DOI: 10.13009/EUCASS2022-4920
9TH EUROPEAN CONFERENCE FOR AERONAUTICS AND SPACE SCIENCES (EUCASS)

Analysis of some nonlinear features in the flight dynamics of a jet airplane

KOLB Sebastien⋆
⋆*Centre de Recherche de l'Ecole de l'Air (CREA)*
BA 701, F-13661 Salon Air
sebastien.kolb@ecole-air.fr

Abstract

Some issues and features related to nonlinear dynamics are examined for a classical model of jet airplane in this study. Concretely these nonlinear features may imply unexpected events and it is then important to evaluate if there are dangerous or not. Normally they are linked to the real behaviour and physics of an aircraft but may also explain how a simulator (unexpectedly) reacts in certain circumstances.

Here the analysed model is the Bizjet full-scale aircraft which is provided by Professor R. F. Stengel for study purposes (with characteristics closed to a Learjet). In order to perform the nonlinear analysis, the Matcont toolbox is mainly employed. Amongst others in this work, state jumps are observed and give rise to a nonzero bank angle during classical longitudinal flights. The phenomenon is described mathematically by means of a bifurcation diagram which exhibits some real (pitchfork) bifurcations. In order to cope with this problem, the dangerous zone is delimited. There may also be some Hopf bifurcations which create periodic orbits suddenly. the predicted behaviours are illustrated and corroborated with time simulations. Moreover, this type of T-tail aircraft can be responsible for a deep stall phenomenon. By calculating the basin of attraction, we try to delimit the zone of initial conditions leading to such a phenomenon. More generally a diagnosis of the deep stall prone configurations is made (especially aft centred aircrafts) in order to avoid them. A loss-of-control can besides present an important transient nature. In this case, the qualitative results depend no more on the local properties and other ways must be employed to diagnose correctly the situation during the upstream analysis part which improves understanding (and permits to avoid numerous time simulations).

Finally, one hope is to contribute to the pre-design of a jet airplane so as to avoid such events and to diminish the associated risks. In order to be mathematically rigorous (for researchers) and meaningful (for aerospace engineers or pilots), for every addressed critical points, the mathematical analysis and the underlying physics are both exposed. The point-of-view of the pilot is also investigated and proposals of training exercises (on simulator) for loss-of-control are formulated.

Introduction

A classical approach of the analysis and control of flight dynamics is based on the linear theory like most of the engineering training. It is a powerful method, since it permits to diagnose the stability, to separate several modes and potential decoupling, to diagnose their characteristics, etc. Nevertheless some issues may appear because of the nonlinear dynamics cannot be retrieved and expertised such a way. Unfortunately these nonlinear features may imply unexpected events and it is then necessary to assess how hazardous there are in the design phase but also if some further training is adequate for a well educated pilot or engineer. Someone disagnosing such a behaviour wonders always if it is really the real physics or if the modelling needs some improvements. But it is already a first step to perform the qualitative analysis and to explain how and why it can happen with a real aircraft or a simulator and try to think on the possible reactions in these circumstances.

In what follows, the attention is focused on classical flights such as a pure longitudinal flight. Even there, suddenly the aircraft may stabilised itself in another equilibrium or sustained periodical oscillations may happen. Some conditions for which it may happen are explicited. The bare airframe is first examined but also some aircraft-pilot couplings. A simple rate limited actuator may be at the origin of some flying qualities cliffs. Indentifying how these phenomenons are met like an aggressive piloting or if it can happen requires a nonlinear analysis. Nowadays these

DOI: 10.13009/EUCASS2022-4920

points are taken into account in the design phase but it can always be a bad surprise to face. Some experience with this type of nonlinear phenomenons reveals to be helpful in order to be able to take some reasonable decisions, but sometimes only a well designed or well known aircraft can permits to cope with these problems.

After presenting the aircraft model with some general considerations, several flight cases (open-loop and with pilot-in-the-loop) are studied essentially with nonlinear dynamical system theory (bifurcation theory).

1. Aircraft model

The reference airplane for this study is the Bizjet full-scale aircraft provided by Professor R. F. Stengel in both his homepage and classical textbook[8] and with characteristics closed to a Learjet.

