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Optimal Time Terrestrial Mapping Scheduling for Spaceborne Push-broom Imagers

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Abstract
In this paper, we propose a Mixed-Integer Linear Programming (MILP) based method for generating an optimal image acquisition schedule. We especially consider a Terrestrial Mapping problem, which is defined as finding an acquisition schedule for obtaining images of a certain predefined area. We consider an Electro-Optical (EO) satellite with a push-broom type imager, occupying a Sun-Synchronous Orbit (SSO). By exploiting the properties of SSO, the terrestrial mapping problem is converted into a MILP problem, which could be efficiently solved using commercial solvers. As a result, we propose multiple variations of the terrestrial mapping problem which could be optimized in the MILP framework.

1. Introduction

Optimally allocating satellite imaging capability is essential in operating satellite systems concerning its immense operation cost. When it comes to terrestrial mapping, it is of utmost importance to meticulously schedule the imaging time and area since the field of regard of the satellite is limited per orbit. If one can exploit the imaging chances to the extent possible, it can directly reduce the operation hour and cost of the satellite system. However, since the possible combination of the satellite image schedules is infinite, one should carefully manipulate the problem into a solvable format.

This paper concentrates on the minimum acquisition time problem of satellite imagery when mapping the predefined terrestrial landmass. Here, a terrestrial mapping problem is defined as an optimal scheduling problem where the goal is to acquire a set of images of the given area of landmass as soon as possible. It can be thought of as a practical variation of a set covering problem, which considers the ways to cover the whole set (entire mapped area) with finite subsets (strips of satellite images). The landmass of interest could be a city, a country, or any random polygon with a width greater than the swath width of the imager. The satellite system of interest is an Electro-Optic (EO) push-broom imager on a circular Sun-Synchronous Orbit (SSO), acquiring strips of images with fixed roll angles per each pass. (Fig. 1) Here, the roll angle of the satellite controls the lateral positioning of the acquired strip of the image with respect to the ground track of the satellite orbit.

A number of prior researches on optimal satellite operation scheduling exist [1, 2], but the targets were mostly limited to point targets. To the authors' knowledge, the terrestrial mapping problem is rarely covered in the literature, which highlights the novelty of this work.

The minimum-time terrestrial mapping problem is naturally complex due to the curse of dimensionality. For instance, imagine a brute-force method for finding a minimum-time full-coverage schedule. Let us assume that we discretize possible roll angles to 30 candidates, and try to find the minimum time solution for the first 10 orbits. Already the number of possible combinations of roll angles reach 30^{10}, which is impossible to be appraised with a limited amount

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of computational resources. Henceforth, the crux of solving the terrestrial mapping problem is to formulate it into a solvable format, which requires some creative thinking.

We suggest a scheduling framework based on the Mixed Integer Linear Programming (MILP) technique. Thanks to the property of MILP, the generated optimization problem can be solved using widely used commercial solvers within a reasonable computational effort. Furthermore, formulating the problem in the MILP framework enables the user to easily manipulate the problem into a minimum-time full coverage schedule, a minimum-time partial coverage schedule, or even a minimum-time coverage schedule with a priority on high-value targets. The problem is further generalized considering the swath width of the imager and the overlap of the acquired strip images due to the Earth's curvature. The generated schedules propose optimal imaging time slots and optimal roll angles of the satellite.

