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Aerodynamic Parameter Estimation for Fixed-wing UAV Using Iterated Extended Kalman Fliter

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Abstract

Aerodynamic parameter estimation is crucial for creating precise simulation environments and achieving more accurate aircraft dynamic modeling. However, dynamics of the aircraft involve non-linearity, and the presence of noise from measurement sensors further complicates parameter estimation. One of the nonlinear filtering techniques, Extended Kalman Filter (EKF), can filter out the noise and enable parameter estimation. The EKF linearizes nonlinear system using a first-order Taylor series approximation. But, for systems with highly non-linearity, the errors caused by linearization can be significant, making estimation hard. To reduce such linearization errors, methods such as Iterated EKF and Unscented Kalman Filter (UKF) have been proposed. In this paper, the estimation of aerodynamic coefficients for a fixed-wing aircraft is conducted using the previously introduced filters, and their performance is compared. The results show that under the same condition, IEKF provides more accurate parameter estimation.

1. Introduction

Unmanned Aerial Vehicles (UAVs) have widely employed as flight test platforms by applying research on control. When utilizing such platforms, one of the most crucial aspects is how to model the dynamics of UAVs.[2] In aircraft modeling, particularly for the establishment of an accurate model in preparation for flight experiments, aerodynamic coefficients play a important role in creating a sophisticated simulation environment. In order to determine these aerodynamic coefficients, traditional commercial manned aircraft have used methods such as Computational Fluid Dynamics (CFD) or wind tunnel testing. However, these are not suitable for UAVs with relatively short design cycle due to their high cost and computational power requirements.[11] Instead, parameter estimation and system identification techniques that utilize experimental data to estimate aerodynamic coefficients offer a simpler and relatively more accurate alternative to aforementioned methods, making them suitable for application in UAVs.
There are numerous methods available for parameter estimation and system identification. Among them, recursive estimation, which utilizes only the latest sample data without requiring the entire time data, is considered a promising approach for estimating the aerodynamic coefficients of UAVs. Real-time estimation of aerodynamic coefficients is achieved by obtaining data from sensors mounted on UAVs. However, the measured flight data is contaminated with various sources of noise, such as process noise, sensor noise and biases. Hence, in order to estimate the aerodynamic coefficients, the application of noise filtering techniques is also necessary to mitigate the effects of noise. Filtering techniques are a type of probabilistic methods and are predominantly employed for state estimation in the presence of noise.[7] In the aerospace industry, a Kalman Filter (KF) stands out as one of the most widely used for addressing noise filtering issues.[12] The Kalman Filter,[10] as an linear estimator that employs the state-space equations of the system, is an optimal data processing algorithm with a recursive structure. Nonetheless, the majority of systems in various industrial fields exhibit non-linearity, leading to the impracticability of applying traditional Kalman filters. This has resulted in the emergence of a need for nonlinear filtering techniques.
The Extended Kalman Filter (EKF), which is the most widely employed nonlinear noise filtering technique in today, is utilized to address the non-linearity of systems and reduce the impact of noise. The EKF approximates non-linearity by computing the Jacobian of the system dynamics and measurement equations around the current estimates, using a first-order Taylor series expansion. If the non-linearity is severe, the EKF may not approximate it well, leading to linearization errors that can affect the estimation performance or convergence of the filter. Despite its limitations, the EKF is successfully employed not only for estimating aerodynamic coefficients but also in other aerospace applications.[3,4,14]

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To overcome the challenges posed by the EKF, several alternative algorithms, such as the Unscented Kalman Filter (UKF)[9] and the Iterated Extended Kalman Filter (IEKF),[5,13] have been developed and being applied.
By updating the linearization point using the current estimate, the IEKF addresses the limitations of the EKF and reduces the linearization error. In each iteration, the measurement Jacobian is recalculated based on the current estimate, allowing for a more precise linearization of the nonlinear system. This iterative refinement process continues until the desired accuracy or convergence is achieved.
The UKF, introduced by Julier and Uhlmann, is based on the Unscented Transform (UT).[8] Instead of directly approximating the nonlinear functions using Taylor series, the UKF utilize the UT. This transform selects a set of sigma points that are carefully extracted to capture the mean and covariance of the probability distribution. These sigma points are then transformed through the nonlinear functions, providing an approximation of the transformed non-linearity.
In this paper, we compare the performance of various nonlinear filters for aerodynamic coefficient estimation using simulation data from a fixed-wing aircraft.
The paper is organized as follows. In section 2, the algorithms of the introduced filters are explained, followed by the presentation of the UAV model and simulation environment utilized for aerodynamic coefficient estimation. In the last section, Section 3, results of parameter estimation from various filters are compared and analyzed.