For such a study, the mathematical model is written under the form of an autonomous ODE (Ordinary Differential Equation)

$$\dot{X} = F(X, U) \tag{1}$$

whose state vector X and control vector U are

$$\begin{aligned} X &= \{u, v, w, p, q, r, \phi, \theta, h\} && (2) \\ U &= \{\delta_x, \delta_e, \delta_a, \delta_r\} && (3) \end{aligned}$$

For the function of dynamics F, its expression is derived from the Newton's laws which are applied to an aircraft and lead to

$$\begin{aligned} m\left(\left(\frac{d\vec{V}_a}{dt}\right)_{R_b} + \vec{\Omega}_{R_b/R_0} \wedge \vec{V}_a\right) &= \vec{R}_{aero} + \vec{F}_{prop} + m\vec{g} && (4) \\ \left(\left(\frac{dI \cdot \vec{\Omega}_{R_b/R_0}}{dt}\right)_{R_b} + \vec{\Omega}_{R_b/R_0} \wedge I \cdot \vec{\Omega}_{R_b/R_0}\right) &= \vec{M}_{aero} && (5) \end{aligned}$$

The equations of translation and of rotation are written in the body-fixed frame R_b. The external forces comes from gravity, aerodynamics and propulsion. A possible contribution of propulsion to moment is neglected.

The model is quite realistic for all the static features (aerodynamic coefficients C_Z, C_X, Cm) and dynamical aspects (aerodynamic stability derivatives) and a wide range of AoA is covered (including post-stall). The textbook[8] furnishes all the details concerning the diverse modelling parts of the Bizjet and is very instructive.

2. Longitudinal flight

The pure longitudinal flight represents the most elementary phase in the exploitation and conception of an aircraft. A steady trimmed rectilinear flight with a vertical plane of symmetry for the aircraft has zero roll, sideslip and yaw angles and rates and no lateral speed component but forces in the vertical plane and (pitch) moment perpendicular to it are driving the aircraft. It corresponds to level flight or cruise for example. Nevertheless the pseudo-equilibria of climb and descent may also be taken into account by neglecting the equation corresponding to the (slow) dynamic of altitude h. The classical theory states powerful results such as the existence of one unique equilibrium solution once two parameters are fixed (two controls or two states for example) and the good predictability of the handling qualities via linear criteria.

Nevertheless a pure longitudinal flight which looks apparently standard can also be at the origin of some surprises and some sudden behavioural changes which can lead to losses of stability and control or degraded handling qualities. Two types of pathological cases are here observed, in one situation with multiple equilibria and in another one with periodic oscillations.

DOI: 10.13009/EUCASS2022-4920

2.1 Nonzero bank angle (Pitchfork bifurcation)

Normally when remaining in the flight envelope, the behaviour of the aircraft must remain intuitive in the sense that the classical theory is verified for pure longitudinal flight. Nevertheless it seems possible to find control values for which multiple equilibria exist. The computation (with the Matcont toolbox[2]) of the bifurcation diagram which plots the equilibrium state in function of the control paramater allows to catch such features. The figures 1 illustrate two situations where pitchfork bifurcations are at the origin of multiple equilibria. The practical consequence is the existence of a range of elevator angles δ_e for which the classical flight point is unstable whereas the aircraft stabilizes itself at a nonzero bank angle ϕ.

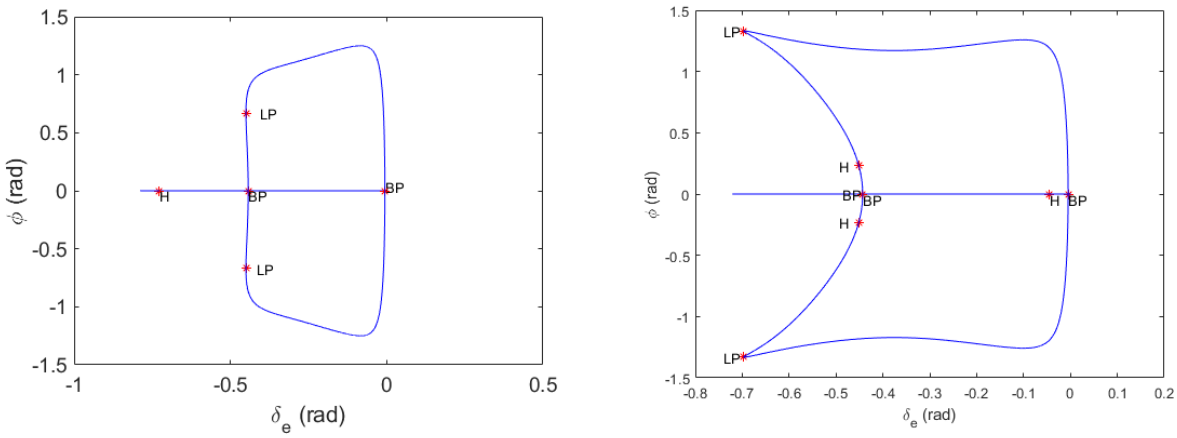

Figure 1: Pitchfork bifurcations: sudden apparition of stabilized flight with nonzero bank angle ϕ for $\delta_x = 58.6\%$ and an altitude of 10500 m (left) and for $\delta_x = 26.9\%$ and an altitude of 1000 m (right)

