positive direction of the orbital angular momentum vector of the leader, and O_Ly-axis is defined by the right-handed Cartesian frame. Define $\boldsymbol{\rho}_i = [x_i\ y_i\ z_i]^{\mathrm{T}}$ and $\boldsymbol{v}_i = [\dot{x}_i\ \dot{y}_i\ \dot{z}_i]^{\mathrm{T}}$ as the position and velocity of the player i relative to the leader spacecraft, where the subscript $i = P$ or E denotes the pursuer or the evader. For the underactuated case without along-track control, the dynamics of pursuit-evasion games can be derived as

$$\ddot{\boldsymbol{\rho}}_i = \boldsymbol{F}_i(\boldsymbol{\rho}_i, \boldsymbol{v}_i) + \hat{\boldsymbol{u}}_i \tag{1}$$

with

$$\boldsymbol{F}_i = \begin{bmatrix} 2\dot{\theta}_L\dot{y}_i + \dot{\theta}_L^2 x_i + \ddot{\theta}_L y_i + n_0^2 R_L - n_i^2(R_L + x_i) \\ -2\dot{\theta}_L\dot{x}_i + \dot{\theta}_L^2 y_i - \ddot{\theta}_L x_i - n_i^2 y_i \\ -n_i^2 z_i \end{bmatrix}$$

where $\hat{\boldsymbol{u}}_i = [u_{ix}\ 0\ u_{iz}]^{\mathrm{T}}$ is the control acceleration for the player $i = P$ or E, the terms u_{ix} and u_{iz} are, respectively, the the radial and normal accelerations. $n_0 = (\mu_E/R_L^3)^{1/2}$ and $n_i = (\mu_E/R_i^3)^{1/2}$ are, respectively, the mean angular velocity of the leader and the player, while μ_E denotes the gravitational constant of Earth. θ_L is the argument of latitude of the leader spacecraft, which satisfies $\dot{\theta}_L = n_0$ and $\ddot{\theta}_L = 0$ in circular orbits.

Since the relative distance between the player i and the leader is considerably shorter than their orbital radius, Eq. (1) is conventionally linearized as

$$\dot{\boldsymbol{X}}_i = \boldsymbol{A}\boldsymbol{X}_i + \boldsymbol{B}\boldsymbol{u}_i \tag{2}$$

with

$$\boldsymbol{A} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 3n_0^2 & 0 & 0 & 0 & 2n_0 & 0 \\ 0 & 0 & 0 & -2n_0 & 0 & 0 \\ 0 & 0 & -n_0^2 & 0 & 0 & 0 \end{bmatrix} \tag{3}$$

$$\boldsymbol{B} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}^{\mathrm{T}} \tag{4}$$

where $\boldsymbol{u}_i = [u_{ix}\ u_{iz}]^{\mathrm{T}}$.

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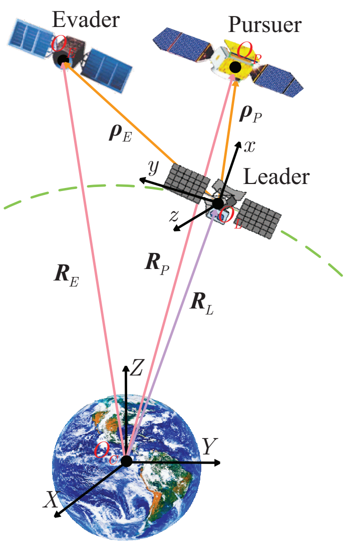

Figure 1: Coordinate frames of the pursuit-evasion game.