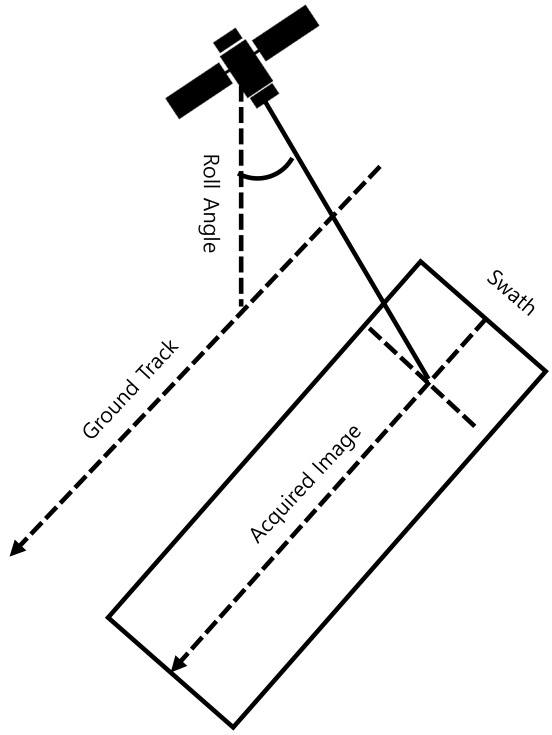

Figure 1: Illustration of a Push-Broom Imager

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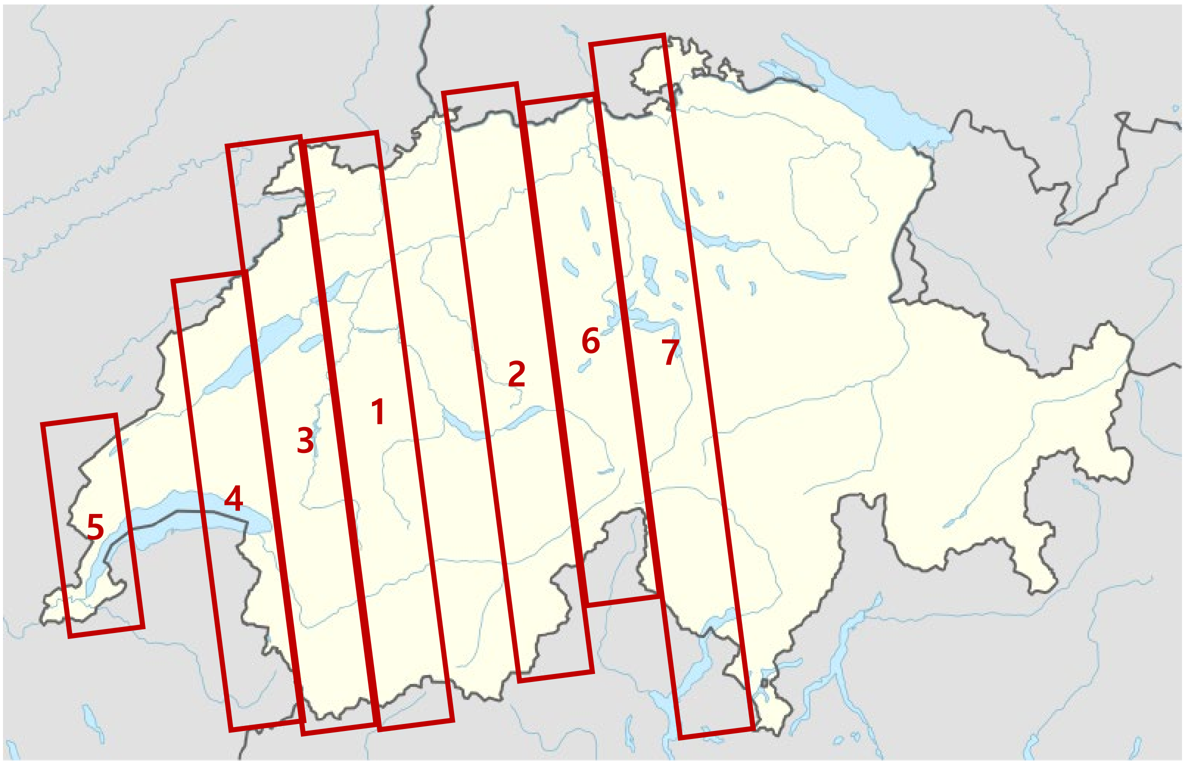

Figure 2: Illustration of a Terrestrial Mapping in Progress, with Switzerland being the target area.