From the point of view of mathematics and flight dynamics, the pitchfork bifurcation[6] is respectively associated to a zero real eigenvalue and the loss of stability of the spiral mode. The pilot may be completely astonished because with a little change of elevator angle (and airspeed), the aircraft gets abruptely a nonzero bank angle. The situation at the right of image 1 seems to be the most dangerous since limit cycles exist for far lower values then the critical elevator angle (equivalent to a critical airspeed value). Thus when remaining or thinking as much as possible in the linear (reversible) frame, the knowledge of this critical value is not enough to avoid nonlinear features. Moreover a *hysteresis* phenomenon seems to happen here since when the jump occurs after a little variation of the elevator angle δ_e, a huge correction of this longitudinal control must be done in order to recover and to find again a normal situation (with zero bank angle).

A first case of bifurcation in jet flight dynamics was analysed, the loss of stability was associated to the sign change of a real eigenvalue leading to multiple equilibria but there exist also situations where a pair of complex (conjuguate) eigenvalues is responsible for that.

2.2 Periodic orbits (Hopf bifurcation)

Longitudinal flight consists mainly of stabilized equilibrium points whose characteristics are fixed so as to remain in the flight envelope and to verify good flying qualities. Nevertheless apparently periodic orbits may also appear suddenly. The underlying mathematical frame is first briefely exposed before dealing with some flight cases.

A Hopf bifurcation[6] correspond to an equilibrium whose Jacobian matrix has a pair of complex conjuguate eigenvalues λ of zero real part i.e. $\mathbb{R}(\lambda) = 0$. In the field of nonlinear dynamics, the major problem is the possible creation of limit cycles in addition to the loss of stability. Once the locus of Hopf bifurcation points is determined, it is possible to explain the diverse encountered situations and to separate the set of conditions, control parameters leading to different behaviours. For a longitudinal flight, the pilot uses the two controls of elevator and throttle $U = \{\delta_x, \delta_e\}$ which are sufficient to cover all the flight domain (admissible conditions) since piloting $\{\delta_x, \delta_e\}$ is equivalent to fixing $\{V_a, h\}$. Therefore these control parameters are varied and employed to draw a graph summurasing the diverse situations.

DOI: 10.13009/EUCASS2022-4920

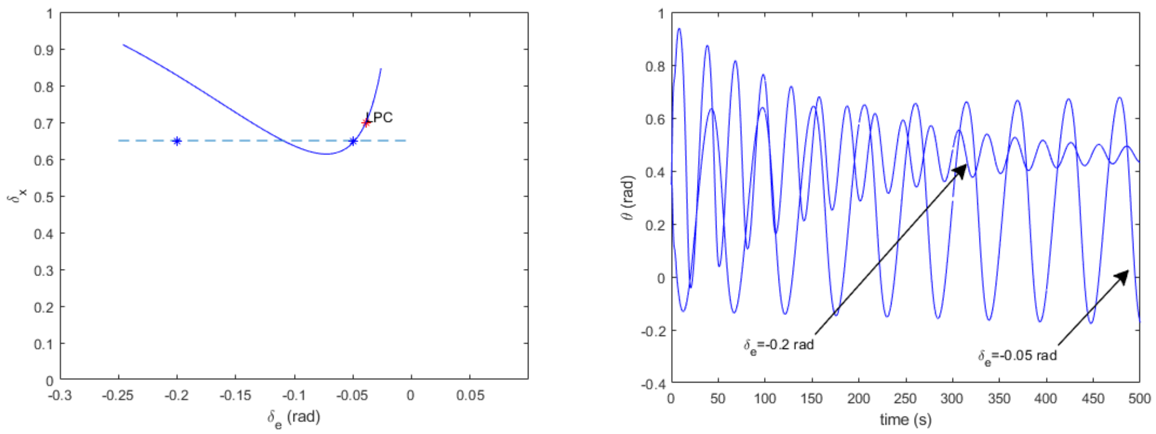

Figure 2: Locus of Hopf bifurcations (left) traced in the plane of longitudinal controls δ_e (elevator angle) and δ_x (throttle) and time simulations (right) for a fixed thrust throttle $\delta_x = 65\%$ and elevator angles δ_e lower and higher than the (Hopf) bifurcation value

In the (left) figure 2, the locus of the Hopf bifurcations permits to delimit the critical control parameters for which a change of stability occurs and where periodical orbits may be created thus rendering the aircraft behaviour non-standard and non-intuitive. Indeed a pilot acquainted with classical flight dynamics[3] may be surprised when meeting such persistent periodic oscillations whereas operating points are almost always stable equilibrium points (or possibly unstable but with a very low divergence). In any case, the handling qualities are degraded. From the flight dynamics viewpoint, the phugoid mode seems to be responsible for that since it becomes suddenly unstable and the observed oscillation period is of the same order of its (habitual) magnitude. The time simulations of the pitch angle θ (right figure 2) illustrate one case for which the oscillations are damped out and another one for which a permanent periodic orbit is observed.

