# Anti Sloshing Control Law Design For Launchers Using A Model-Free Approach

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# Abstract

The performances of a launcher's mission in terms of trajectory following accuracy, energy consumption and reliability depend on the ability of the control design to reject or reduce perturbations. After separation of the first stage, short boosts of the upper stage's thruster allow to reach the expected trajectory. Maneuvers are currently performed by orientating the nozzle which can lead to unknown propellant slosh dynamics and may affect the vehicle stability and controllability. In this paper, an adaptive approach is presented to derive an anti sloshing control strategy for a launcher's second stage subject to short boosts in space. Our objective is to mitigate the impact of disturbing phenomena to preserve a level of performances similar to the one obtained for the nominal launcher model. As the model characteristics are usually partially unknown or could evolve during the trajectory's phases, adaptive solutions have been sought for to stabilize their evolution, and expand the operating range of the controller<sup>1,2</sup>. A new control design based on the developments in the field of model-free control (MFC) approach<sup>3,4,5,6</sup> is proposed here. The results obtained can be compared to those of more traditional methods which have certified results on launchers such as  $\mathcal{H}_{\infty}$  synthesis<sup>7,8</sup>. Simulations demonstrate the improvement of the disturbance rejection and the recovery from initial sloshing conditions with the ultra-local adaptive model of the model-free controller.

# 1. Introduction

Robust and adaptable control laws are of paramount interest for the sake of future spatial missions. Being able to adapt to the evolution of environment and guarantee the stability and the safety of the system in presence of perturbations could lead to new possibilities. Relaxing constraints insuring robustness would widen the scope of possibilities during the design phase. Recent improvements on European engines lead to study reignitions in space to control the launcher's attitude or other characteristics such as the sloshing phenomena. This effect, generated by the movement of ergols in the tanks, needs to be controlled to avoid uncertainties in attitude or resonance with the spacecraft's motion. Propellant sloshing is considered as one of the major sources of perturbations that should be accounted for in the design of the attitude control laws. In classical design of linear control law for a launcher stage, this perturbation is not represented in the dynamics model. One relies on robust linear control<sup>9,10</sup> to limit this perturbation impact on the launcher stability at the potential cost of higher conservatism. Therefore gains in efficiency could be sought for by introducing a model of the dynamics of sloshing in the design phasis but this induced movement is proved to be difficult to represent accurately. Accounting for this phenomenon can be performed via adaptive methods. Several approaches have been designed for this purpose, such as those of  $2^{1}$ , reference control with a switched adaptive strategy 11,12, adaptive augmenting control algorithms which uses data to adjust the total loop gain on-line<sup>13</sup> or with model-free adaptive control by building a virtual equivalent dynamical linearization data model<sup>3</sup>. Using a computational fluid dynamics (CFD) approach to tackle the sloshing issue leads to an important computational burden and a complex study beforehand<sup>14</sup> . An easier representation of sloshing is usually introduced using equivalent mechanical model such as spring-mass or pendulum as they provide a fair compromise between accuracy of the representation and model complexity<sup>15,2,16,17</sup> but they may not reflect all the coupling dynamics at stake. Several methods have been suggested to design a controller based on these models which avoids amplification of the fuels' movement e.g. Lyapunov-based nonlinear control<sup>18</sup>. They rely on a good knowledge of the physical phenomena involved which can be an major hurdle for future missions and launcher's design.

As the available knowledge on the sloshing phenomenon is most often sparse, it is of interest to design the control law with less dependency on the knowledge of a complete model and control the system with the data obtained online. The main goal here is to find the best compromise between precision and robustness in a disturbed context while

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adapting to unmodeled dynamics. In this paper, the focus is set on the model-free control method initially introduced by Fliess and Join<sup>19,4</sup>. This type of control allows to consider systems with a poor modeling knowledge and to handle perturbations, saturations and uncertainties. Such model-free controllers began to be applied to practical problems quite recently and have given successful results, see e. g.<sup>20,21,22,23</sup>. Their ability to handle unmodeled perturbations have already been illustrated in<sup>24,25</sup> but not yet for a launcher upper stage's control.

The main contribution of this paper consists in the development of a model-free controller for controlling the attitude and the sloshing phenomena of a launcher's third stage. The results obtained using model-free control are compared with those obtained using a structured  $H_{\infty}$  control law as developed in<sup>26</sup> with an observer, already used for miscellaneous applications such as the VEGA launcher<sup>27</sup>.

