Thrust Analysis by Length Reduction of an ED Nozzle with Fixed Expansion Ratio

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Abstract

This study numerically analyzed the thrust of the ED nozzle according to the length. The nozzle length was shortened by 10%, 20%, and 30% based on the ED nozzle with the throat angle of 30°. Alternatively, the nozzle length was shortened by 13% and 27% by increasing the throat angle. As a result, the thrust of the ED nozzle with the nozzle length shortened by 27% and 30% was similar to the basic reference ED nozzle. Therefore, it was confirmed that the length of the ED nozzle having the fixed expansion ratio may be reduced while maintaining the thrust.

1. Introduction

1.1 Altitude Compensating Nozzle

In the growing space market, the performance improvement of the launch vehicles is required for competitiveness. Methods for improving the performance of the launch vehicles include a method of increasing a characteristic velocity(C^*), which is a characteristic of a combustion chamber, and a method of increasing a thrust coefficient(C_F), which is a characteristic of a nozzle. It takes a lot of budget and time to develop a technology to increase the characteristic velocity, which is the characteristic of the combustion chamber. Therefore, we focused on the method of increasing the thrust coefficient, which is a characteristic of the nozzle. In particular, since the altitude compensating nozzle can improve the thrust coefficient only by changing the nozzle, the economic burden in technology development can be reduced.

Conventional bell nozzles used in rocket engines have a fixed nozzle exit pressure due to a fixed expansion ratio. Therefore, at the altitudes lower than the design altitude, overexpansion occurs due to the atmospheric pressure higher than the nozzle exit pressure, and at the altitudes higher, underexpansion occurs due to the atmospheric pressure lower than the nozzle exit pressure. By changing the expansion ratio of the altitude compensating nozzle, it is possible to minimize the performance loss caused by the difference between nozzle exit pressure and the atmospheric pressure according to the altitude in a bell nozzle that is commonly used. Altitude compensating nozzles include aerospike nozzle, dual-bell nozzle, and ED(expansion deflection) nozzles. Table 1 is a summary of the advantages and disadvantages of the ED nozzle, the aerospike nozzle, and the dual-bell nozzle among the altitude compensating nozzles.

Altitude compensation nozzle type	Expansion Deflection(ED)	Aerospike	Dual-bell
Advantages	 Operational at lower chamber pressures No Significant increase in engine mass Shorter nozzle length compared to bell nozzle 	 Reduction of vehicle base drag No moving parts and gimballing due to the entirely fixed geometry 	• Lower cooling requirement rather than ED nozzle and aerospike nozzle
Disadvantages	 High local thermal load in the nozzle throat High cooling requirement Nozzle flow complex 	 Higher mass and thermal load due to larger component(plug) Interaction of external vehicle flow with nozzle flow 	 Difficult to optimize nozzle contour design Increased engine mass due to additional extension of the nozzle

Table 1: Advantages and disadvantages of ED nozzle, aerospike nozzle and dual-bell nozzle.

1.2 Expansion Deflection Nozzle

Among the altitude compensating nozzles, the ED nozzle has a pintle, which is the center of the nozzle. By the pintle, the flow field inside the nozzle is changed according to the altitude, and the effective area at the nozzle exit changes accordingly. As shown in Fig. 1(a), at low altitudes, an open mode is formed in which the external flow enters the inside of the nozzle. It can be seen that the effective area of the nozzle exit of the ED nozzle is small in the open mode. As the altitude increases, the shear layer after the nozzle throat moves to the center of the nozzle and reattaches to the centreline of the nozzle. A reattachment shock wave is generated at the reattachment point of the shear layer. Therefore, at high altitudes, it becomes an closed mode in which a closed recirculation region is formed behind the pintle due to the pintle and shear layer as shown in Fig. 1(b). In the closed mode, the external flow does not enter the nozzle and has a larger effective area at the nozzle exit than in the open mode.



Figure 1: Flow Field in the ED nozzle (a) and open mode and (b) closed mode[2].