From a rough engineering point of view, the major objective consists in avoiding these pathological situations but they need to be predicted correctly nevertheless. If an avoidance is not possible, the nonlinear analysis contributes to assess how hazardous it is. Indeed all this situations are not catastrophic and sometimes just knowing them or forgetting its routine experience to apply a convenient strategy is sufficient to cope with it.

In the first section, some phenomena occurring during a pure longitudinal flight were scrutinized. Diverse types of behaviours were exposed and predicted thanks to nonlinear analysis. After examining an apparently safe and trouble-free phase, the next section will deal with an hazardous phenomenon, the deep stall.

3. Deep stall

For an aircraft with a T-tail like the Bizjet which is studied here, a problematic phenomenon of deep stall may be encountered. Effectively once the aircraft is stalled, the wake of the main wing may disturb the aerodynamics of the horizontal tail (placed at the top of the vertical tail), implying a loss of the pitch control efficiency. Consequently, the standard pitch-down manoeuvre employed so as to leave the stall is useless in this situation. Unfortunately the aircraft may be stuck in an equilibrium state at high angle-of-attack.

After presenting the classical method to diagnose the deep stall proneness based on the aerodynamic coefficient of static pitching moment, a study in the (α, q) plane is accomplished. Even if it may be incomplete, it furnishes valuable information on the conditions leading to deep stall and on significant indicators. Later some results or differences coming from the global approach (with the whole longitudinal model) are also mentioned.

3.1 Characteristic equation of pitching moment

A simplified modelling of the deep stall concentrates on the (α, q) variables. Even if it is imperfect, it allows to get valuable information on the susceptibility of an airplane configuration. as a first step, the analysis work focuses its

DOI: 10.13009/EUCASS2022-4920

attention to the dynamical system

$$\begin{cases} \dot{\alpha} &= q + \ldots \\ \dot{q} &= \frac{1}{I_{YY}} Cm(\alpha, q, \delta_e) \frac{1}{2} \rho S c V^2 \end{cases} \tag{6}$$

where the aerodynamic coefficient of pitching moment is approximated by

$$Cm(\alpha, q, \delta_e) = Cm_{static}(\alpha, \delta_e) + Cm_q(\alpha) \frac{c}{V} q \tag{7}$$

The static part of the pitching moment permits to diagnose the configurations for which an equilibrium ($Cm = 0$) which is stable ($Cm_\alpha = \frac{\partial Cm}{\alpha} < 0$) at high angle-of-attack exists. Especially for an aircraft which is already suspected to meet such problems and examined for aft-centered configurations may encounter this problem. Once the existence of deep stall is proved for a certain configuration, then delimiting the (exact or approximate) conditions which lead to deep stall is important. Some information and calculations are exposed next on these topics.

3.2 Basin of attraction

For a two-dimensionsional sufficiently regular (autonomous differential) dynamical system, trajectories cannot cut themselves.[6] Therefore in the (α, q) plane, the stable manifold of the saddle-point at medium AoA permits to delimit the conditions for which an aircraft converges towards the equilibrium at high angle-of-attack.[5] Examining the dynamics of the subsystem (α, q) is maybe restrictive and based on a too strong decoupling (at high AoA), nevertheless it gives valuable information concerning the deep stall susceptibility[7] in a quite direct manner.

For the calculation of this section, a static margin variation of 30% (aft-centered configuration) is taken towards the reference airplane. The static pitching moment has 3 equilibria. The classical stable one at low AoA $\alpha_{LOW} = 0.19\ rad$, an unstable one at medium AoA $\alpha_{MEDIUM} = 0.32\ rad$ and the stable one at high AoA $\alpha_{HIGH} = 0.49\ rad$ corresponding to deep stall.