2.2 Controllability and feasibility analysis

By using the linear system theory,[27] the rank of the controllability matrix for the case without along-track thrust is $\text{rank}(\boldsymbol{B}, \boldsymbol{AB}, \ldots, \boldsymbol{A}^5\boldsymbol{B}) = 5 < 6$, namely, the control pair $(\boldsymbol{A}, \boldsymbol{B})$ is uncontrollable. Moreover, the uncontrollable system can be decomposed into a controllable variable $\bar{\boldsymbol{X}}_{ic} = [\dot{y}_i\ z_i\ y_i\ \dot{x}_i\ \dot{z}_i]^{\mathrm{T}}$ and an uncontrollable one $\bar{X}_{iu} = \dot{y}_i/2n_0 + x_i$.

$$\dot{\bar{\boldsymbol{X}}}_{ic} = \bar{\boldsymbol{A}}_{ic}\bar{\boldsymbol{X}}_{ic} + \bar{\boldsymbol{A}}_{iu}\bar{X}_{iu} + \bar{\boldsymbol{B}}\boldsymbol{u}_i \tag{5}$$

with

$$\bar{\boldsymbol{A}}_{ic} = \begin{bmatrix} 0 & 0 & 0 & -2n_0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ n_0/2 & 0 & 0 & 0 & 0 \\ 0 & -n_0^2 & 0 & 0 & 0 \end{bmatrix}, \ \bar{\boldsymbol{A}}_{iu} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 3n_0^2 \\ 0 \end{bmatrix}, \ \bar{\boldsymbol{B}} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{6}$$

Note that the theoretical condition for the follower spacecraft to fly around the leader in a natural formation around the leader is $\dot{y}_i = -2n_0x_i$.[27] That is to say, despite that the pursuit-evasion game is feasible in the absence of along-track control, the uncontrollable variable of the player i remains at zero throughout the game.

$$\bar{X}_{iu}(t_0) = x_i(t_0) + \dot{y}_i(t_0)/2n_0 = \bar{X}_{iu}(t_f) = x_i(t_f) + \dot{y}_i(t_f)/2n_0 = 0. \tag{7}$$

where t_0 and t_f refer to the initial time and the terminal time, respectively.

2.3 Problem statement

This paper investigates the incomplete information pursuit-evasion game problem for the underactuated spacecraft without along-track control. To be specific, the underactuated game strategy is developed so that the pursuer can capture the evader under the condition that the pursuer does not have access to the accurate information of the opponent.

3. Underactuated spacecraft pursuit-evasion game

Define $\boldsymbol{e} = [\dot{e}_y\ e_z\ e_y\ \dot{e}_x\ \dot{e}_z] = \bar{\boldsymbol{X}}_{Pc} - \bar{\boldsymbol{X}}_{Ec}$ as the error state between the pursuer and the evader. From Eq. (5), the error dynamics of pursuit-evasion games is governed by

$$\dot{\boldsymbol{e}} = \bar{\boldsymbol{A}}_{ic}\boldsymbol{e} + \bar{\boldsymbol{B}}(\boldsymbol{u}_P - \boldsymbol{u}_E) \tag{8}$$

where $\boldsymbol{u}_P = [u_{Px}\ u_{Pz}]^{\mathrm{T}}$ and $\boldsymbol{u}_E = [u_{Ex}\ u_{Ez}]^{\mathrm{T}}$.

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3.1 Complete information game strategy

To ensure the pursuer can capture the evader, we assume that the control acceleration of the pursuer is greater than that of the evader. Meanwhile, the pursuer and the evader can access each other's state information and control parameters in the complete information scenario.

In a zero-sum equilibrium game, the pursuer tries to intercept the evader with minimal cost, while the evader tries to get rid of the pursuer with minimal cost. Thus, the cost function of the pursuit-evasion game is set as

$$\min_{\boldsymbol{u}_P}\max_{\boldsymbol{u}_E} J = \frac{1}{2}\boldsymbol{e}^{\mathrm{T}}\boldsymbol{S}\boldsymbol{e} + \frac{1}{2}\int_{t_0}^{t_f}(\boldsymbol{e}^{\mathrm{T}}\boldsymbol{Q}\boldsymbol{e} + \boldsymbol{u}_P^{\mathrm{T}}\boldsymbol{R}_P\boldsymbol{u}_P - \boldsymbol{u}_E^{\mathrm{T}}\boldsymbol{R}_E\boldsymbol{u}_E)\mathrm{d}t \tag{9}$$

where $\boldsymbol{S}$, $\boldsymbol{R}_P = \mathrm{diag}(r_P,\ r_P)$, and $\boldsymbol{R}_E = \mathrm{diag}(r_E,\ r_E)$ are positive definite matrices. $\boldsymbol{Q}$ is a positive semidefinite matrix.