This paper presents in section 2 the case of study, its objectives and the considered dynamics for the launcher and sloshing phenomena. In section 3, details on the model-free control method are given such as its goal, structure and associated parameters. Results are provided in section 4 for two cases: a command on the attitude and a recovery from initial conditions with uncertainties and perturbations. Those are then compared to the ones obtained with a structured  $H_{\infty}$  synthesis. Perspectives and conclusions on the possibilities to widen the operating range of the controller are given in section 5.

# 2. Case of study and dynamics

# 2.1 Context

In a changing environment with different launchers configurations with for example possible upper stage reignitions during the Ariane 6 flight, new control laws need to be developed to adapt to perturbations and uncertainties and thus to increase precision with which the trajectory is followed. Constellations in particular need the upper stage thruster to perform short boosts to reach the expected target and precisely deliver every payload.

The problem tackled in this paper is the design of the control law for a launcher upper stage in the exoatmospheric phase. The third stage and the remaining payloads are propulsed by the main engine during short boost phases which are followed by orbital phases.



Figure 1: Exo-atmospheric phase with Ariane 6's third stage

Acceleration phases lead to perturbations as they induce motion of the ergols in the two tanks called sloshing. This phenomenon is not well damped and interacts with the launcher's dynamics. Sloshing as an oscillatory movement can be modeled as a set of individual oscillation mode, each one represented under the form of a pendulum with a specific mass and length<sup>2,16</sup>. An active control of this perturbation is needed to avoid adding constraints or antisloshing devices which would increase the total mass.

The control law sought for aims at reaching the expected launcher attitude while taming the sloshing in the tanks. The control is effected by deflection of the nozzle. Some uncertainties must also be considered as initial conditions can be different from the ones expected by the guidance algorithm or the existence of a deflection bias whose amplitude is unknown and might need to be estimated. Adaptive possibilities are useful to match the real physics and to bridge the gap with a simplified model. This nonlinear system must account for actuator saturations, design constraints, uncertainties and perturbations.

# 2.2 Objectives

Several objectives to be met by the control design need to be tackled on this flight phase. The main ones can be summarized by:

- The control of the sloshing phenomena by actively attenuating the ergols' motion in the tanks by resetting the pendulums angles.
- The control of the launcher's attitude: a reference input can be given to orientate the stage and the associated payloads. While this is not the main objective in this study, a precise and fast attitude tracking is being sought. The focus will be set on the response time and the overshoot to a step entry.
- Ensuring the stability of this nonlinear system while recovering from initial conditions or estimating a deflection bias  $\beta_{bias}$ . The effective deflection denoted  $\beta_r$  can be written as:

$$\beta_r = \beta_c + \beta_{bias}$$

with  $\beta_c$  the deflection control input. This bias appears as a disturbance after the controller.

- Minimizing the transverse acceleration. A close to zero acceleration needs to be obtained at the end of the boost.
- Handling the saturations inherent to the device design. The orientation of the main engine is subject to mechanical constraints leading to an angular rate limit and a maximal deflection angle.
- Uncertainties on the physical parameters which do not allow to perfectly represent the full dynamics. The pendulums are not a sufficient model to represent the sloshing issue and an uncertainty on the pendulums' lengths might need to be taken into account.
- Compensate delays. Actuator and computational delays are introduced in the system representation for a more realistic evolution of the system.

The model's complexity has been progressively increased to match those objectives. Miscellaneous mission parameters, the sloshing phenomena (one mode and then two modes), uncertainties and different delays were finally taken into account.

# 2.3 Sloshing model and state-space representation

Without loss of generality, in order to simplify the calculations and simulations, a planar description of the problem is considered in this paper.

The launcher is represented on figure 2 with two pendulums corresponding to the two sloshing modes. The figure 2 is using the notations:

- *i*: represents the sloshing mode taken into consideration: i = 1 for the simplified case with one sloshing mode,  $i \in \{1, 2\}$  for the full problem with two modes
- coordinate system representing the chosen vertical and horizontal axis respectively denoted  $x_0$  and  $z_0$  $(x_0, y_0, z_0)$ :

coordinate system associated to the launcher  $(x_E, y_E, z_E)$ : coordinate system associated to pendulum *i* 