2. MILP Formulation of the Terrestrial Mapping Problem

In this section, we present the method to formulate the terrestrial mapping problem in MILP format. To facilitate the formulation, we introduce the assumptions below.

- The satellite orbits in a perfect SSO. i.e. The inclination of the satellite is kept constant, and the local time of the revisit is also maintained. Furthermore, the eccentricity is kept zero (the orbit is circular) hence the altitude of the satellite is always constant, too.
- The Earth oblateness is ignored. The Earth surface is assumed to be a perfect sphere.
- The swath width of the image is kept constant regardless of the roll angle. (This is a strong assumption. A discussion regarding this assumption is given in the remarks.)

By these assumptions, we can simplify the terrestrial mapping problem into a MILP format, which enables us not only to solve the terrestrial mapping problem in an acceptable amount of computing resources but also allows us to modify the problem into multiple applications. We first propose the basic full coverage formulation (section 2.1) to give the blueprint of our approach. Then we suggest possible variations of the terrestrial mapping problem, including the fastest partial coverage method, prioritizing certain targets, and introducing some orbital or temporal constraints. Finally, we provide the method to incorporate the effect of the Earth curvature, which is crucial in solving the terrestrial mapping problem in high latitudes.

Remark 1. We assumed that the swath width of the image is constant in order to simplify the problem. However, the swath width of the image is highly dependent of the roll angle for actual EO satellites. Though reluctant, we introduce this assumption in order to smoothly convert the problem into MILP. By choosing the minimum swath width given with respect to nadir pointing (roll angle equal to zero), we can safely guarantee that the satellite will certainly acquire the designated imaging area. The authors are aware that lifting this assumption would lead to a better solution of the terrestrial mapping problem, and striving to provide an answer.

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2.1 Basic Full Coverage Formulation

In order to convert the terrestrial mapping problem into a MILP form, we first slice the target area into multiple strips parallel to the direction of orbital inclination, with equal swath width. We label each strips from S_1 to S_N. For the simplicity, we label the strips from the left to the right. Similarly, we label each revisits from R_1 to R_M, starting from the earliest revisit. Next, we define the **binary** variable (i.e. the variable takes only the values of 0 or 1) X_{ij} to encode the correspondence between S_i and R_j. If the image of the strip S_i is taken during the revisit R_j, we set X_{ij} to be 1, and otherwise 0. In this way, we can express any image acquisition scheduling using X_{ij}. Furthermore, let us define a set of sets P_1 to P_M which encodes the visibility of the strips in the revisit R_j. For instance, if the strip S_2, S_3, and S_4 are visible from the revisit R_j and the others are unobservable due to the roll angle limitation, we set $P_j = \{2, 3, 4\}$. Furthermore, we define the value A_i to denote the size of the intersection between the strip S_i and the target area. (i.e. A_i denotes the km^2 of the target area covered by taking strip S_i) The definitions are depicted in Fig. 3.

Using these definitions, we can define an optimizable MILP formulation which results in a minimum-time (minimum-revisit) image acquisition schedule. First, we consider the constraints given below.

- The number of strips taken from a single revisit cannot exceed one.

$$\sum_{i=1}^{N} X_{ij} \leq 1 \; \forall j \tag{1}$$

- Each strips must be assigned to exactly one revisit. (Full coverage)

$$\sum_{j=1}^{M} X_{ij} = 1 \; \forall i \tag{2}$$

- Visibility of the strip1s must be considered.

$$X_{ij} = 0 \text{ for } i \notin P_j \;\; \forall j \tag{3}$$

Next, we consider the minimum-time (or minimum revisit) objective. Colloquially stating, we try to minimize the index j where at least one X_{*j} is non-zero. By exploiting the binary property of X_{ij}, we can convert such objective into a minimization of certain maximum function:

$$\text{minimize} \quad \max(j \cdot X_{ij}) \tag{4}$$

Note that the value jX_{ij} is equal to j only in the case when X_{ij} is one. Otherwise, due to the binary property of X_{ij}, the value becomes zero. Hence, the function max(jX_{ij}) is equal to the maximum index j where at least one X_{ij} is nonzero. Yet, it is also observable that the objective function is nonlinear. We convert the above nonlinear objective into a set of a linear objective function and a constraint, by introducing an additional fictitious variable s. Here, s does not have to be an integer variable.