2. Nonlinear Filtering

2.1 Extended Kalman Filter (EKF)

If we consider a general continuous-time nonlinear stochastic dynamic and a discrete-time nonlinear measurement model, the can be expressed as follows.

$$\begin{aligned} \dot{x} &= f(x, u, t) + w \\ z_k &= h_k(x_k) + v_k \\ w(t) &\sim N(0, Q) \\ v_k &\sim N(0, R_k) \end{aligned} \tag{1}$$

where x is the state variable vector with an initial value of x_0 at time t_0, u is the input vector of the system, and z_k is the measurement vector at time t_k. The continuous-time process noise w and the discrete-time measurement noise v_k are assumed to be independent Gaussian white noise with zero mean and covariance Q and R, respectively.
The EKF is a filter based on the first-order Taylor series approximation, which approximates the mean and covariance during the update process. The EKF consists of two steps: time update and measurement update. The time update, also known as prediction, is referred to as the process of using the previous time step's state vector, covariance matrix to compute a predicted state vector and covariance matrix at the current time step, using the system model. The measurement update, also known as correction, is performed after the time update stage. It involves adjusting the predicted state vector and covariance matrix using the measured values. As a result, estimated state vector and covariance matrix for the next time step are calculated. Mathematically, it can be represented as follows.

1) Time update

$$\begin{aligned} \hat{x}_k(-) &= f(\hat{x}_{k-1}(+), u_{k-1}) \\ P_k(-) &= F P_{k-1}(+) F^T + Q_{k-1} \end{aligned} \tag{2}$$

where F is the Jacobian matrix of f with respect to x, and is evaluated at $\hat{x}_k(-)$. The P_k represents the error covariance matrix. The "-" denotes a priori estimate of state vector before processing the measurement. While the "+" represents a posteriori estimate of state vector after processing the measurement.

2) Measurement update

$$\begin{aligned} S_k &= H P_k(-) H^T + R_k \\ K_k &= P_k(-) H^T S_k^{-1} \\ \hat{x}_k(+) &= \hat{x}_k(-) + K_k\left(z_k - h_k(\hat{x}_k(-))\right) \\ P_k(+) &= (I - K_k H) P_k(-) \end{aligned} \tag{3}$$

where H is the Jacobian matrix of $h_k(x_k)$ with respect to x, it is also evaluated at $\hat{x}_k(-)$. When constructing the EKF using the aforementioned process, several initial values need to be set beforehand. When there is less information

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available about the estimated states, it is recommended to set a larger initial value for the error covariance matrix, denoted as P_0. In other words, as P_k increases, the estimation error also increases, and P_k decreases, the estimation error decreases. The process noise covariance Q and measurement noise covariance R should also be defined prior to the estimation process. Since Q can be challenging to set, it needs to be adjusted through experimentation. The determination of R is specific to the sensors used for measurements and can be obtained from the sensor manufacturer. The Euler integration method can be used for the computation of the EKF algorithm in (1).

2.2 Iterated Extended Kalman Filter (IEKF)

The EKF linearizes the nonlinear system equations by expanding them using a Taylor series and neglecting higher-order terms, considering only the first-order terms for linearization. However, if the system model and measurement model exhibit strong non-linearity, there is a potential for significant degradation in the performance of the EKF due to linearization errors.

To address the inaccuracies caused by theses errors, various techniques have been proposed, and the IEKF is one of them. Both EKF and IEKF linearize the system model around the priori estimate, $\hat{x}_k(-)$, but there is a difference in their approach. In EKF, the measurement model linearization is performed only once, using the first-order Taylor series expansion for the measurement update step. On the other hand, IEKF takes an iterative approach by updating the linearization point of the measurement model iteratively based on the most recent posteriori estimate, $\hat{x}_k(+)$. In other words, when $\hat{x}_k(+)$ is obtained, the IEKF relinearizes the measurement model around this estimate and get a new posteriori estimate of $\hat{x}_k(+)$. By iteratively performing this process to acquire an improved estimate, it mitigates errors caused by the linearization. The iteration process is terminated when the maximum predetermined number of iterations is reached or when the change in values between consecutive iterations becomes negligible. We can describe the algorithm for IEKF[6] as follows:

Step 1: (initialization) Set the iteration i = 0, and set the predictive estimate

$$\begin{aligned} \hat{x}_k^0(+) &= \hat{x}_k(-) \\ P_k^0,(+) &= P_k(-) \end{aligned} \tag{4}$$

Step 2: (measurement update iterations) Compute the Jacobian matrix at the best state estimate available, the Kalman gain, and the next iteration of the state estimate as

$$\begin{aligned} H_k^i &= \left.\frac{\partial h_k(x)}{\partial x}\right|_{x=\hat{x}_k^i} \\ K_k^i &= P_k(-)\left[H_k^i\right]^T\left(H_k^i P_k(-)\left[H_k^i\right]^T + R_k\right)^{-1} \\ \hat{x}_k^{i+1}(+) &= \hat{x}_k(-) + K_k^i\left(y_k - h(\hat{x}_k^i(+)) - H_k^i(\hat{x}_k(-) - \hat{x}_k^i(+)\right) \end{aligned} \tag{5}$$

Repeat Step 2 with i = i + 1 until a stopping condition is met.

Step 3: (updating the state and covariance matrix) Save the last iteration as the new filtering mean and compute the covariance matrix based on the last iteration.

$$\begin{aligned} \hat{x}_k(+) &= \hat{x}_k^i(+) \\ P_k(+) &= (I - K_k^i H_k^i)P_k(-) \end{aligned} \tag{6}$$

In the above algorithm, $\hat{x}_k^i$ denotes the posteriori estimate of x_k obtained after conducting the relinearization of the measurement model. Similarly, K_k^i and H_k^i represent the Kalman gain and the Jacobian matrix of the measurement model, respectively, computed using the estimate $\hat{x}_k^i$ in the current iteration.

2.3 Unscented Kalman Filter (UKF)

One of the another method to address the problem of linearization errors in EKF is the UKF. If EKF resolves the non-linearity in system and measurement models through linearization, UKF, on the other hand, eliminates the linearization process and approximates non-linearity using Unscented Transform (UT) method introduced by Julier and Uhlmann.[8] The UT is a method that approximates the nonlinear transformation of the mean and covariance using a small number of specially selected samples. That is, it calculates the predicted values of the state variable and error covariance without directly using the system model equation, but instead utilizing a representative subset of data.

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To perform the UT, the first step is to determine the $2n_x + 1$ sigma points, which are a set of samples. These sigma points are essential for capturing the statistical properties of the distribution. Additionally, weights need to be assigned to each sigma point to represent their relative importance. Here, n_x represents the number of system and parameters begin estimated. The UKF may appear quite different from the EKF algorithm, but apart from using UT instead of linearization, the iterative structure of prediction and correction remains the same. In the prediction phase, the estimation and error covariance prediction are computed using the sigma points and UT, without directly relying on the system model. In the EKF algorithm, the covariance of measurement error was not required. However, in UKF, it is necessary for calculating Kalman gain, and it is also obtained using the sigma points and UT, rather than directly the measurement model. Finally, the covariance matrix (P_{xz}), between the state and the measurement is computed, followed by the calculation of the Kalman gain. After that, the estimated state and the error covariance are updated.
In summary, the UKF algorithm can be described as follows:

Step 1 : Initialize $(\hat{x}_0, P_0)$

Step 2 : Calculation sigma points and weights

$$\begin{aligned}
&\chi_0 = \bar{x}, \quad W_0 = \frac{\kappa}{(n_x+\kappa)} \\
&\chi_i = \bar{x} + \left(\sqrt{(n_x+\kappa)P_{xx}}\right)_i, \quad W_i = \frac{1}{2(n_x+\kappa)}, \quad i = 1, ..., n \\
&\chi_{i+n} = \bar{x} - \left(\sqrt{(n_x+\kappa)P_{xx}}\right)_i, \quad W_{i+n} = \frac{1}{2(n_x+\kappa)}, \quad i = 1, ..., n
\end{aligned} \tag{7}$$