According to the differential game theory, the underactuated saddle-point strategy pair corresponds to the underactuated game strategy. Firstly, the Hamiltonian function is formulated as

$$\mathcal{H} = \boldsymbol{\lambda}^{\mathrm{T}}(\bar{\boldsymbol{A}}_{ic}\boldsymbol{e} + \bar{\boldsymbol{B}}(\boldsymbol{u}_P - \boldsymbol{u}_E)) + \frac{1}{2}(\boldsymbol{e}^{\mathrm{T}}\boldsymbol{Q}\boldsymbol{e} + \boldsymbol{u}_P^{\mathrm{T}}\boldsymbol{R}_P\boldsymbol{u}_P - \boldsymbol{u}_E^{\mathrm{T}}\boldsymbol{R}_E\boldsymbol{u}_E) \tag{10}$$

where $\boldsymbol{\lambda}$ is the adjoint variable. Then, the terminal condition has the expression of

$$\Phi = \frac{1}{2}\boldsymbol{e}^{\mathrm{T}}\boldsymbol{S}\boldsymbol{e} \tag{11}$$

Notably, the initial error state is set to $\boldsymbol{e}(t_0)$.

Furthermore, the saddle-point solution also follows the stability conditions

$$\begin{cases} \boldsymbol{u}_P = -\boldsymbol{R}_P^{-1}\bar{\boldsymbol{B}}^{\mathrm{T}}\boldsymbol{\lambda} \\ \boldsymbol{u}_E = -\boldsymbol{R}_E^{-1}\bar{\boldsymbol{B}}^{\mathrm{T}}\boldsymbol{\lambda} \end{cases} \tag{12}$$

the adjoint equation,

$$\dot{\boldsymbol{\lambda}} = -\frac{\partial\mathcal{H}}{\partial\boldsymbol{e}} = -(\bar{\boldsymbol{A}}_{ic}^{\mathrm{T}}\boldsymbol{\lambda} + \boldsymbol{Q}\boldsymbol{e}) \tag{13}$$

and the terminal boundary condition

$$\boldsymbol{\lambda}(t_f) = \frac{\partial\Phi}{\partial\boldsymbol{e}} = \boldsymbol{S}\boldsymbol{e}(t_f) \tag{14}$$

In addition, $\boldsymbol{\lambda}$ and $\boldsymbol{e}$ obey the linear feedback form $\boldsymbol{\lambda} = \boldsymbol{P}\boldsymbol{e}$, where $\boldsymbol{P} = \boldsymbol{P}^{\mathrm{T}}$ is a positive definite matrix. In this way, substituting Eq. (8) into the time derivative of $\boldsymbol{\lambda}$ leads to

$$\dot{\boldsymbol{\lambda}} = -\bar{\boldsymbol{A}}_{ic}^{\mathrm{T}}\boldsymbol{\lambda} - \boldsymbol{Q}\boldsymbol{e} \tag{15}$$

Substituting Eq. (12) into Eq. (15) yields a underactuated differential Riccati equation, given by

$$\dot{\boldsymbol{P}} + \bar{\boldsymbol{A}}_{ic}^{\mathrm{T}}\boldsymbol{P} + \boldsymbol{P}\bar{\boldsymbol{A}}_{ic} - \boldsymbol{P}(\bar{\boldsymbol{B}}\boldsymbol{R}_P^{-1}\bar{\boldsymbol{B}}^{\mathrm{T}} - \bar{\boldsymbol{B}}\boldsymbol{R}_E^{-1}\bar{\boldsymbol{B}}^{\mathrm{T}})\boldsymbol{P} + \boldsymbol{Q} = \boldsymbol{0} \tag{16}$$

Thus, the underactuated saddle-point strategy pair is derived as

$$\begin{cases} \boldsymbol{u}_P = -\boldsymbol{R}_P^{-1}\bar{\boldsymbol{B}}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{e} \\ \boldsymbol{u}_E = -\boldsymbol{R}_E^{-1}\bar{\boldsymbol{B}}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{e} \end{cases} \tag{17}$$

The saddle point strategy pair in Eq. (17) is the underactuated game strategy pair. Players will perform the orbital pursuit-evasion game driven by Eq. (17). Since the maximum acceleration of the pursuer is greater than the evader, the pursuer will eventually capture the evader.