$$\text{minimize} \quad s \tag{5}$$

$$\text{subject to} \quad s \geq j \cdot X_{ij} \; \forall i, j \tag{6}$$

Here, we can observe that the value s is equal to max(jX_{ij}) at the optimum. If s is greater than max(jX_{ij}), one can reduce s to max(jX_{ij}) without any constraint violation. If s is less than max(jX_{ij}), it violates the constraints, so the current value of s cannot be admitted. Hence by contradiction, one can show that s actually reaches the value of max(jX_{ij}) at the optimal point.

In conclusion, combining Eq. (1 – 3, 5, 6) defines a MILP that minimizes the number of revisits required for a full coverage of the target area. By solving the defined MILP using commercial solvers, one can find an optimal image acquisition schedule.

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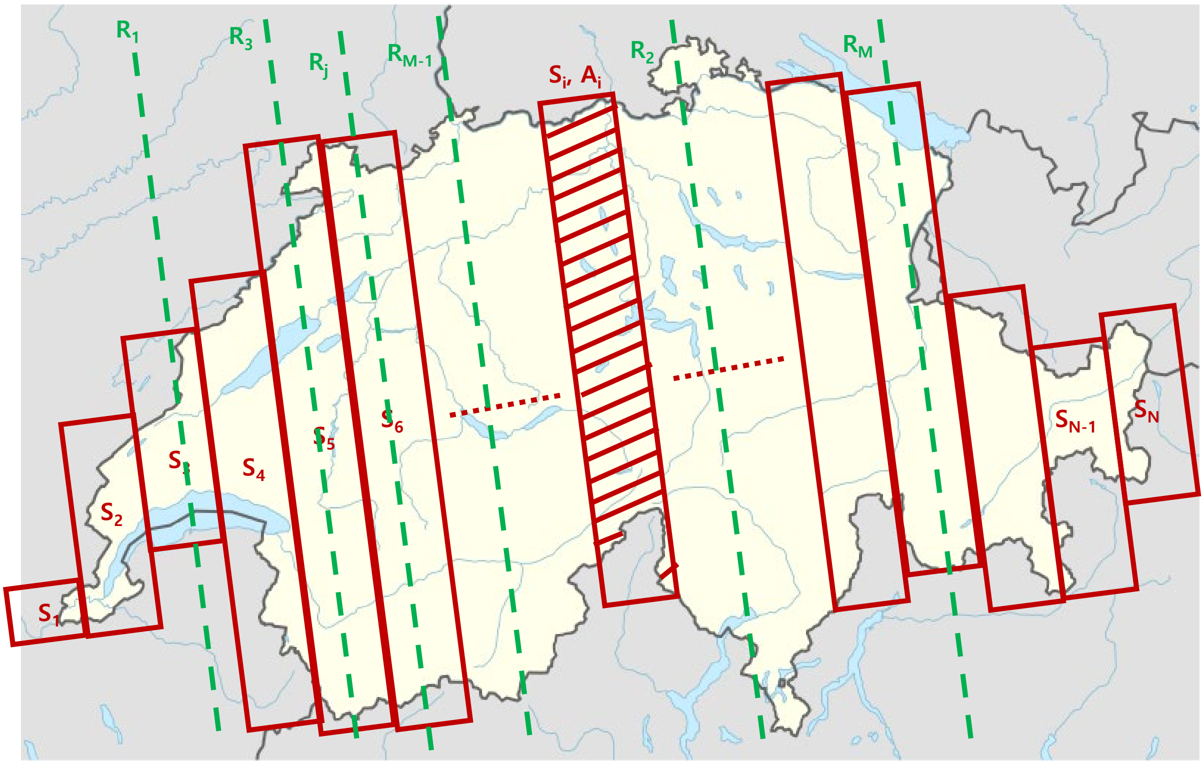

