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Diagnosis and Prognosis of Structural Damage Growth with Parameter Sharing in Fleet Maintenance Digital Twins

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Abstract
In this study, two categories of uncertain parameters, fleet-shared and standalone, in individual digital twins are considered within a fleet. a sharing step is added in the particle filter, allowing the distributions of fleet-shared parameters to be shared among different individuals. Gaussian copula is used to measure and recover correlation among states and parameters before and after sharing. Preliminary validation using several hypothesized specimens illustrates that our approach. can improved the accuracy in parameter calibration, leading to improved diagnosis and prognosis. Further validation on complex structures with complicated damage growth mechanisms is left as future work by the authors.

1. Introduction

Fatigue fracture resulting from crack growth under cyclic loads significantly impacts the structural integrity of aeronautical and mechanical structures. In order to ensure integrity, several engineering applications have implemented various methods such as safe-life, fail-safe, and damage tolerance. However, these methods primarily focus on fleet management in a deterministic manner, often disregarding discrepancies between individual entities. Since the 1970s, individual aircraft tracking (IAT) has been employed in structural integrity management. Building upon the concept of IAT, the Airframe Digital Twin (ADT) was proposed by NASA and AFRL in 2010 [1]. The ADT serves as a virtual representation of the physical component and aims to account for uncertainties in fatigue damage evolution, as well as track the remaining useful life of structures.

Particle filter (PF)-based Bayesian updating approach has been widely used as the framework for the ADT [2,3]. In PF, the uncertain model parameters are combined with the damage state as the augmented state vector, and the state-parameter estimation is conducted using damage state observations to improve the performance of damage diagnosis and prognosis [4]. With correlations in the joint distribution of augmented states, the model parameters are calibrated using the damage state observations. However, due to the correlation between the parameters, different combination of parameters may lead to the similar damage evolution, making it difficult to find the true parameters value. If the particle filters are generated for each individual and the model calibrations are conducted separately, various calibration results may be obtained.

In a fleet where all aircraft have the same structural form, there is similarity in the structural damage state under similar loads and environmental histories [5]. In this study, one difference in the model parameters at the fleet level in the digital twin model was considered, and more specifically, the model parameters were divided into fleet-share and

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standalone parameters. In our approach, the standalone parameters are updated as in the conventional particle filter, while a sharing step to share the distributions of fleet-shared parameters among different individuals is added. Several hypothesized specimens with simple crack growth processes are used to validate the proposed approach preliminarily, and results show that our approach can improved the accuracy in parameter calibration, leading to improved diagnosis and prognosis.

2. Conventional Particle Filter

In previous study, the particle filter, or dynamic Bayesian network for complex systems, are constructed for each structure as the digital twin model, which are used to conduct the diagnosis and prognosis and then schedule the inspection and maintenance [6]. For the crack growth problem, the state of the system is denoted by crack length a. The fatigue crack growth modeled by Pairs law [7] is as follows:

$$\frac{da}{dN} = C(\Delta K)^m \tag{1}$$

where $\frac{da}{dN}$ are the increments of crack lengths per cycle, ΔK is the ranges of stress intensity factor, C and m are material parameters, which are considered to be uncertain.

The augmented state vector is defined as $\boldsymbol{x}_k = [a_k, \log C_k, m_k]$, and the state function can be obtained:

$$\boldsymbol{x}_k = \begin{bmatrix} \log C_k \\ m_k \\ a_k \end{bmatrix} = \begin{bmatrix} \log C_{k-1} + \omega_{1,k} \\ m_{k-1} + \omega_{2,k} \\ a_{k-1} + e^{\omega_k} C_k\big(\Delta K(a_{k-1})\big)^{m_k} \Delta N \end{bmatrix} \tag{2}$$

where $\omega_{1,k}$ and $\omega_{2,k}$ are the noise term in the evolution of the $logC$ and m， respectively, ω_k is the noise of the crack growth following Gaussian distribution $N\left(-\frac{\sigma_\omega^2}{2}, \sigma_\omega^2\right)$, leading to $E(exp(\omega_k)) = 1$.