It is notable that the game strategy (17) is deduced under the assumption of complete information. Then, the incomplete information game strategy will be given.

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3.2 Incomplete information game strategy

In an incomplete information game, the pursuer cannot access accurate information about the evader. Conversely, to explore the effect of incomplete information on the game strategy, it is assumed that the evader has access to the complete information of the pursuer.

From a practical point of view, the pursuer can measure the opponent's relative position and velocity by using Doppler effect- or laser-based sensors.[28] Also, observer-based approaches can be applied to measure the acceleration of maneuvering targets.[29] Therefore, we assume that the control acceleration of the evader at k time step estimated by the pursuer is $\hat{\boldsymbol{u}}_E = \boldsymbol{u}_E(k) + \boldsymbol{U}$, where $\boldsymbol{u}_E(k)$ is the real acceleration and $\boldsymbol{U}$ is the standard normal distribution $\boldsymbol{U} \sim N(\boldsymbol{0}_{2\times1}, \boldsymbol{J})$, $\boldsymbol{J}$ is a positive definite matrix.

Then, substituting the second term of Eq. (17) into Eq. (8) yields that

$$\dot{\boldsymbol{e}} = \boldsymbol{A}_{PE}\boldsymbol{e} + \boldsymbol{B}\boldsymbol{u}_P \tag{18}$$

where $\boldsymbol{A}_{PE} = \bar{\boldsymbol{A}}_{ic} + \bar{\boldsymbol{B}}(\boldsymbol{R}_E)^{-1}\bar{\boldsymbol{B}}^{\mathrm{T}}\boldsymbol{P}$.

Also, the error state $\boldsymbol{e}$ is generally observed by the following equation.

$$\boldsymbol{Y} = \boldsymbol{I}_{5\times5}\boldsymbol{e} \tag{19}$$

Eqs. (18) and (19) have the discrete-time expressions, given by

$$\begin{cases} \boldsymbol{e}(k) = \boldsymbol{\Phi}(k/k-1)\boldsymbol{e}(k-1) + \bar{\boldsymbol{B}}\boldsymbol{u}_P(k-1) + \boldsymbol{W}(k-1) \\ \boldsymbol{Y}(k) = \boldsymbol{I}_{5\times5}(k)\boldsymbol{e}(k) + \boldsymbol{V}(k) \end{cases} \tag{20}$$

where $\boldsymbol{W}(k-1) \sim N(\boldsymbol{0}_{5\times1},\ \boldsymbol{G})$ and $\boldsymbol{V}(k) \sim N(\boldsymbol{0}_{5\times1},\ \boldsymbol{M})$ are Gaussian noises. $\boldsymbol{G}$ and $\boldsymbol{M}$ are positive definite matrices. $\boldsymbol{\Phi}_h(k+1/k)$ is the state transition matrix, formulated by

$$\boldsymbol{\Phi}(k/k-1) = e^{-\boldsymbol{A}_{PE}T_g} \tag{21}$$

where T_{PE} indicates the sampling time.

The Kalman filter is used to estimate the discrete-time system (20).