- $(u_i, v_i, w_i)$ :
  - launcher's attitude  $\theta$ :
  - deflection bias  $\beta$ :
  - $A_i$ : attachment point of pendulum associated to the mode *i*
  - $B_i$ : center of the mass of the pendulum *i*
  - G: center of mass without the sloshing masses
  - T: point of application of the thrust P
  - pendulum length of sloshing mode i  $l_{p,i}$ :
  - $X_{\Omega}$  : coordinates of the point  $\Omega$
  - $L_i$ : algebraic distance between the pendulum attachment point and the launcher's center of mass without sloshing masses for the mode *i*:  $(L_i = |X_{A_i} - X_G|)$
  - $L_t$ : algebraic distance between the nozzle center of rotation and the launcher's center of mass without sloshing masses  $(L_t = |X_T - X_G|)$

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## 2.3.1 Sloshing model

As shown in several studies  $^{2,28,29,16}$ , sloshing phenomena can be quite well approximated by a pendulum or by a spring-mass system. The parameters for a pendulum representation are :

- the length  $l_{p,i}$  of the pendulum
- the mass  $m_i$  attached to it
- the position of the jointure point  $X_{A_i}$

The focus will be set on a case with two modes which allow a sufficiently representative configuration of the launcher to study the effects of sloshing and the possible mutual interaction of its modes.



Figure 2: Sloshing model with oscillating masses

# 2.3.2 State space representation

In order to find a state space representation of the system, the following notations are introduced:

- M: launcher's mass without sloshing masses
- I: launcher transverse inertia with sloshing masses
- $m_i$ : sloshing mass of the mode *i*
- $\gamma$ : launcher's acceleration  $\left(=\frac{P}{M+\sum m_i}\right)$ .
- $\omega_i$ : eigen-pulsation of sloshing mode *i* under acceleration  $\left(\omega_i = \sqrt{\frac{\gamma}{L_{p,i}}}\right)$
- $\xi_i$ : natural damping of the sloshing mode *i* under acceleration  $\left(\xi_i = \xi_i^{1g} \sqrt{\frac{9.81}{\gamma}}\right)$

A first representation of the system dynamics as a double integrator can be written as :

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$$I\ddot{\theta} = \overrightarrow{TG} \wedge \overrightarrow{F} = (X_G - X_T) P \sin\beta$$
<sup>(1)</sup>

$$\iff \ddot{\theta} = K_1 \beta \text{ with: } K_1 = \frac{P(X_G - X_T)}{I}$$
(2)

While this representation is too simplistic for an industrial study, it will be used in order to test and build the first steps of the model-free controller.

The small angles approximation is taken into account to find the following results.

By applying theorems of kinetic moment and resultant,  $\ddot{\alpha_1}$ ,  $\ddot{\alpha_2}$ ,  $\ddot{\theta}$  and  $\ddot{z}$  can be expressed as functions of state parameters.

With the following state vector:

$$X = \begin{pmatrix} \alpha_1 & \dot{\alpha_1} & \alpha_2 & \dot{\alpha_2} & \dot{z} & \theta & \dot{\theta} \end{pmatrix}^T$$
(3)

A state space representation can be given of the dynamics:

$$X = AX + BU$$
  

$$Y = CX + DU$$
(4)

The state matrix A is defined as:

$$A = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ a_{11} & da_{11} & a_{12} & da_{12} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ a_{21} & da_{21} & a_{22} & da_{22} & 0 & 0 & 0 \\ c_1 & dc_1 & c_2 & dc_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ g_1 & dg_1 & g_2 & dg_2 & 0 & 0 & 0 \end{pmatrix}$$

with:

$$\begin{split} a_{i,j} &= -\frac{\gamma}{l_{p_i}} \left( \delta_{i,j} + \frac{m_j}{M} - \frac{m_j L_j (l_{p,i} - L_i)}{I} \right) \\ da_{i,j} &= -2 \xi_j \frac{\sqrt{\gamma l_{p_j}}}{l_{p_i}} \left( \delta_{i,j} + \frac{m_j}{M} + \frac{m_j (l_{p,j} - L_j) (l_{p,i} - L_i)}{I} \right) \\ c_i &= \frac{m_i \gamma}{M} \\ dc_i &= \frac{2 m_i \xi_i}{M} \sqrt{\gamma l_{p,i}} \\ g_i &= -\frac{\gamma m_i L_i}{I} \\ dg_i &= -2 \xi_i \sqrt{\gamma l_{p_i}} \frac{m_i (l_{p,i} - L_i)}{I} \end{split}$$