To effectively track the evolution process of state vector $\boldsymbol{x}_k$, the following two tasks need to be accomplished by Bayesian inference:

Forward propagation: predict the state vector $\boldsymbol{x}_k$ according to the state variables $\boldsymbol{x}_{k-1}$ at the previous time step and the state transition between the two adjacent time steps:

$$p(\mathbf{x}_k \mid \mathbf{y}_{1:k-1}) = \int p(\mathbf{x}_k \mid \mathbf{x}_{k-1}) p(\mathbf{x}_{k-1} \mid \mathbf{y}_{1:k-1}) \mathrm{d}\mathbf{x}_{k-1} \tag{3}$$

Backward inference: updating the joint probability distribution of the state variables $\mathbf{x}_k$ when observation $\mathbf{y}_k$ is available:

$$p(\mathbf{x}_k \mid \mathbf{y}_{1:k}) = \frac{p(\mathbf{y}_k \mid \mathbf{x}_k) p(\mathbf{x}_k \mid \mathbf{y}_{1:k-1})}{p(\mathbf{y}_k \mid \mathbf{y}_{1:k-1})} \tag{4}$$

where $p(\mathbf{y}_k \mid \mathbf{x}_k)$ is observation likelihood, $p(\mathbf{y}_k \mid \mathbf{y}_{1:k-1})$ is a normalization constant.

In the Bayesian filtering framework, the posterior probability density function (PDF) is often unavailable in closed form for many cases. To approximate the posterior PDF, particle filtering utilizes N_s samples, also known as "particles," to represent the distribution, as shown in Eq. (5).

$$p(\boldsymbol{x}_k \mid \boldsymbol{y}_{1:k}) \approx \sum_{i=1}^{N_s} \widetilde{w}_k^{(i)} \delta\big(\boldsymbol{x}_k - \boldsymbol{x}_k^{(i)}\big) \tag{5}$$

where N_s is the number of particles, δ is the Dirac delta function, $\boldsymbol{x}_k^{(i)}$ is the i_{th} particle, $\widetilde{w}_k^{(i)}$ is the importance weight of i_{th} sample.

These particles are drawn from an importance density $q\big(\boldsymbol{x}_k \mid \boldsymbol{x}_{k-1}^{(i)}, \boldsymbol{y}_k\big)$ that is supposed to be similar to the desired posterior PDF$p(\boldsymbol{x}_k \mid \boldsymbol{y}_{1:k})$ and easy to be sampled from. The most commonly used importance density is the transition PDF $p\big(\boldsymbol{x}_k \mid \boldsymbol{x}_{k-1}^{(i)}\big)$, which simplifies the weight updating equation as given by Eq. (6).

$$w_k^{(i)} \propto w_{k-1}^{(i)} p\big(\boldsymbol{y}_k \mid \boldsymbol{x}_k^{(i)}\big) \tag{6}$$

where $p\big(\boldsymbol{y}_k \mid \boldsymbol{x}_k^{(i)}\big)$ is the measurement likelihood. The weight is then normalized with Eq. (7):

$$\widetilde{w}_k^{(i)} = \frac{w_k^{(i)}}{\sum_{i=1}^{N_s} w_k^{(i)}} \tag{7}$$

There are two challenges in the PF: 1) degeneracy, meaning that all but one particle will have negligible weights after a few iterations; and 2) sample impoverishment, meaning the loss of sample diversity.

After weight updating, a resampling procedure is typically performed to address the issue of particle degeneracy. This involves discarding particles with small weights and duplicating those with large weights. However, the resampling procedure can lead to the particle impoverishment problem, where particles become the same replications of a few

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particles after several resampling steps. This can result in limited possible crack growth paths for filtering, particularly for the model parameter component $\{\boldsymbol{\mu}_k^i\}_{i=1}^N$, which can significantly affect filtering and prognostic outcomes.
To address this problem, the regularized particle filter (RPF) [8] introduces a continuous approximation of the posterior PDF $p(\boldsymbol{x}_k \mid \boldsymbol{y}_{1:k})$ using the kernel density method [9],, as expressed in Eq. (8).

$$p(\boldsymbol{x}_k \mid \boldsymbol{y}_{1:k}) \approx \sum_{i=1}^{N_s} \tilde{w}_k^{(i)} K_h\left(\boldsymbol{x}_k - \boldsymbol{x}_k^{(i)}\right) \tag{8}$$

where $K_h(\cdot)$ is the rescaled kernel function given by $K_h(\boldsymbol{x}_k) = \frac{1}{h^{n_x}} K\left(\frac{x_k}{h}\right)$, h is the kernel bandwidth, n_x is the dimension of the state vector. After the resampling procedure, particles are randomly drawn from this continuous approximation of the posterior PDF. This regularization step is used to increase the diversity of the particles, thereby preventing the particle impoverishment problem.