Figure 3. Definition of Strip S, Revisit R, and its Area A (shaded)

2.2 Optimal Partial Coverage Formulation

The full coverage formulation provided in Sec. 2.1 could be extended to cover a partial coverage case. Here, the partial coverage problem refers to a problem with a goal to cover more than certain percent (e.g. 50%) of the total target area with minimum time. For some applications, a time required for a full coverage could be substantially longer compared to, for instance, a 90% coverage. In order to balance the percentage of the covered area versus the total time spent, one may find it useful to utilize the partial coverage form. Interestingly, a subtle change of the full coverage formulation yields a partial coverage form, which shows the extensibility of the suggested MILP approach.

First, the constraint Eq. (2) is modified to allow uncovered strips.

- Each strip must be assigned to exactly one revisit, or not assigned at all. (Partial coverage)

$$X_{ij} \leq 1 \text{ for } i \in P_j \quad \forall j \tag{7}$$

Then, an area constraint is added to ensure more than A^* km^2 of the total area A_{total}.

- The sum of the area covered by the strips must exceed the predefined goal.

$$\sum_{i,j} A_i X_{ij} \geq A^* \tag{8}$$

Here, note that the summation of A_iX_{ij} with respect to j gives either A_i when the strip S_i is assigned to certain revisit (at most one X_{ij} is nonzero), or zero when the strip S_i is not assigned to any revisit (all X_{ij} are zero). Hence, the double summation with respect to i, j gives the total area covered only by the assigned strips.

In summary, Eq. (1, 5-8) gives a MILP for minimum-time partial coverage form. Note that setting $A^*=A_{total}$ results in an alternative full-coverage problem.

2.3 Prioritizing Certain Target

While the partial coverage form could be useful, some users might want to force the acquisition of certain targets of high importance. For example, the schedule requirement could be: “find a minimum-time acquisition scheduling which covers more than 70% of the Swiss territory while including Geneva, Lausanne, and Bern”. This could be

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achieved by introducing additional constraints. Let us define a set Q which contains the index of high-priority strips. For instance, if strip 2, 3, and 6 contain Geneva, Lausanne, and Bern, set $Q = \{2, 3, 6\}$. Then, applying the constraint below enables us to force the acquisition of the strips embedded in Q.

$$\sum_{j=1}^{M} X_{ij} = 1 \; \forall i \in Q \tag{9}$$

2.4 Adding Revisit Constraints

We may further extend the problem to incorporate certain revisit constraints. It could be useful, for instance, when acquisition of two adjacent strips in neighbouring revisits is required. Let us consider the case of taking the strip S_{i_1} and S_{i_2} with a revisit gap of j^*. i.e. If the strip S_{i_1} is taken at the revisit R_{j_1}, we wish the strip S_{i_2} to be acquired at the revisit R_{j_2} where $j_2 = j_1 + j^*$. Since j_1 is undetermined a priori, the revisit constraint must be expressed indirectly using a multiplication of two binary variables.

$$\sum_{j=1}^{M} X_{i_1,j} X_{i_2,j+j^*} = 1 \; \forall j \le M - j^* \tag{10}$$

Unfortunately, the multiplication of variables is nonlinear, hence the constraint Eq. (9) cannot be handled in the MILP framework. Yet, a standard trick (sometimes referred as the big-M method) could be utilized to convert the given constraint into a set of linear constraints. The big-M method could be applied when a variable (either a floating point number or an integer) with a bound on its value is multiplied with a binary variable.