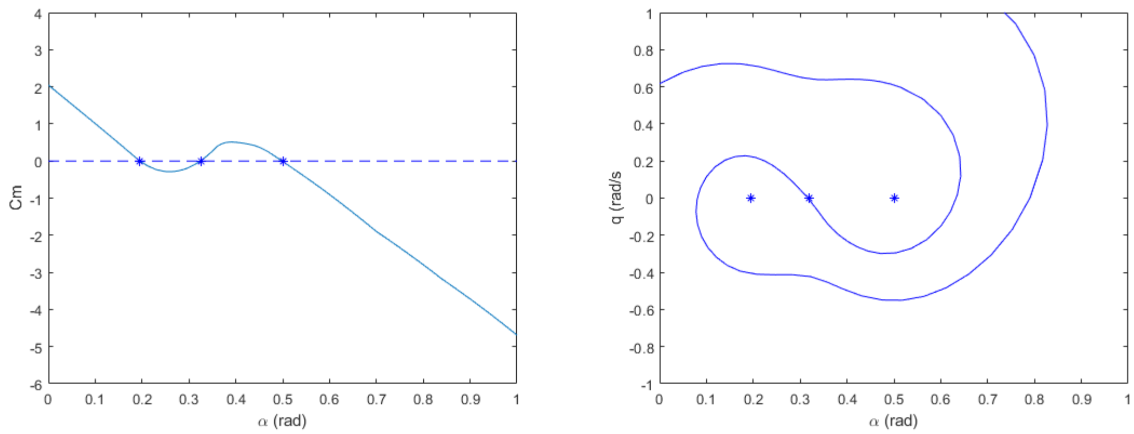

Figure 3: Aerodynamic coefficient of static pitching moment Cm_{static} (left) and basin of attraction in the (α, q) plane (right)

The time simulations (figure 4) illustrate the two situations. One situation for which it is locked at high AoA (left figure) and another one for which the aircraft stabilizes itself at the classical low AoA (right figure) even if the initial value of AoA is quite high. The initial conditions are either outside or inside the basin of attraction drawn in figure 3. The predictions deduced from the diagrams of the figure 3 are well correlated with the time simulations. Indeed they show the convergence to the equilibrium at low or high AoA respectively with values corresponding to zero pitching moment and initial conditions inside or outside the basin of attraction.

The study of the decoupled model (α, q) is interesting, but there remain some discrepencies with the global longitudinal flight model which will be evoked next.

DOI: 10.13009/EUCASS2022-4920

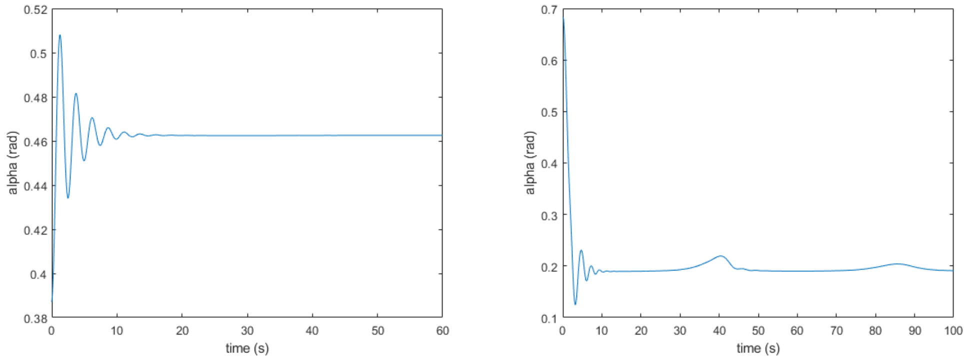

Figure 4: Time simulations for initial conditions inside (left) and outside (right) the basin of attraction of the equilibrium at high angle-of-attack

3.3 Some differences for flights at high angle-of-attack

When examining such post-stall flights, an awareness of some differences must be kept in mind so as to be able to have a reference (or not) with the standard cases of the usual flight envelope.[3] The short period mode at high angle-of-attack has apparently modal characteristics (pulsation, period, damping) which look like the ones at low angle-of-attack. The traditional closed forms seem to estimate correctly their values. Nevertheless all the (longitudinal) components are sollicited and the (α, q) decoupling which is generally made at low AoA is not possible. This point may be important for the design of a recovery procedure. Besides the phugoid mode (of the classical subsonic flight) disappears almost.

Moreover there is a huge negative flight-path-angle $\gamma < 0$ because of the considerable drag once stalled. This fact remains one of the major problem for flights at high AoA and implies often undesired strong descent rate. The level flight is not possible (the equilibrium calculation of the whole longitudinal model must take it into account).

After analysing some phenomenons linked to the dynamics of the bare aircraft which reveals that the handling qualities can be degraded due to the presence of multiple equilibria or periodic orbits, the following topics deal with issues coming from closed loop system and especially pilot in the loop.