3. Parameter share scheme

In this study, the parameters within digital twins are categorized into two groups. The first group comprises fleet-share parameters, which implies that their true values are consistent across different individuals within a fleet. It is common that during the construction of a simulation model, there exists a certain bias between the predicted and actual responses due to limitations inherent in the emulation tools. The second group consists of standalone parameters, which exhibit variations among individuals and are attributed to uncertainties arising from the manufacturing process.
As illustrated in Figure 1, the posterior distribution of damage states, considering all uncertain model parameters, is initially updated using the conventional Bayesian update method. Subsequently, the updated distribution of the fleet's shared parameters is propagated to the remaining individuals within the fleet. Notably, since the parameter distributions are represented as multidimensional particles, the correlation between dimensions is lost when the values of a particular dimension of the particles are directly substituted. Therefore, it becomes necessary to restore this correlation after sharing, a process facilitated by the introduction of the Gaussian Copula function in the subsequent section.

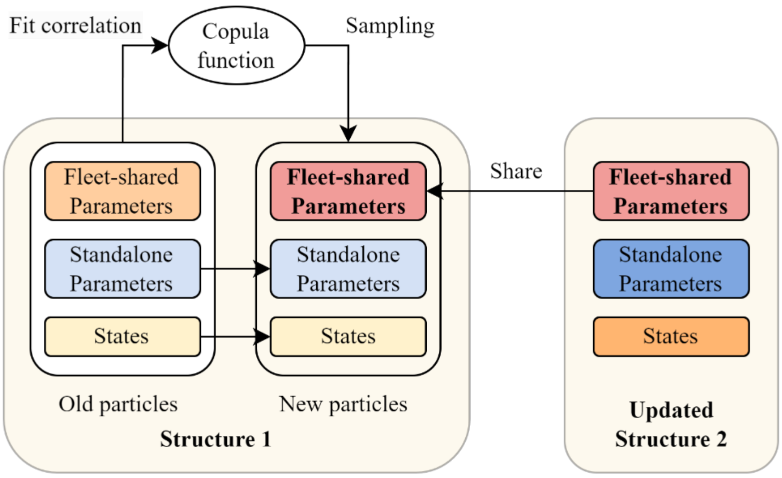

Figure 1 Parameter share scheme.

3.1 Gaussian copula

A copula [10] is a multivariate distribution with uniform margins on the unit interval. Gaussian copula function is easily constructed and fitted to approximate the multi-dimensional joint distribution and thus adopted here. The formula of the Gaussian copula function is as follows:

$$F_R^{\text{Gauss}}(u) = \Phi_R\left(\Phi^{-1}(u_1), \ldots, \Phi^{-1}(u_d)\right) \tag{9}$$

where Φ^{-1} is the inverse function of the standard normal distribution, $\mathbf{R}$ is the covariance matrix.
In each individual, the damage size and parameters possess prior distributions at initialization, and with the inclusion of inspection updates, a joint distribution is formed, whose correlation parameters are then be fitted by the Gaussian copula. It is important to note that the conversion of each marginal distribution into a Gaussian distribution is only for fitting the correlation matrix of the Gaussian Copula. The Copula is only used to characterize the relationship between the CDFs of the marginal distributions of multiple variables, so the samples generated by sampling with the Copula still follow the original marginal distribution, not the Gaussian distribution.

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3.2 Flowchart to Integrate the Parameter Sharing in the Particle Filter

With the assistance of the Gaussian copula, the correlation between the state and parameters are retained, that solves the problem for the parameter share. Here the full process of the parameter share scheme is introduced, as shown in Figure 2.