Step 3 : Prediction of the estimated state and error covariance

$$[\hat{x}_k(-), P_k(-)] \longleftarrow UT\,(f(\chi_i), W_i, Q) \tag{8}$$

Step 4 : Prediction of the measured values and error covariance

$$[\hat{z}_k, P_z] \longleftarrow UT\,(h(\chi_i), W_i, R) \tag{9}$$

Step 5 : Calculation Kalman gain

$$\begin{aligned}
P_{xz} &= \sum_{i=1}^{2n+1} W_i \{f(\chi_i) - \hat{x}_k(-)\}\{h(\chi_i) - \hat{z}_k\}^T \\
K_k &= P_{xz}P_z^{-1}
\end{aligned} \tag{10}$$

Step 6 : Calculation of the estimated states and error covariance

$$\begin{aligned}
\hat{x}_k(+) &= \hat{x}_k(-) + K_k(z_k - \hat{z}_k) \\
P_k(+) &= P_k(-) - K_k P_z K_k^T
\end{aligned} \tag{11}$$

where χ_i is the sigma points, W_i is the weights, $\sqrt{P_{xx}}$ is the ith column of the matrix square root of P_{xx} and κ is an arbitrary design constant.

2.4 State space model of UAV

The flight data was obtained by modeling the Aerosonde UAV using parameters from Small Unmanned Aircraft Theory and Practice[1] and simulating it in MATLAB/Simulink. The data was acquired under trimmed conditions. To estimate parameters using the filters mentioned earlier, a state space model is required to generate predicted values and compare them with measurements. The system model employed the longitudinal dynamics of an aircraft, and the measurements model is as follows:

$$\begin{aligned}
\dot{V} &= -\frac{\bar{q}S}{m}C_D - g\sin(\theta-\alpha) + \frac{F_T}{m}\cos(\alpha+\sigma_T) \\
\dot{\alpha} &= -\frac{\bar{q}S}{mV}C_L + q + \frac{g}{V}\cos(\theta-\alpha) - \frac{F_T}{mV}\sin(\alpha+\sigma_T) \\
\dot{\theta} &= q \\
\dot{q} &= \frac{\bar{q}S\,\bar{c}}{I_{yy}}C_m + \frac{F_T}{I_{yy}}(l_{Tx}\sin\sigma_T + l_{Tz}\cos\sigma_T)
\end{aligned} \tag{12}$$

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where the aerodynamic coefficients are modeled as follows.

$$\begin{aligned} C_L &= C_{L_0} + C_{L_\alpha}\alpha + C_{L_{\delta e}}\delta_e + C_{L_q}\frac{q\bar{c}}{2V} \\ C_D &= C_{D_0} + C_{D_\alpha}\alpha + C_{D_{\delta e}}\delta_e \\ C_m &= C_{m_0} + C_{m_\alpha}\alpha + C_{m_{\delta e}}\delta_e + C_{m_q}\frac{q\bar{c}}{2V} \end{aligned} \tag{13}$$

The measurement equations are the following:

$$\begin{aligned} V_m &= V \\ \alpha_m &= \alpha \\ \theta_m &= \theta \\ q_m &= q \\ \dot{q}_m &= \frac{\bar{q}S\,\bar{c}}{I_{yy}}C_m + \frac{F_T}{I_{yy}}(l_{Tx}\sin\sigma_T + l_{Tz}\cos\sigma_T) \\ a_{xm} &= \frac{\bar{q}S}{m}C_x + \frac{F_T}{m}\cos\sigma_T \\ a_{zm} &= \frac{\bar{q}S}{m}C_z - \frac{F_T}{m}\sin\sigma_T \end{aligned} \tag{14}$$

where the longitudinal and vertical force coefficients, C_x and C_z, are provided as follows.

$$\begin{aligned} C_x &= C_L\sin\alpha - C_D\cos\alpha \\ C_z &= -C_L\cos\alpha - C_D\sin\alpha \end{aligned} \tag{15}$$

In (6), V is the airspeed, α is the angle of attack, θ is the pitch angle, q is the pitch rate, δ_e is the elevator deflection. $\bar{q}\,(= 1/2\rho V^2)$ is the dynamic pressure, F_T is the thrust, m is the mass of the UAV, S is the wing area, $\bar{c}$ is the chord length. σ_T represents the inclination angle of the engines, I_{yy} is the moment of inertia, ρ is the density of air.