The input matrix B:

$$B = \begin{pmatrix} 0 & b_1 & 0 & b_2 & -\frac{T}{M} & 0 & \frac{TL_t}{I} \end{pmatrix}^T$$

with:

$$b_i = -\frac{T}{ML_i} \left( \frac{1-ML_t(l_{p,i}-L_i)}{I} \right)$$

The output matrix C:

$$C = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

And the feedthrough matrix D:

 $D = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ 

# 3. Control Design with a model-free method

Instead of tuning a gain online as performed by adaptive control laws, the solution presented herein uses the model-free control scheme to evaluate the unmodeled parts of the dynamics while rejecting perturbations<sup>19,4</sup>. In this approach, the previous equations of motion and launcher's parameters are not used in the controller design. The plant behavior in terms of input-output relationship is assumed to be well-approximated by a system of ordinary differential equations, e. g. a second-order equation between output y and input u. Using the implicit function theorem, the model-free controller can be expressed as a linear function of the reference output trajectory's derivatives, a local approximation of the structural information and a PID control on the tracking error.

## 3.1 MFC Objective

In this study, the main objective sought for in the model-free method is the capacity to reject perturbations and unknown deviations from the rigid dynamics. With a control law based on an ultra-local model, a better control of sloshing modes might be achieved. Its capacity of estimating perturbations and its adaptability to uncertainties while controlling sloshing modes are the main characteristics which are evaluated here. The focus is put here on the evaluation of the model-free controller performances for our study case. Stability guarantees are not investigated but will be considered in future work, particularly by using the work of Delaleau<sup>30</sup>.

## 3.2 Structural basis of MFC

Since the plant behavior is assumed to be well approximated by a system of ordinary differential equations, the dynamics can be written as:  $f_i(y, \dot{y}, \ddot{y}, ..., u, \dot{u}, ...) = 0$  where  $f_i$  is a sufficiently smooth function.

The implicit function theorem yields that the equations can be synthesized in the form:

$$\frac{d^n y}{dt^n} = \phi(t) + \zeta \ u \tag{5}$$

where *n* is the derivation order chosen by the user,  $\phi(t)$  represents the structural information and  $\zeta$  is a non-physical constant parameter. The quantity  $\phi(t)$  is time-varying and gathers the parts that need to be compensated such as perturbations, uncertainties or unknown parts of the model.

The value of n is kept as a design parameter but here is set to two. The considered model is assumed to be approximated ultra-locally by a second order differential equation so by:

$$\ddot{\mathbf{y}}(t) = \phi(t) + \zeta \, u(t) \tag{6}$$

In the Laplace domain, equation (6) can be rewritten as:

$$s^{2}Y(s) - sy(0) - \dot{y}(0) = \frac{\phi(t)}{s} + \zeta U(s)$$
<sup>(7)</sup>

After differentiating it two times with respect to s:

$$2Y(s) + 4s\frac{dY(s)}{ds} + s^2\frac{d^2Y(s)}{ds^2} = \frac{2\phi(t)}{s^3} + \zeta\frac{d^2U(s)}{ds^2}$$
(8)

To avoid derivative terms leading to increased noises, equation (8) is multiplied by  $\frac{1}{s^3}$ :

$$\frac{2Y(s)}{s^3} + \frac{4}{s^2}\frac{dY(s)}{ds} + \frac{1}{s}\frac{d^2Y(s)}{ds^2} = \frac{2\phi(t)}{s^6} + \frac{\zeta}{s^3}\frac{d^2U(s)}{ds^2}$$
(9)

All terms are of the form  $\frac{1}{s^{y}} \frac{d^{n}F}{ds^{n}}$ . We use the Cauchy's formula:

$$\frac{1}{s^{\gamma}}\frac{d^{n}F}{ds^{n}} \longleftrightarrow \frac{(-1)^{n}}{(\gamma-1)!} \int_{0}^{t} (t-\tau)^{\gamma-1}\tau^{n}f(\tau)d\tau$$
(10)

Transformed back to time domain and on an integration window of duration *T*, the expression of  $\phi(t)$  can be approximated by:

$$\hat{\phi}(t) = \frac{60}{T^5} \int_0^T \left( T^2 - 6T\tau + 6\tau^2 \right) y(\tau) d\tau - \frac{30\zeta}{T^5} \int_0^T (T - \tau)^2 \tau^2 u(\tau) d\tau \tag{11}$$

Estimate  $\hat{\phi}(t)$  is obtained by integration over the time interval [t - T, t]. The size of the window is of paramount importance in order to obtain a representative result.