The full coverage formulation provided in Sec. 2.1 could be extended to cover a partial coverage case. Here, the partial coverage problem refers to a problem with a goal to cover more than certain percent (e.g. 50%) of the total target area with minimum time. For some applications, a time required for a full coverage could be substantially longer compared to, for instance, a 90% coverage. In order to balance the percentage of the covered area versus the total time spent, one may find it useful to utilize the partial coverage form. Interestingly, a subtle change of the full coverage formulation yields a partial coverage form, which shows the extensibility of the suggested MILP approach. We introduce a new variable p_j to hold the value of $X_{i_1,j} X_{i_2,j+j^*}$. Moreover, we use a large constant value Ω, which must be greater than the maximum bound of $X_{i_2,j+j^*}$.

$$\sum_{j=1}^{M} p_j = 1 \; \forall j \le M - j^* \tag{11}$$

$$p_j \le \Omega X_{i_1,j} \; \forall j \le M - j^* \tag{12}$$

$$p_j \ge -\Omega X_{i_1,j} \; \forall j \le M - j^* \tag{13}$$

$$p_j \ge X_{i_2,j+j^*} - \Omega\left(1 - X_{i_1,j}\right) \forall j \le M - j^* \tag{14}$$

$$p_j \le X_{i_2,j+j^*} + \Omega\left(1 - X_{i_1,j}\right) \forall j \le M - j^* \tag{15}$$

At optimum, the value p_j is always equal to the value of $X_{i_1,j} X_{i_2,j+j^*}$, hence Eq. (11) can be a proxy of Eq. (10) while maintaining the linearity. Note that if $X_{i_1,j}$ is zero, Eq. (12, 13) becomes $p_j = 0$, while Eq. (14, 15) gives an inactive boundary constraint on p_j. (Since the value Ω is significantly large) For the case when $X_{i_1,j}$ is one, Eq. (12, 13) becomes inactive, and Eq. (14, 15) becomes $p_j = X_{i_2,j+j^*}$.

In summary, adding Eq. (11-15) to the MILP problem activates the revisit constraints.

2.5 Adding Temporal Constraints

Similar to section 2.4, we can also add timing constraints. Specifically, we can impose the MILP problem to consider either (1) the maximum time gap between two strips, or (2) the minimum time gap between two strips. Let us define the time T_j to denote the time of the revisit R_j. Furthermore, let us denote the maximum or the minimum time gap required as T_{max} and T_{min} respectively.

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With naïveté, the maximum time constraint could be defined as below which contains a nonlinear absolute value function.

$$\left|\sum_{j=1}^{M} X_{i_1,j}T_j - \sum_{j=1}^{M} X_{i_2,j}T_j\right| \leq T_{max} \tag{16}$$

However, the absolute value function less than a constant could be divided into two linear inequalities.

$$\sum_{j=1}^{M} X_{i_1,j}T_j - \sum_{j=1}^{M} X_{i_2,j}T_j \leq T_{max} \tag{17}$$

$$\sum_{j=1}^{M} X_{i_1,j}T_j - \sum_{j=1}^{M} X_{i_2,j}T_j \geq -T_{max} \tag{18}$$

Here, Eq. (17, 18) replaces Eq. (16) with linear inequalities. Please be aware that the big-M method is not required since Similarly, the minimum time constraint could be naively defined with an absolute value function.

$$\left|\sum_{j=1}^{M} X_{i_1,j}T_j - \sum_{j=1}^{M} X_{i_2,j}T_j\right| \geq T_{min} \tag{19}$$

Note that the sign of the inequality is in reverse. Due to this fact, one cannot simply divide the absolute value function into two linear inequalities. We introduce a new **binary** variable Δ to convert Eq. (19) into two linear inequalities, distinct from Eq. (20, 21).

$$\sum_{j=1}^{M} X_{i_1,j}T_j - \sum_{j=1}^{M} X_{i_2,j}T_j \geq T_{min} - \Delta\Omega \tag{20}$$

$$-\left(\sum_{j=1}^{M} X_{i_1,j}T_j - \sum_{j=1}^{M} X_{i_2,j}T_j\right) \geq T_{min} - (1-\Delta)\Omega \tag{21}$$

Note that when Δ is zero, Eq. (20) is active while Eq. (21) becomes inactive. When Δ is one, Eq. (21) becomes active and Eq. (20) becomes inactive.