2.5 Application of the filter

To estimate parameters using filter methods, it is necessary to augment the existing system state vector by expanding the unknown variables and defining them as additional state vectors in the system model. Assume that The unknown parameters for estimating aerodynamic coefficients are constants with zero derivatives and they can be expressed as a vector as follows:

$$X = [C_{D_0}, C_{D_\alpha}, C_{L_0}, C_{L_\alpha}, C_{m_0}, C_{m_\alpha}] \tag{16}$$

Therefore, by augmenting the existing state variables and the unknown parameters in (16), we can represent the new state vector as x_{new}:

$$x_{new} = \begin{pmatrix} x \\ X \end{pmatrix} \tag{17}$$

Now, by utilizing (17), we can create the augmented and (1) is substituted with (18).

$$\begin{aligned} \dot{x} &= f(x_{aug}, u, t) + w \\ z_k &= h_k(x_{aug}) + v_k \end{aligned} \tag{18}$$

It is difficult to estimate unknown parameters using filters without excitation. Therefore, to provide excitation, we applied double 3-2-1-1 elevator input in longitudinal motion at different times, as shown in Figure 1.
The flight simulation data and the estimated responses through the filter are shown in Figure 2. In airspeed estimation, the IEKF provides accurate estimates, while EKF and UKF show slightly deviated estimation results, but overall all the filters provide good estimations.
Next, The results of the aerodynamic parameter estimation is displayed. The initial values for estimating aerodynamic

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coefficient using each filter are the same, and their results are illustrated in Figure 3. It can be observed that the IEKF performs better in aerodynamic parameter estimation than the EKF and the UKF under the same conditions and with limited measurements. Figure 4, which represents the estimation errors, illustrates that the IEKF exhibits faster convergence time and minimal error compared to other two filters.

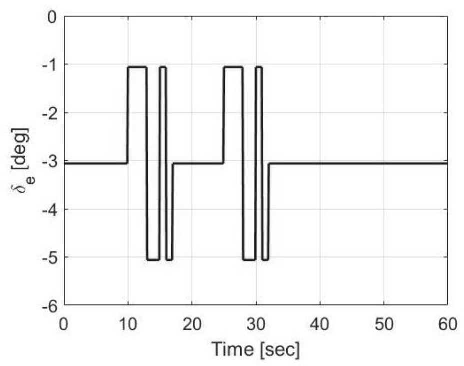

Figure 1: Deflection angle

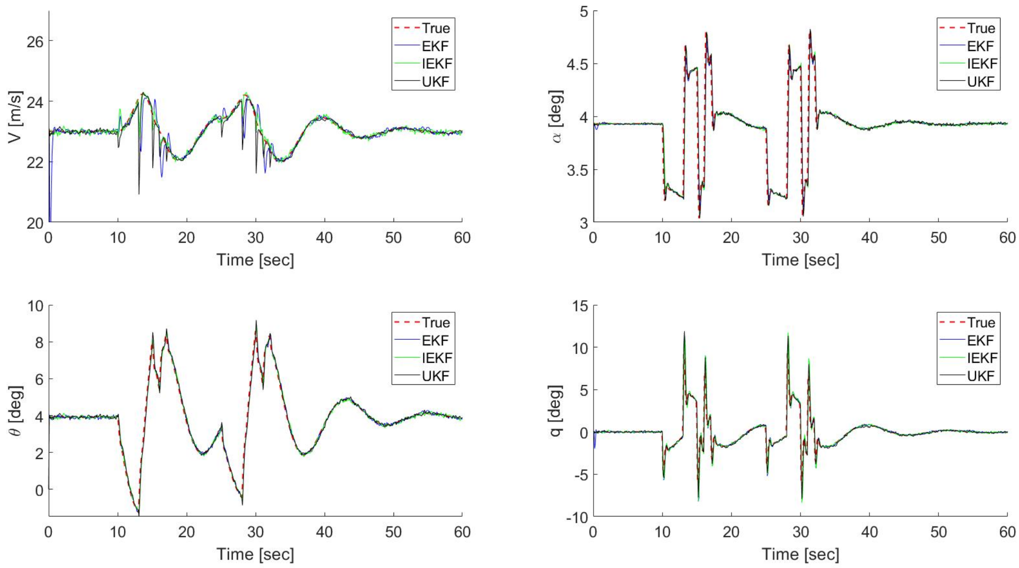

Figure 2: Estimated response

3. Conclusion

In this paper, three nonlinear recursive filters are used to perform aerodynamic coefficient estimation, and their performance is compared. The filters used in this study are EKF, Iterated EKF, UKF. The result indicates that the IEKF exhibits remarkable performance in the estimation process compared to the other two filters under the same conditions. However, a high-order approach of EKF, known as IEKF, suffers from the drawback of longer computation time compared to the other two filters, as it involves iterative measurement updates until a termination condition is met.
Future work will involve considering all aerodynamic coefficients of the Aerosonde UAV and applying methods from an optimization viewpoint to enhance the performance of parameter estimation, including improving the convergence and computation time of IEKF.