For numerical implementation of the controller, the expression of the estimate  $\hat{\phi}(t)$  needs to be discretized. A trapezoidal approximation gives:

$$\hat{\phi}(t) = \frac{60}{T^5} \sum_{k=0}^{N} a_k \left( T^2 - 6kT_s + 6(kT_s)^2 \right) y(k) - \frac{30\zeta}{T^5} \sum_{k=0}^{N} a_k \left( T - kT_s \right)^2 (kT_s)^2 u(k)$$
(12)

with:

$$a_{k} = \begin{cases} T_{s}/2, & k = 0 \text{ and } k = NT_{s} \\ T_{s}, & k = 1, \dots, n-1 \end{cases}$$

where  $T_s$  is the sampling period and N is chosen such as  $N.T_s = T$ .

A robust controller can be designed based on the estimation of the unmodeled dynamics and perturbations given by  $\hat{\phi}(t)$ . The loop is closed via an intelligent PID controller, or i-PID controller. The resulting control expresses as a linear function of the reference output trajectory's derivatives, a local approximation of  $\phi(t)$  and a PID control on the tracking error. The controller form taken into account is:

$$u(t) = \frac{-\hat{\phi}(t) + \ddot{y}^* - K_I \int e - K_D \dot{e} - K_P e}{\zeta}$$
(13)

where  $\ddot{y}^*$  is the second order derivative of the reference output trajectory  $y^*$ , e is the error between the measured output y and the reference value and  $K_p$ ,  $K_i$ ,  $K_d$  are gains values. The value of  $\hat{\phi}(t)$  is an estimate of  $\phi(t)$ . Algebraic estimators are considered to find those values, as developed in<sup>31</sup>.

#### 3.3 Approach parameters

There are three parameters to tune in this method:

- ζ : this non physical constant has a great impact on the system's behavior and can modify the damping on θ and α<sub>i</sub>. We notice the divergence of the control law for values too far from one. The parameter ζ is chosen in order to guarantee that ζu and ÿ have the same order of magnitude.
- $T_s$ : the step of the integration window.
- N: the number of points considered in this window.

The integration window T is computed by multiplying N and  $T_s$  and should correspond to the attitude's period. The values of  $T_s$  and N are then codependent. Several trials are made to find the values matching with the study case and are presented in the next section.

Another parameters of this study are the tuning gains of the PID controller. Those are determined by a structured  $H_{\infty}$  synthesis approach with the same dynamics taking into account two sloshing modes. The PID gains are tuned considering no perturbation since the unknown parts of the dynamics will be taken into account in the  $\phi(t)$ .

# 4. Simulation Results

#### 4.1 Approach

The model-free controller being built step-by-step for reuse purposes, the first step consists of a simplified rigid model without uncertainties, delays and sloshing. The results presented here consider the highest complexity: with parameters uncertainties, saturations, delays and sloshing modes. The results will shed light on the resonance between the modes and the difficulties to reach a null sloshing angle with two modes.

The values of  $\zeta$  and of the integration window *T* can be difficult to choose since no constraint is explicitly given on those. Their choice was made after several tests and evaluation of the magnitude of the attitude's motion period, of  $\ddot{y}$  and *u*.

The first simulation uses a first-order step response as an entry to help controlling the launcher's attitude with a consign. The other test tries to recover from several uncertainties and perturbations by bringing the attitude back to zero.

# 4.2 Features

The initial conditions consist of having non zero sloshing angles at the beginning of the boost. A small deflection bias is considered in the simulations and a first order Padé approximant is used to represent the delays.

Uncertainties on some parameters will be considered in this study to deviate from the nominal model. The thrust, inertia, attachment point and length of the pendulums representing the sloshing modes have added percentages on their original value.

## 4.3 Structured $H_{\infty}$ Synthesis and Observer

The theory developed by Gahinet and Apkarian<sup>26</sup> allowed to develop a structured analysis algorithm giving as outputs the PID gains and the Luenberger estimator gains to match some criteria. As explained beforehand, the main objectives here are to minimize the response time and the overshoot while stabilizing the system via phase control. An exclusion contour is defined to guarantee this characteristic. The observer used here is only considered to tackle the deflection bias issue. Its behavior can be assimilated to an integrator, allowing to reject the disturbance impacting the system after the command. The integral gain is then set to zero.

The values used for the proportional derivative controller gains are computed by structured  $H_{\infty}$  analysis.