4. Feedback loop with rate limiting

The feedback loop permits to modify the behaviour of the bare airframe. It can be performed by an automatic control system or by the pilot himself. The issue exposed next concerns the presence of a rate limited actuator which may give rise to periodic orbits (or even bifurcation of periodic orbits). In particular, rate limiting is an important source of pilot induced oscillations, mainly because of the sudden increase of phase lag leading to handling qualities cliffs. Well known examples are the accidents of the Saab JAS 39 Gripen and Lockheed Martin YF-22 Raptor during flight tests or air shows in the nineties. One major issue was that linear design and analysis are unable to catch such triggers and behaviours. To cope with these mishaps, some nonlinear methodologies were developped so as to diagnose the critical cases and behaviours during design phases, upstream of the first flights. It permits to avoid such (category II) PIO tendencies and to design adapted filters (phase compensation schemes) such that no (substancial) overall phase lag is added into the system.

The mathematical method employed to determine the periodic (steady state) solutions of the nonlinear differential equation is based on the harmonic balance whose characteristic equation is:

$$1 + L(j\omega)\, N(A, \omega) = 0 \tag{8}$$

where $L(j\omega)$ is a separable linear part and $N(A, j\omega)$ is the describing function of a simple nonlinear element associated to a dead-zone, a saturation, a hysteresis, a rate limited actuator for example. If a solution (A, ω) exists, then there is a periodic orbit of pulsation ω and (first-order approximaton of the) amplitude A.

DOI: 10.13009/EUCASS2022-4920

For a rate limited element,[1,4] the describing function depends only on the ratio $\frac{\omega_{onset}}{\omega} = \frac{R}{A\omega}$ with $\omega_{onset} = \frac{R}{A}$ (where R is the rate limit) and for the completely saturated case:

$$N(A,\omega) = \frac{4}{\pi}\frac{\omega_{onset}}{\omega}e^{-j\arccos\left(\frac{\pi}{2}\frac{\omega_{onset}}{\omega}\right)} \tag{9}$$

The flight case studied here has low speed and altitude (at a Mach $\mathcal{M} = 0.3$ and an altitude $h = 1000\ m$) and is linked to a low speed cruise flight just before the landing phase for example. The actuator is constrained by a rate limit $R = 10°/s$. A very aggressive pilot is chosen according to the crossover model theory,[1,4] that is to say a pilot gain $K_p = -3.5$ which is associated to a phase angle $\phi_c = -160°$ (the phase angle value when the amplitude curve crosses $0\ dB$). The selected pilot model is a pure gain -the simplest model- but it is already sufficient to obtain some interesting indications, otherwise a Neal-Smith or more sophisticated model can be employed so as to represent the neuro-muscular system (perception and reaction).

The figure 5 sums up the command channel for the longitudinal axis of pitch (near equilibrium). The linear part $L(j\omega)$ contains the bare airframe plus the pilot (a pure gain -the simplest model-), the nonlinear part $N(A, j\omega)$ corresponds to the rate limit.