The ED nozzle was devised by Rao in 1960, and through research, two key characteristics were confirmed: a shorter length than the bell nozzle and the altitude compensation[3-4]. In the late 1960s, Wasko[5] and Muller et al. [6] conducted research on the ED nozzle. Wasko conducted a study on the performance of the ED nozzle including the external flow effect in the transition region[5]. Mueller et al. conducted a study on the section where flow separation occurred inside the ED nozzle[6]. Although interest in ED nozzle began in the 1960s, it was difficult to conduct research due to the complexity of the flow field inside the ED nozzle and numerical analysis. As the technique of numerical analysis developed, Taylor and Hempsell[7-11] conducted a study on the design parameters

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of the ED nozzle in the 2000s. As shown in Table 2, various studies on the ED nozzle conducted by research institutes in various countries since the 1990s are briefly summarized[7-21]. Research was conducted mainly on the flow field and performance of the nozzle according to the design parameters of the ED nozzle. Among them, EADS and ESTEC conducted a numerical study on the payload gain at the upper stage of the launch vehicle by applying the ED nozzle to the engine at the upper stage of Ariane 5[17]. Through the study, the thrust-to-weight ratio of the engine with the short ED nozzle was increased compared to the conventional engine with the bell nozzle.

Research institute	Country	Nature of Study	Focus of study
German Aerospace	Cormony	Numerical and	Transition and flow characteristics with
Center(DLR)	Germany	Experimental	various nozzle pressure ratio
University of Pristol	United kingdom	Numerical and	Flow characteristics of design variables
University of Briston	United kingdom	Experimental	for optimal design
University of New South	Australia	Numerical and	Thrust efficiency according to various
Wales(UNSW)	Australia	Experimental	ED nozzle shapes
			Shape Modelling for applying the ED
EADS, ESTEC	Europe	Numerical	nozzle concept to the upper stage of
			launch vehicle
College of Engineering	India	Numerical	• Flow characteristics with pintle base
Trivandrum	Inula	Numericai	shape
		Numerical and	• Flow and thrust characteristics of a
University of Beihang	China	Experimentel	nozzle combined with the ED nozzle and
_		Experimental	the dual-bell nozzle
University of Hashin China		Numerical	• Flow pattern in the large expansion ratio
University of Harbin	Ciina	numerical	ED nozzle

Table 2: Overseas studies on the ED nozzle.

In Korea, Chaungnam National University has conducted research on the ED nozzle since 2015[22-46]. Table 3 briefly summarizes the major studies on ED nozzle conducted at Chungnam National University. The ED nozzle was mainly applied to the KSLV-II to study the flow field inside the ED nozzle and thrust performance according to the design parameters of the ED nozzle. Among them, the possibility of length reduction by applying the ED nozzle to the first stage of the KSLV-II has been confirmed[37]. The length reduction method used in the previous study is to reduce the nozzle length by 10% while fixing other design parameters.

Year	Nature of Study	Focus of study	
2015	Numerical	One-dimensional analysis for the ED nozzle using matlab	
2016	Numerical	• Throttling and altitude compensating effect of the ED nozzle	
2017 Numerical and Experimental		• Flow characteristics of the ED nozzle at low nozzle pressure ratios	
		 Performance analysis according to the minimum distance 	
		• Transition characteristics of dual-bell nozzle and ED nozzle combined	
2019	Numerical and	shape	
2018	Experimental	• Flow characteristics according to the nozzle throat angle and pintle	
		inflection angle	
2010 Numerical and		 Thrust and Flow characteristic of 3D printed ED nozzle 	
2019	Experimental	 Thrust and Flow characteristic according to the pintle radius 	
2021	Numerical and	Thrust of 3D printed 2D Planar ED nozzle	
2021	Experimental	• Thrust and Aerodynamic load according to the pintle shape	

Table 3: Studies of Chungnam National University on the ED nozzle.