$$\begin{cases} \hat{\boldsymbol{e}}(k/k-1) = \boldsymbol{\Phi}(k/k-1)\hat{\boldsymbol{e}}(k-1) + \bar{\boldsymbol{B}}\boldsymbol{u}_P(k-1) \\ \hat{\boldsymbol{e}}(k) = \boldsymbol{K}(k)(\boldsymbol{Y}(k) - \boldsymbol{H}(k)\hat{\boldsymbol{e}}(k/k-1)) + \hat{\boldsymbol{e}}(k/k-1) \\ \boldsymbol{K}(k) = \boldsymbol{F}(k/k-1)\boldsymbol{H}^{\mathrm{T}}(k)(\boldsymbol{H}(k)\boldsymbol{F}(k/k-1)\boldsymbol{H}_h^{\mathrm{T}}(k) + \boldsymbol{M}(k))^{-1} \\ \boldsymbol{F}(k/k-1) = \boldsymbol{M}(k-1) + \boldsymbol{\Phi}(k/k-1)\boldsymbol{F}(k-1)\boldsymbol{\Phi}^{\mathrm{T}}(k/k-1) \\ \boldsymbol{F}(k) = \boldsymbol{K}(k)\boldsymbol{M}(k)\boldsymbol{K}^{\mathrm{T}}(k) + (\boldsymbol{I}_{5\times5} - \boldsymbol{K}(k)\boldsymbol{H}(k))\boldsymbol{F}(k/k-1)(\boldsymbol{I}_{5\times5} - \boldsymbol{K}(k)\boldsymbol{H}(k))^{\mathrm{T}} \end{cases} \tag{22}$$

where $\boldsymbol{K}(k)$, $\boldsymbol{F}(k)$, and $\boldsymbol{F}(k/k-1)$ are positive definite matrices.

As can be seen, the control parameter $\boldsymbol{R}_E$ of the evader should be given to estimate the error state. Here, an indirect estimation approach of the control parameter $\boldsymbol{R}_E$ for spacecraft pursuit-evasion game is developed.

Step 1: $\hat{\boldsymbol{e}}(k)$ is transformed into $\hat{\boldsymbol{e}}'(k) = \boldsymbol{I}_{5\times5}\hat{\boldsymbol{e}}(k)$.

Step 2: Eq. (16) multiplied by $\hat{\boldsymbol{e}}'(k)$ yields that

$$\dot{\boldsymbol{P}}\hat{\boldsymbol{e}}'(k) + \bar{\boldsymbol{A}}_{ic}^{\mathrm{T}}\boldsymbol{P}\hat{\boldsymbol{e}}'(k) + \boldsymbol{P}\bar{\boldsymbol{A}}_{ic}\hat{\boldsymbol{e}}'(k) - \boldsymbol{P}(\bar{\boldsymbol{B}}\boldsymbol{R}_P^{-1}\bar{\boldsymbol{B}}^{\mathrm{T}} - \bar{\boldsymbol{B}}\boldsymbol{R}_E^{-1}\bar{\boldsymbol{B}}^{\mathrm{T}})\boldsymbol{P}\hat{\boldsymbol{e}}'(k) + \boldsymbol{Q}\hat{\boldsymbol{e}}'(k) = \boldsymbol{0} \tag{23}$$

Step 3: Provided that $\hat{\boldsymbol{u}}_E(k)$ is given. In this sense, $\boldsymbol{R}_E(k)$ and $\boldsymbol{P}$ are unknown. $\hat{\boldsymbol{u}}_E(k)$ is transformed into

$$\hat{\boldsymbol{u}}'_E(k) = \begin{bmatrix} 0 & 0 & 0 & \hat{u}_{Ex} & 0 \\ 0 & 0 & 0 & 0 & \hat{u}_{Ez} \end{bmatrix} \tag{24}$$

where $\hat{u}_{Ex}$ and $\hat{u}_{Ez}$ are, respectively, the estimated values of u_{Ex} and u_{Ez}.

Step 4: Substituting $\hat{\boldsymbol{u}}_E(k) = -\boldsymbol{R}_E^{-1}(k)\bar{\boldsymbol{B}}^{\mathrm{T}}\boldsymbol{P}\hat{\boldsymbol{e}}(k)$ into Eq. (23) and then multiplied by $\hat{\boldsymbol{e}}'^{-1}(k)$ yields that

$$\dot{\bar{\boldsymbol{P}}} + \bar{\boldsymbol{A}}_{ic}^{\mathrm{T}}\bar{\boldsymbol{P}} + \bar{\boldsymbol{P}}\bar{\boldsymbol{A}}_{ic} + \boldsymbol{Q} - (\bar{\boldsymbol{P}}\bar{\boldsymbol{B}}\boldsymbol{R}_P^{-1}\bar{\boldsymbol{B}}^{\mathrm{T}}\bar{\boldsymbol{P}} - \bar{\boldsymbol{P}}\bar{\boldsymbol{B}}\hat{\boldsymbol{u}}_E(k)\hat{\boldsymbol{e}}^{-1}(k)) = \boldsymbol{0} \tag{25}$$

Step 5: Solving $\bar{\boldsymbol{P}}$ in Eq. (25).