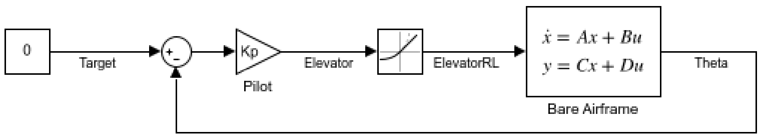

Figure 5: Aircraft - Pilot system

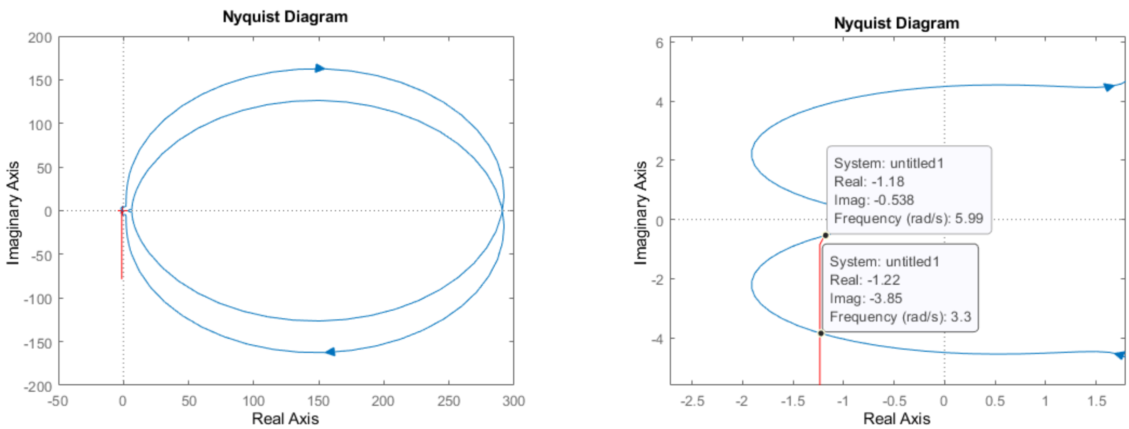

Figure 6: Nyquist diagram: bare airframe plus pilot $L(j\omega)$ (blue) and negative inverse of the describing function $-1/N(A,\omega)$ (red) with a zoom having some supplementary values indicated at the intersections

The analysis of the possible intersections of $L(j\omega)$ and $-1/N(A,\omega)$ on the Nichols chart (figure 6) gives the following results concerning existence (equation 8) and stability. The describing function method predicts a stable limit cycle whose pulsation $\omega \approx 3.3 rad/s$ (period $T = \frac{2\pi}{\omega} \approx 1.9s$) and an unstable limit cycle whose pulsation $\omega \approx 6 rad/s$. By finishing the resolution of the harmonic balance equation, it is possible to know that for the elevator angle δ_e, the stable limit cycle has an amplitude of 0.27 and the unstable limit cycle an amplitude of 0.05. The time simulations of figure 7 confirm these predictions.

Moreover the existence of an unstable periodic orbit has a consequence which is the convergence to zero when the initial conditions are small as illustrated by time simulations on figure 8 (another possibility to visualize the unstable limit cycle would be to make backward time integration).

In this particular case, since there exist some pilot gains for which no limit cycles exists and then a range of pilot gains (for aggressive enough pilots), for which there are two limit cycles, one stable and one unstable. The apparition

DOI: 10.13009/EUCASS2022-4920

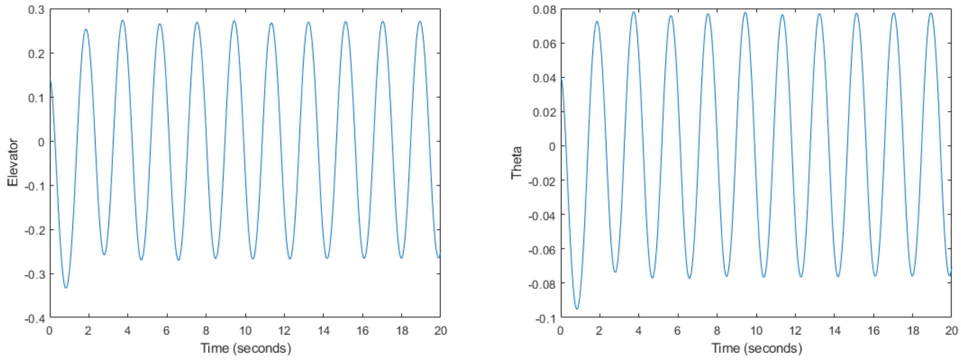

Figure 7: Time simulations for initial conditions converging to the stable limit cycle: elevator angle δ_e (left) and pitch angle θ (right)

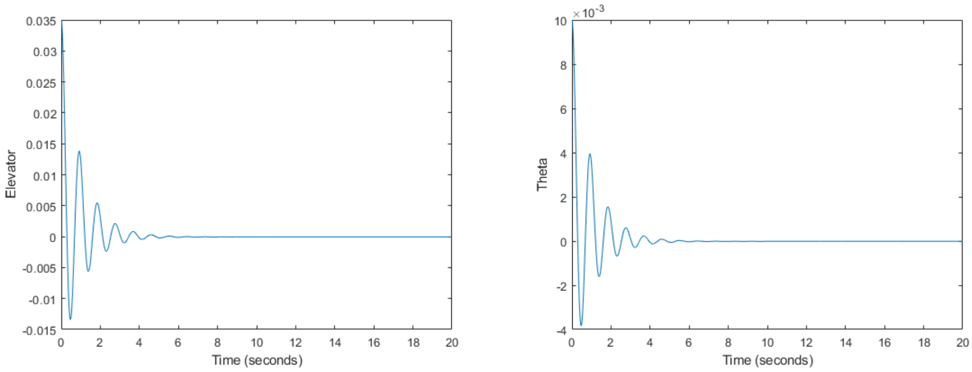

Figure 8: Time simulations for initial conditions inside the unstable limit cycle (convergence to zero equilibrium): elevator angle δ_e (left) and pitch angle θ (right)

of this periodic features come from a *saddle-node bifurcation of limit cycles*.[6] The practical consequence is that the pilot must be careful, because if he becomes too aggressive then the coupled system may abruptly change of behaviour and pilot induced oscillations may appear.