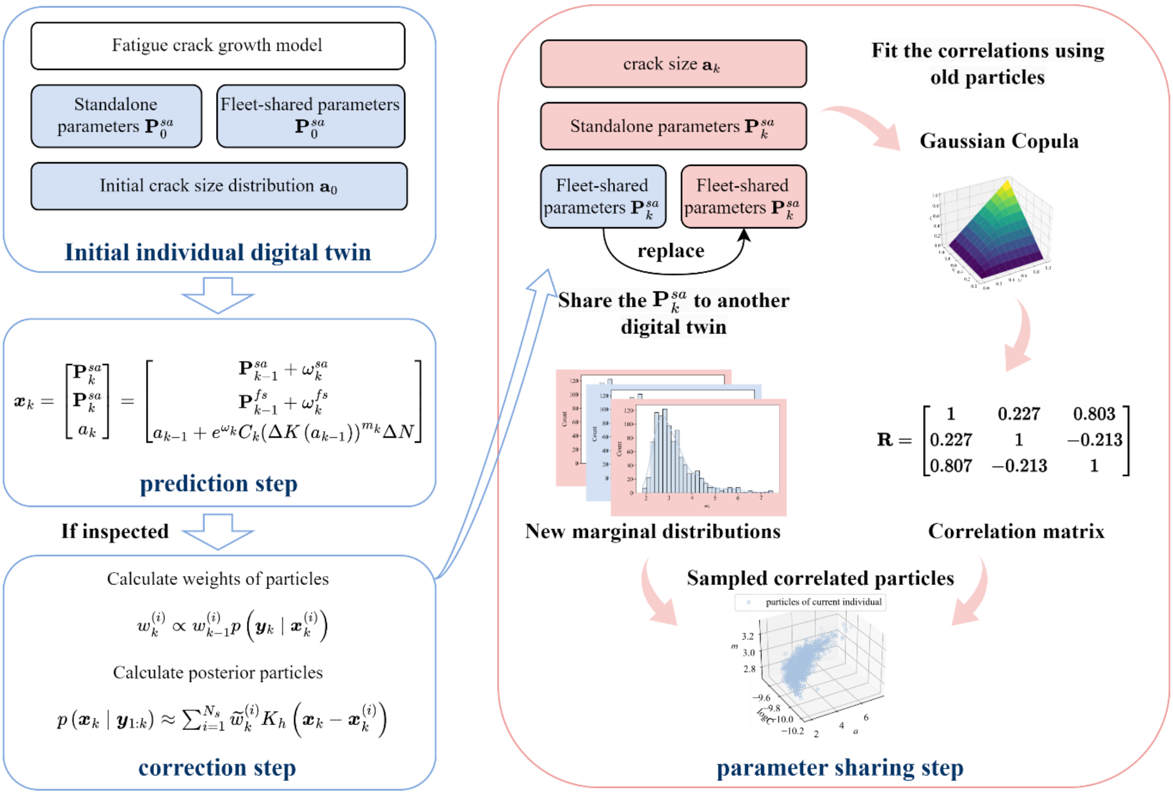

Figure 2 Flowchart of the particle filter with the parameter sharing.

4. Application to a hypothesized crack growth process

In this study, a simple crack growth process at the center of an infinitely large plate is used to validate the proposed method. The plate is subjected to a bidirectional uniform positive pressure σ, as shown in Figure 3.

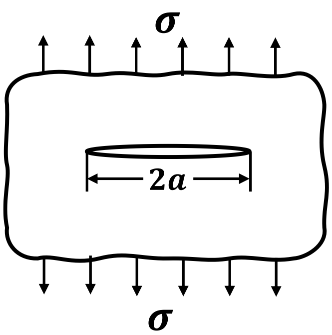

Figure 3 Simple crack in an infinite body subjected to a bidirectional uniform positive pressure.

The stress intensity factor range, denoted by ΔK, is calculated by:

$$\Delta K = \Delta\sigma\sqrt{\pi a} \tag{10}$$

where $\Delta\sigma$ is the stress range, a is the crack length.
The Paris law [7] is adopted as the crack growth model.

$$\frac{da}{dN} = C(\Delta K)^m \tag{11}$$

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where $\frac{da}{dN}$ are the increments of crack lengths per cycle, C and m are material parameters in the Paris law, which are uncertain.
Due to variations in the initial crack growth and material parameters, the crack growth histories of the three specimens differ. The true parameters of the three specimens are presented in Table 1. Considering that for the same material, m is observed to be essentially constant in the fatigue crack growth test, while C varies with the specimen, therefore in this calculation, we consider m as a fleet-shared parameter and C as an standalone parameter.
The inspections of each specimen were performed at different cycles, generated through random sampling, and the obtained observations were used to update the crack length and parameters distribution.

Table 1: True parameters of the three hypothesized specimens

Specimen	**1**	**2**	**3**
$a_{0,true}$	2.0	2.05	1.95
$logC$	-9.82	-9.85	-9.79
m	3.0	3.0	3.0

The parameter setting of the proposed approach and the traditional particle filter are shown in Table 2.