Step 6: Defining $\boldsymbol{R}_E(k) = \mathrm{diag}(\hat{r}_{E1}(k),\ \hat{r}_{E2}(k))$. $\hat{r}_{E1}(k)$ and $\hat{r}_{E2}(k)$ can be found via $\hat{\boldsymbol{u}}_E(k) = -\boldsymbol{R}_E^{-1}(k)\bar{\boldsymbol{B}}^{\mathrm{T}}\bar{\boldsymbol{P}}\hat{\boldsymbol{e}}(k)$.

Therefore, the pursuer's game strategy under incomplete information can be expressed as

$$\bar{\boldsymbol{u}}_P(k) = -\boldsymbol{R}_P^{-1}\bar{\boldsymbol{B}}^{\mathrm{T}}\bar{\boldsymbol{P}}\hat{\boldsymbol{e}}(k) \tag{26}$$

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4. Simulation

The underactuated pursuit-evasion game with incomplete information is simulated in this section. The control parameters for the pursuer and evader are chosen as: $\boldsymbol{Q} = \boldsymbol{S} = \mathrm{diag}(0,\ 1,\ 1,\ 0,\ 0)$, $\boldsymbol{R}_P = \mathrm{diag}(1.1 \times 10^6,\ 1.1 \times 10^6)$, $\boldsymbol{R}_E = \mathrm{diag}(1.6{\times}10^6,\ 1.6{\times}10^6)$, $\boldsymbol{W} = \mathrm{diag}(2.5{\times}10^{-5},\ 10^{-2},\ 10^{-2},\ 2.5{\times}10^{-5},\ 2.5{\times}10^{-5})$, $\boldsymbol{J} = \mathrm{diag}(1{\times}10^{-4},\ 1{\times}10^{-4})$, $\boldsymbol{V} = \mathrm{diag}(5{\times}10^{-4},\ 10^{-3},\ 10^{-3},\ 5{\times}10^{-4},\ 5{\times}10^{-4})$, $\boldsymbol{G} = \boldsymbol{M} = \boldsymbol{F}(0) = \mathrm{diag}(2.5{\times}10^{-7},\ 10^{-6},\ 10^{-6},\ 2.5{\times}10^{-7},\ 2.5{\times}10^{-7})$. Table 1 lists the orbital elements of the leader.[27] Also, the initial position of the pursuer and the evader are, respectively, set to $\boldsymbol{\rho}_P(t_0) = [1500\ 600\ 600]^{\mathrm{T}}$m and $\boldsymbol{\rho}_E(t_0) = [0\ 0\ 0]^{\mathrm{T}}$m. The initial velocity of the pursuer and the evader are, respectively, set to $\boldsymbol{v}_P(t_0) = [-5\ \ -3.5\ 0]^{\mathrm{T}}$m/s and $\boldsymbol{v}_E(t_0) = [0\ 0\ 0]^{\mathrm{T}}$m/s.