## 4.4 Results

## **4.4.1** Resulting $\theta$ control input with one sloshing mode



Figure 3:  $\theta$  control input with one sloshing mode

While the requested  $\theta$  value is reached with a similar overshoot and time response compared to the  $H_{\infty}$  synthesis, the damping in  $\alpha$  is much faster, needing only one period of time to drastically reduce its amplitude.

# **4.4.2** Resulting $\theta$ control input with two sloshing modes

In the case when all sources of uncertainties have been integrated, it can be observed that the control inputs make the sloshing phenomena dissipate quickly, the corresponding angles are brought to zero in a period of time and  $\dot{z}$  reaches zero during the boost. If a sloshing mode is added, its angle does not converge to zero and this oscillation induces a

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Figure 4:  $\theta$  control input with two sloshing modes

small oscillation in  $\alpha_1$  as well as shown in figure 4. The attenuation of the first angle and the maintain of a second one at a constant and small amplitude with the MFC method is an admissible result for applications.

## 4.4.3 Recovery from initial conditions with one sloshing mode

Another objective can be to recover from initial conditions and a deflection bias while keeping  $\theta$  close to zero, see figure 5. In this configuration, the main goal is again to compensate the bias while reaching a null attitude while mitigating the sloshing effects.

With a single mode, the sloshing angle is well controlled by reaching zero in about 5 s. This quick control needs wider variations in  $\theta$  at the beginning of the simulation.

# 4.4.4 Recovery from initial conditions with two sloshing modes

In this case, the recovery from initial conditions is similar to the one obtained with a  $H_{\infty}$  synthesis. A quicker mitigation of sloshing modes is to observe, with a second sloshing mode not controlled as in the  $\theta$  command. A wider range of attitude is reached in order to reach zero again.

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Figure 5: Recovery from initial conditions with one sloshing mode

## 4.4.5 Results conclusion

For both use cases, a limit is quickly reached for the saturation on the attitude angular rate with the PD-controller. The model-free continues to control the system on a greater range of values: the structured synthesis diverges for an angular rate saturation three times larger than with the model-free controller (the attitude saturation being never reached).

The first simulations were made with a complex problem taking into account various uncertainties and perturbations e. g. delays, a deflection bias, .... Such tests led to a quick control of the sloshing phenomena, particularly with only one sloshing mode. A clear difference is apparent with two sloshing modes: the oscillations of the first mode are maintained by the second mode which is not amplified but not controlled. The two sloshing modes dynamics are more challenging to control but we observe that the chosen design leads to a clear damping of the first mode sloshing angle although oscillations of small amplitude can still be observed.

The sloshing phenomena is well damped with a MFC controller, quicker than with the structured  $H_{\infty}$  method but the attitude have more difficulties to match the target with a command on the attitude. The transverse velocity  $\dot{z}$  is brought to zero in this case too. If the main goal is only to recover from the uncertainties and perturbations without any command, the MFC and  $H_{\infty}$  controllers seem similar. While the attitude reaches slightly greater values, the model-free method allows more possibilities on the angular rate saturation with a two times lower limit before diverging.

# 5. Conclusion and future work

This study presented the application of a recent method of control on an industrial use case. The use of an ultra-local representation of the dynamics led to evaluate the unknown parts of the system and control while not supposing any specific dynamics in the design. Several design parameters need to be tested and carefully chosen in order to find an adapted solution for the problem. Model-free control proved to be useful for a launcher's attitude control in an uncertain and disturbed environment. While recovering from initial conditions or perturbations, the sloshing phenomena were mitigated faster than with classical control methods. Applications with model-free control leading to concluding results, this method seems useful for future developments in the domain. An automated way of selecting the miscellaneous design features could help improving its possibilities and could lead to a practical on-board implementation.

While this method led to a fast sloshing control, the different tests held to choose the parameters and in particular  $\zeta$  and  $T_s$  open perspectives to improve the method. This choice of values could be deepened into an adaptive method. This idea comes from a limit on the control with important delays. Changing the value of  $\zeta$  allowed to better satisfy

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Figure 6: Recovery from initial conditions with perturbations and two sloshing modes

the requirements and having an adaptive value could help to improve the robustness of the controller, see<sup>32</sup>. The focus being made on sloshing control with uncertainties and perturbations, the comparison with a structured  $H_{\infty}$  synthesis is not complete and other characteristics could probably be better handled with this widely used method. Further studies should be made on their robustness limits and on the stability proof. The model-free results will be compared to the ones obtained with a reinforcement learning method for sloshing control in the same use case in a future study.

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