Conclusion

The flight dynamics presents some nonlinear features which cannot be examined with the standard linear theory. Normally an aircraft is designed in order to keep an intuitive (linear) behaviour as much as possible. Nevertheless for common flight phases like the pure longitudinal flight, it was shown that for certain conditions and ranges of elevator angles, multiple equilibria can exist as consequence of a real pitchfork bifurcation. In this case, the common theorem which is generally very helpful in flight mechanics and states that once two parameters are fixed, there is one unique achievable state, fails. Moreover a loss of stability due to a Hopf bifurcation may also be responsible for the appearance of periodic orbits in addition to the unstable equilibrium point. They degrade the handling qualities most of the time but the degree of severity can be very different. At the end, a phenomenon of pilot induced oscillations was triggered by a rate limited actuator. Even if the aggressiveness of pilot inputs seems a key factor, design problems can be avoided thanks to such type of nonlinear analysis (involving bifurcation of limit cycles for example).

In this study, classical flight phases were scrutinised and may encounter uncommon behaviours. Predicting them is helpful for a real aircraft but also for a simulator, since it allows to understand which phenomenon is occurring and eventually how to cope with it. This type of study can contributes to the analysis and comprehension of different topics. With the on-going mastery and technological progresses, engineers and researchers will try to enlarge more and more the usable flight envelope. Besides UAV are commonly employed nowadays and demand a less conservative approach towards safety with regards to the requirements for an aircraft transporting passengers, thus they can explore post stall manoeuvres more likely for example.

DOI: 10.13009/EUCASS2022-4920

Nomenclature

α	angle-of-attack (AoA)
β	sideslip angle
δ_x	thrust throttle
δ_a	aileron angle (roll control)
δ_e	elevator angle (pitch control)
δ_r	rudder angle (yaw control)
γ	flight-path-angle
ρ	air density
ϕ	bank angle
θ	pitch angle
ψ	azimut
Ω	rotation vector
b	span
c	chord
g	Earth's gravitational acceleration
h	altitude
m	mass
p	roll rate
q	pitch rate
r	yaw rate
(u, v, w)	airspeed components in the body-fixed frame $\mathcal{R}_b$
$I = \begin{bmatrix} I_{XX} & 0 & -I_{XZ} \\ 0 & I_{YY} & 0 \\ -I_{XZ} & 0 & I_{ZZ} \end{bmatrix}$	inertia matrix
(C_X, C_Y, C_Z)	aerodynamic force coefficients
(Cl, Cm, Cn)	aerodynamic moment coefficients
R_X	drag
R_Z	lift
$\mathcal{R}_a$	aerodynamic frame
$\mathcal{R}_b$	body-fixed frame
$\mathcal{R}_0$	Earth frame
S	(main wing) reference area
V_a	airspeed

References

[1] AGARD. *Flight Control Design - Best Practices*. Number RTO-TR-029. North Atlantic Treaty Organization, 2000.

[2] A. Dhooge, W. Govaerts, and Yu. A. Kuznetsov. Matcont: A matlab package for numerical bifurcation analysis of odes. *ACM Trans. Math. Softw.*, 29(2):141–164, jun 2003.

[3] B. Etkin and L.D. Reid. *Dynamics of Flight: Stability and control*. John Wiley & Sons, Inc., 3rd edition, 1996.

[4] G. P. Gilbreath. Prediction of pilot-induced oscillations (pio) due to actuator rate limiting using the open-loop onset point (olop) criterion. Master's thesis, Air Force Institute of Technology, Wright-Patterson Air Force Base, Ohio, 2001.

[5] M.G. Goman, G I. Zagainov, and A.V. Khramtsovsky. Application of bifurcation methods to nonlinear flight dynamics problem. *Progress in Aerospace Scineces*, 33:539–586, mars 1997.

[6] John Guckenheimer and Philip Holmes. *Nonlinear oscillations, dynamical systems, and bifurcations of vector fields*, volume 42 of *Applied Mathematical Sciences*. Springer-Verlag, New York, 1983.

[7] Sebastien Kolb, Olivier Montagnier, Laurent Hetru, and Thierry M. Faure. Real-time detection of an aircraft deep stall and recovery procedure. *Journal of Guidance, Control, and Dynamics*, 42(5):1185–1194, 2019.

[8] Robert F. Stengel. *Flight dynamics*. Princeton University Press, Princeton, NJ, 2004.