Table 1: Orbital elements of the leader

Orbit elements	Value	Unit
Semi-major axis	6878137	m
Eccentricity	0	-
Inclination	42	deg
Right ascension of ascending node	-60	deg
Argument of latitude	30	deg

Fig. 2 depicts the game trajectory with incomplete information, where s_P and s_E refer to the initial position of the pursuer and the evader, respectively. In the absence of along-track thrust, the pursuer also gradually approaches the evader. The time history of the relative distance $\mathbf{e}_d$ is given in Fig. 3. In this paper, the terminal condition of the pursuit-evasion is set to the relative distance close to 100 m. Here, the relative distance was reduced to 100 m after 334 s. On the one hand, the radial thrust can drive the player in the *xy*-plane. On the other hand, the normal relative position converges faster than the two directions. Fig. 4 shows the time histories of the radial and normal control acceleration. As can be seen, the pursuer provides greater control acceleration to approach the evader. Meanwhile, the control accelerations of the two players converge to bounded regions close to zero as the relative distance decreases. To be specific, the radial control acceleration converges to the bounded region 10^{-1} m/s^2 around 300 s, while the normal control acceleration approaches the bounded region 10^{-2} m/s^2 at 200 s.

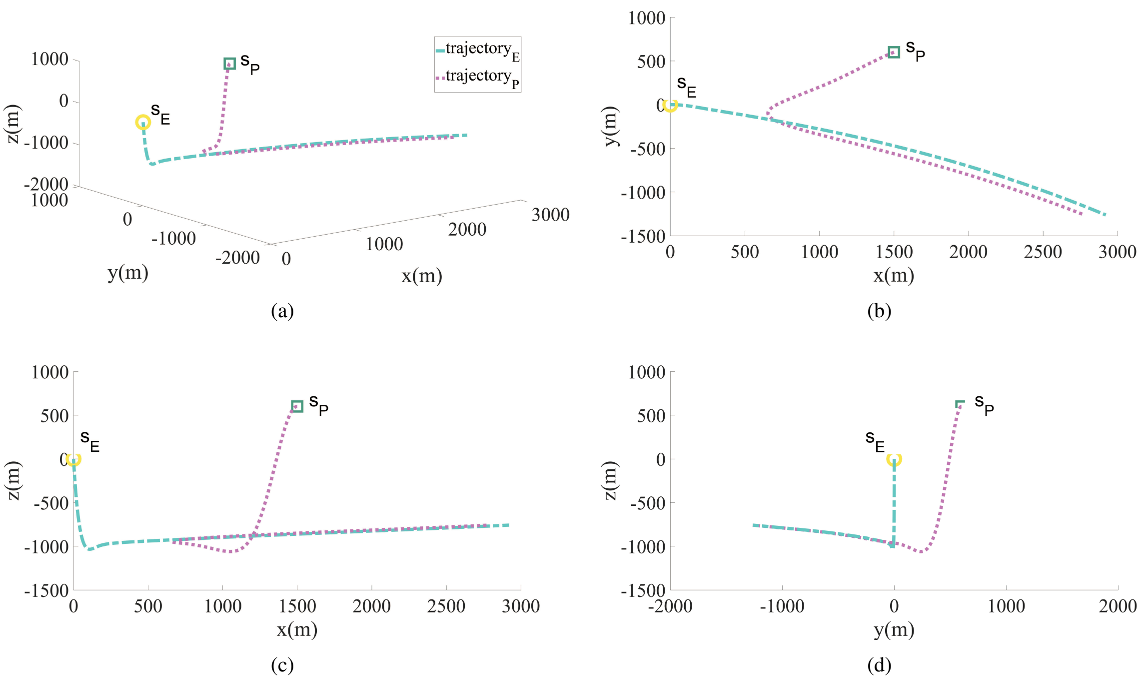

Figure 2: Game trajectories without along-track control. (**a**) Three-dimensional trajectories; (**b**) *xy*-plane; (**c**) *xz*-plane; (**d**) *yz*-plane.