1.3 Methane Rocket Engine

Interest in methane propellant is increasing for the competitiveness of launch vehicles in the space market. As shown in Fig. 2, the specific impulse of the engine to which methane is applied is higher than that of kerosene, which is commonly used for launch vehicles. It also has the advantage of being cheaper than kerosene[48]. In addition, as it is a non-toxic propellant, it is easy to handling, so it is possible to reduce the cost required for facility acquisition and maintenance. Due to these advantages of methane, advanced space countries are developing methane rocket engine

technology. CNES of France and DLR of Germany calculated the thrust performance and system weight by applying methane to the booster rocket engine of the launch vehicle Ariane 5[49-50]. NASA of the United States has conducted combustion tests of the methane rocket engine[51]. In Europe, the Prometheus program is being promoted to reduce launch vehicle costs compared to the existing Vulcain 2.1 engine by using methane[52].



Figure 2: Performance of various propellant [47].

In the previous study, Chungnam National University compared the performance of the conventional kerosene rocket engine and the methane rocket engine based on KSLV-II[53]. As a result of the study, it was confirmed that the specific impulse and thrust of the methane rocket engine were higher than that of the kerosene rocket engine. Also, research has been conducted by combining the methane rocket engine with high performance, easy handling, and inexpensive and the ED nozzle that can improve the performance of the launch vehicle only by changing the nozzle[39, 42]. Through previous studies, basic data on the study of the methane rocket engine to which ED nozzles were applied were secured. Based on this, the thrust performance of the ED nozzle with reduced length was analyzed by applying the ED nozzle to the engine using the methane for the design of the engine of the upper stage of the launch vehicle.

2. Design of the ED nozzle

2.1 Basic ED nozzle

The contour of Case 1, a basic ED nozzle, has 80% of the length of a conical nozzle with a diverging half angle of 15° and is the same as that of the bell nozzle whose initial angle is 30° . The expansion ratio of the basic ED nozzle is 16. The contour of the ED nozzle was designed using the parabolic approximation of Eq. (1)[54].

$$y = a + bx + cx^2 + dx^3 \tag{1}$$

The pintle, the main structure of the ED nozzle, was designed using the ED nozzle throat design method proposed by Taylor and Hempsell[9] in Fig. 3. According to the ED nozzle throat design method, design from the initial reference point, Ω_a to Ω_b , Ω_c , Ω_d in order. The main design parameters for selecting the positions of the reference points are the nozzle throat angle(θ_t), the outer wall radii of curvature(R_w), pintle radius(G_t), the pintle wall radii of curvature(R_p), and the pintle inflection angle(θ_i). The nozzle throat angle is the same as the initial angle of the nozzle contour, so the nozzle throat angle of the basic ED nozzle is 30°. For the nozzle outer wall radii of curvature, 0.382 R_t was used according to Rao's nozzle. The pintle radius is the area that performs the throat in the ED nozzle. Therefore, the pintle radius was calculated to be the same as the throat area of the bell nozzle. The pintle wall radii of curvature was set to be the same as the pintle radius, and the pintle inflection angle was designed to be the maximum. The shape of the designed Case 1 is shown in Fig. 4.



Figure 3: Schematics of E-D nozzle throat modelling method[9].



Figure 4: Basic reference ED nozzle (Case 1).

2.2 Length Reduction of the ED nozzle

In a previous study, in order to reduce the length of the ED nozzle, a method of reducing the nozzle length by 10% was used[37]. In the same way, Case 2~4 were designed by reducing the nozzle length of Case 1, which is the basic nozzle, by 10%. Other than the nozzle length, all design parameters are the same.

The length of the nozzle designed by the parabolic approximation can be shortened by increasing the initial angle of the parabola corresponding to the nozzle throat angle of the ED nozzle. Using this, Case $5 \sim 6$ were designed with nozzle throat angles of 40° and 50° , which are larger than the 30° nozzle throat angle of Case 1, which is the basic nozzle. In Case $5 \sim 6$, the pintle shape differs from Case 1 as the nozzle throat angle increase. Table 4 shows the main specifications of Case $1 \sim 6$, and the length reduction is the ratio of the reduced nozzle length based on the nozzle length of Case 1. Fig. 5 shows the shape of Case $1 \sim 6$.