2.6 Handling Earth Curvature

For the preceding sections, we implicitly assumed that the Earth's surface is totally flat, which resulted in completely parallel positioning of the strips without any overlaps between them. However, overlaps between the strips are inevitable when the curvature is present. When depicted using the Mercator projection, the strips will start to bulge horizontally toward north.

Yet, the overlap between strips can be handled using the inclusion-exclusion principle. For the complete coverage, let us redefine the strips with respect to the southernmost tangent line perpendicular to the inclination angle. The redefined strips shall not overlap at the southernmost end, and when defined correctly, it will start to overlap each other toward north. Let us introduce a new constant B_{ik} to denote the overlapped area between the two strips S_i and S_k. Then, the left-hand side of the constraint Eq. (8) could be modified using the inclusion-exclusion principle. In order to find the total area assigned to any revisit, we first include the sum of area A_i, and then exclude the area of intersection B_{ik}.

$$\sum_{i,j} A_i X_{ij} - \sum_{i,k} B_{ik} \left(\sum_j X_{ij}X_{kj}\right) \geq A^* \tag{22}$$

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We introduce a fictitious variable q_{ijk} to convert the nonlinear multiplication $X_{ij}X_{kj}$ into linear constraints. We again use the big-M method used in section 2.4.

$$\sum_{i,j} A_i X_{ij} - \sum_{i,k} B_{ik} \left(\sum_j q_{ijk}\right) \geq A^* \tag{23}$$

$$q_{ijk} \leq \Omega X_{ij}\ \forall i,j,k \tag{24}$$

$$q_{ijk} \geq -\Omega X_{ij}\ \forall i,j,k \tag{25}$$

$$q_{ijk} \geq X_{ik} - \Omega\left(1 - X_{ij}\right) \forall i,j,k \tag{26}$$

$$q_{ijk} \leq X_{ik} + \Omega\left(1 - X_{ij}\right) \forall i,j,k \tag{27}$$

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In summary, exchanging the constraint Eq. (8) with a set of constraints Eq. (23-27) enables the user to consider the effect of the Earth's curvature.

Remark 2. For extreme cases, not only the adjacent strips, but also the one next to it can intersect each other. (i.e. more than three strips can intersect each other) For instance, it may occur near the north pole. However, most of the targets of high interest are positioned under the latitude of approximately 70 degree. The overlap between more than three strips will never occur in the given latitude range. Note that the ratio of the radius of the Earth at the latitude of 70 degree is around 14,800 km, and it is approximately 18,200km at the latitude of 65 degree. Considering the latitude variation of 5 degree is enough, since most of the EO satellites has maximum time limits (typically less than few hundred seconds) on the continuous image acquisition. The expected maximum horizontal distortion of the strips is approximately +24% when scanning the area in between. In order to have the overlap between three strips, the maximum horizontal distortion should be at least greater than 100% by geometry. As a result, considering the overlap between two strips is enough for meaningful mission objectives.