Table 2: True parameters of the three hypothesized specimens

Parameters	**Meaning**	**Value**
a_0	Prior distribution of the initial crack length	$N\sim(2,0.1^2)$
$logC_0$	Prior distribution of the $logC$	$N\sim(-9.8,0.1^2)$
m_0	Prior distribution of the m	$U\sim(2.5,3.2)$
$\Delta\sigma$	Stress range	25
η	Measurement error	$N(0, 0.5^2)$
ω_a	noise of the crack growth	$N\left(-\frac{0.05^2}{2}, 0.05^2\right)$
ω_1	noise in the evolution of $logC$	$N(0, 0.01^2)$
ω_2	noise in the evolution of m	$N(0, 0.01^2)$
N_s	Number of particles	1000
ΔN	Increment of loading cycle	1000

5. Result and Discussion

Figure 4 depicts the comparison between the diagnostic and prognostic outcomes obtained from the proposed method and the baseline method. It is evident that, in specimen 1 and specimen 3, the mean value of predictions generated by the proposed method demonstrates improvement. Conversely, in specimen 2, the baseline method exhibits better performance. This trend is also reflected in Table 3. Notably, owing to the presence of observation errors, there are instances where the accuracy of the baseline method surpasses that of the proposed method when updated individually. However, overall, the proposed method enhances the accuracy of both structural diagnosis and prognosis.

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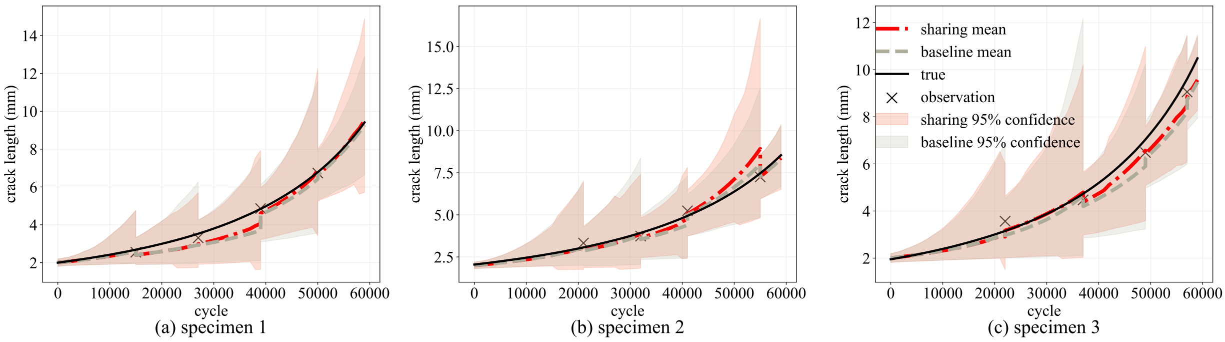

(a) specimen 1 (b) specimen 2 (c) specimen 3

Figure 4 Comparison of the diagnosis and prognosis results between the proposed approach and the baseline approach that updated separately. $\mathrm{m} = 3.0$

Table 3: Comparison of the prognosis accuracy (RMSE)

Method	1	2	3
Proposed	**1.734**	1.762	**1.776**
Baseline	1.998	**1.635**	1.962

Table 4 presents the calibration results of the model parameters. It can be observed that, in most cases, the proposed method outperforms the baseline method for the fleet shared parameter m, while exhibiting comparable calibration deviations. Similar trends are observed for the standalone parameter $logC$. This can be attributed to the existence of a certain correlation between these two parameters. It is essential to note that the comparison of advantages and disadvantages discussed here differs from the evaluation of diagnostic and pre-diagnostic errors. This distinction arises because the calibration results solely reflect the most recent update, whereas the diagnostic and pre-diagnostic accuracy assesses the overall performance.

Table 4: Comparison of the calibration bias of model parameters

Parameter	Method	1	2	3
m	Proposed	0.00836	**0.0109**	**0.00796**
	Baseline	**0.00124**	0.0165	0.0297
$logC$	Proposed	0.00799	**0.00332**	**0.0490**
	Baseline	**0.00204**	0.00412	0.0554

Conclusion

In this study, a novel particle filter-based approach is proposed for conducting the diagnosis and prognosis of structural damage growth in a fleet. The proposed approach incorporates a parameter sharing scheme, wherein standalone parameters are updated using the conventional particle filter, while an additional sharing step is introduced to distribute the fleet-shared parameter distributions among different individuals. To validate the efficiency of the proposed approach, several hypothesized specimens exhibiting simple crack growth processes are employed. The obtained results demonstrate that our approach enhances the accuracy of parameter calibration, thereby improving the diagnosis and prognosis performance. In future studies, further enhancements to the algorithm's stability are warranted, alongside validation on more complex structures and intricate damage evolution scenarios.

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