	Case 1	Case 2	Case 3	Case 4	Case 5	Case6
Nozzle throat angle (deg)	30	30	30	30	40	50
Length reduction	0%	10%	20%	30%	13%	27%

Table 4: Specification of length reduction of Case 1~6.



Figure 5: Schematic of Case 1~6.

3. Numerical Analysis by Length Reduction of the ED nozzle

3.1 Numerical analysis conditions

In the previous study, numerical analysis of burnt gas was performed using frozen flow and equilibrium flow analysis methods[55]. The frozen flow analysis showed lower performance of the nozzle than the equilibrium flow analysis but confirmed the possibility of understanding the performance of the highly calibrated nozzle in terms of time and cost. Therefore, numerical analysis was performed using the frozen flow analysis method. For numerical analysis, burnt gas properties of LO_x/LCH_4 with a mixing ratio of 3.4 were used. The burnt gas properties at the combustion chamber pressure of 2.0 MPa were derived using the NASA CEA code. Eight chemical species with large mole fractions shown in Table 5 were used in consideration of analysis efficiency. Table 6 shows the conditions used for numerical analysis.

Species	Mole Fraction
H ₂ O	0.46467
СО	0.18268
<i>CO</i> ₂	0.11942
H ₂	0.08771
ОН	0.07357
Н	0.02999
02	0.02813
0	0.01373

Fable 5: Chemic	al compositions	of <i>LO_x/LC</i>	H_4 from NAS	A CEA code[56].

Table 6: Computation conditions for analysis.

Configuration		Axisymmetric
Solver		Fluent 17.0, Density based solver
Turbulent model		k-ω SST
Domain Size		$x/r_e = 100, y/r_e = 30$
	Pressure inlet	2.0 MPa, 3386.06 K
Boundary condition	Pressure far-field	at 13 km altitude
	Wall	Adiabatic, no-slip
	Axis	

3.2 Thrust of length reduction of ED nozzles

The thrust of the reduced length ED nozzle using LO_x/LCH_4 as working fluid was calculated using Eq. (2) and is shown in Table 7. From Table 7, it can be seen that the reduction in thrust is reduced when the thrust of Case 2~4, in which the nozzle length of Case 1 is constantly reduced, and the thrust of Case 1 is compared. Among them, Case 4 is similar to Case 1 with a thrust difference of less than 0.3%. Case 5~6, in which the nozzle length was reduced by increasing the nozzle throat angle, also showed the same tendency of thrust. In particular, Case 6 showed higher thrust than Case 1. This is considered to have occurred because the velocity increased as the curvature increased with the shortening of the nozzle length[57].

$$F = \dot{m}v_e + (P_e - P_a)A_e \tag{2}$$

Case	Thrust [N]
Case 1 (Length reduction 0%)	439.68
Case 2 (Length reduction 10%)	430.79
Case 3 (Length reduction 20%)	435.70
Case 4 (Length reduction 30%)	438.44
Case 5 (Length reduction 13%)	436.13
Case 6 (Length reduction 27%)	439.84

Table 7: Thrust and exit velocity of Case 1~6.

4. Conclusion

This research was conducted as a basic study to apply the ED nozzle to the engine of the upper stage of the launch vehicle as a way to secure the competitiveness of the launch vehicle in the growing space market. In addition, it was performed in combination with a methane rocket engine, which is of increasing interest due to its high performance and low price. Numerical analysis was performed on the thrust of the ED nozzle, which was reduced in length by applying the ED nozzle to the engine using methane. For the ED nozzle, the length of the ED nozzle having the length of the 80% bell nozzle was shortened constantly, or the nozzle length was decreased by increasing the nozzle throat angle.

As a result, as the nozzle length was shortened, the decrease in thrust with the ED nozzle having the length of the 80% bell nozzle and the nozzle with the nozzle length reduced by 30% were similar. With a nozzle throat angle of 50° and a 27% reduction in nozzle length, the thrust of the nozzle was improved.

Through this research, it was confirmed that the shortened ED nozzle can be applied to the methane rocket engine. It is expected that the data prepared in this paper can be used as basic data for the design of the ED nozzle applied to the methane rocket engine to improve the performance of the upper stage of the launch vehicle.

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