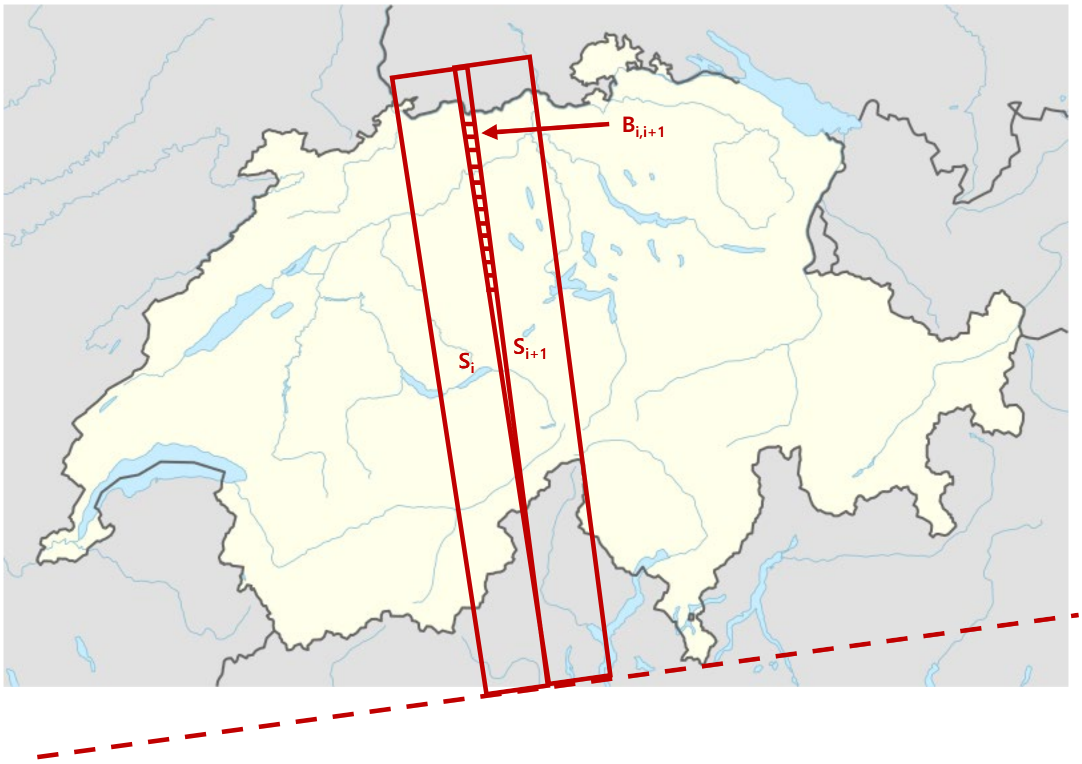

Figure 4. The Effect of the Earth's Curvature on the Overlap Between Strips (Exaggerated for Visibility)

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3. Simulated Examples

In this section, we demonstrate the effectiveness of the suggested MILP approach via simulated examples. For the sake of simplicity, the basic full-coverage form (Sec. 2.1) is used. We show that the suggested MILP is solvable using a reasonable amount of computational resources, within reasonable computation time.

Here, we implement the full-coverage formulation and solve it using Gurobi® optimizer, on a Windows® computer equipped with Intel® i7-12700 processor. (12 cores, 20 threads) We tested the implementation on various numbers of strips(N) and revisits(M) using random problem instances. The computation time is measured by averaging the computation time required over 10 problem instances. We summarize the results on Table. 1.

Table 1. Computation time for various number of strips and revisits (unit: second)

	N = 10	**N = 50**	**N = 100**
M = 50	0.0083		
M = 100	0.014	0.024	
M = 500	0.071	0.10	0.15
M = 1000	0.16	0.21	0.30
M = 5000	1.20	1.54	2.04

It is observable that the required computation time is reasonably short. Furthermore, it is worth noting that the proposed method even scales to 100 strips over 1000 revisits, which shows the excellence of the proposed MILP approach.

4. Conclusions

In this paper, we proposed a MILP based approach on the terrestrial mapping problem for EO satellites orbiting on SSO. The key idea of the suggested approach is to divide the target area into parallel strips a priori and then to find the optimal assignment using MILP. We proposed the basic full-coverage formulation for the terrestrial mapping problem. Moreover, we extended the suggested formulation to include a partial coverage problem, priority constraints, revisit constraints, and time constraints. We also provided a method to consider the effect of the Earth's curvature. We demonstrated the performance using simulated examples, based on commercial MILP solver Gurobi®. The method has been tested on various numbers of strips and revisits, and we showed that the method is tractable on a regular PC with reasonable computation time.

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