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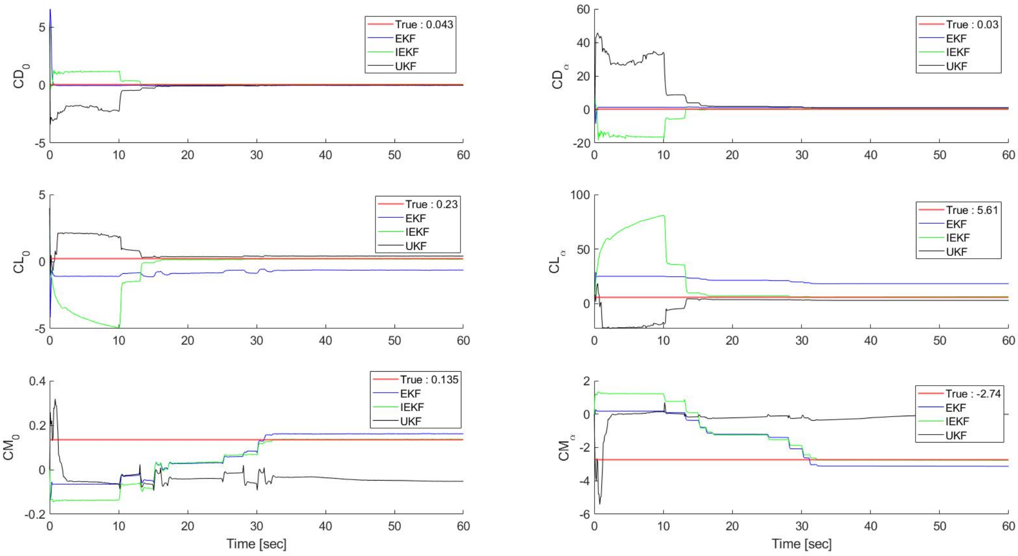

Figure 3: Aerodynamic coefficient estimation

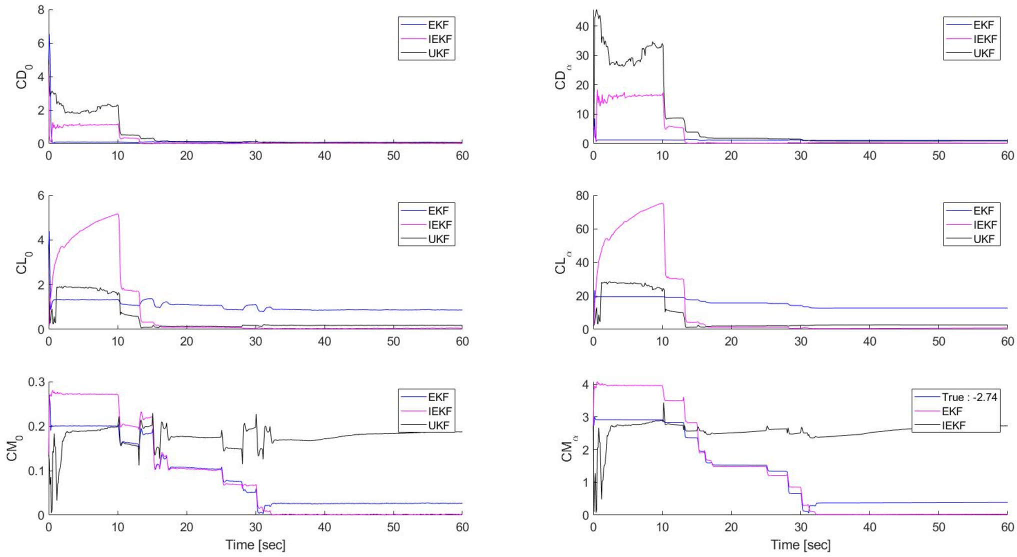

Figure 4: Estimation error of filters

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