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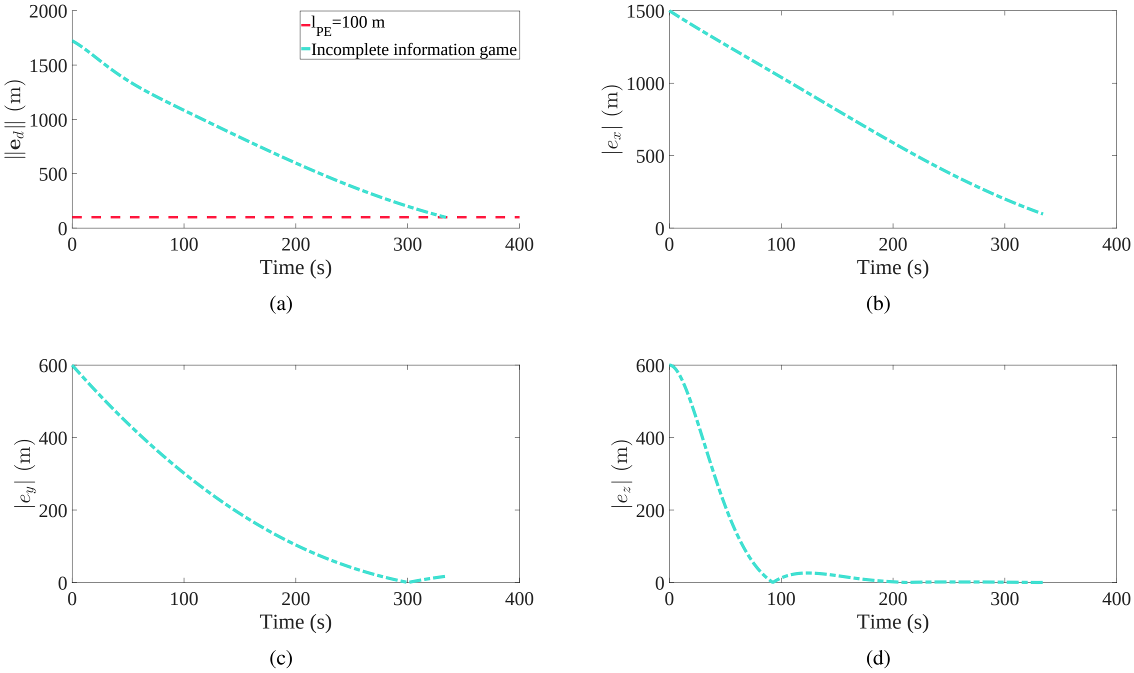

Figure 3: Time histories of relative distances without along-track control. (**a**) Euclidean distances; (**b**) Radial relative distance; (**c**) Along-track relative distance; (**d**) Normal relative distance.

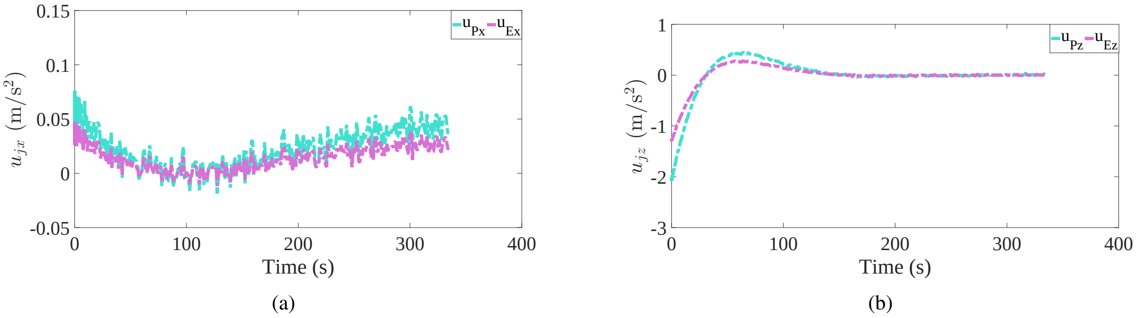

Figure 4: Time histories of control accelerations without along-track control. (**a**) Radial control acceleration with incomplete information; (**b**) Normal control acceleration with incomplete information.

5. Conclusion

The feasibility of underactuated spacecraft pursuit-evasion game in the absence of along-track control is investigated in this paper. Based on differential game theory, the underactuated game strategy is derived to accomplish the three-dimensional orbital game. Considering that the pursuer does not have access to the accurate information of the evader in the actual game scenario, the incomplete information game strategy is derived using the Kalman filter and the indirect estimation approach. The results show that the underactuated game strategy can drive the pursuer to gradually approach the evader. Meanwhile, the control acceleration of the pursuer in the game is greater than that of the evader.

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