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New approach for Weibull parameters boundary definition with unknown suspension times

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Abstract

Weibull Statistical analysis is one of the most widely used methods to predict the reliability characteristics of a population, being capable of providing reasonably accurate failure forecast using a relatively small sample size of test data. Despite it is always preferred to have a complete set of samples (i.e. without suspensions), in daily industry evaluations the presence of suspension times are very common, leading to situations in which the failure modes under analysis may be present in only a sub-set of the population, where the items, and therefore the life of the unfailed units, are unknown. For these cases, a new approach to analyse these particular problems is proposed, providing a straight forward methodology to determine the boundaries of the Weibull parameters linked to a particular population.

1. Introduction

Since Walodi Weibull invented the Weibull distribution in 1937, and wrote a dedicated paper on this subject in 1951, the Weibull distribution has become the leading method in the statistic world for fitting, analysing and predicting life components data [1]. The Weibull method works with extremely small samples, event two or three failures for engineering analysis. Advanced techniques, such as failure forecasting, substantiation of test design, and methods like Weibayes, and the Dauser Shift were developed to overcome many deficiencies in the data [1].

Weibull analysis has been frequently used in aviation industry to assess the reliability of fielded airplane system or components, based on the capacity to quantify reliability (or risk of failure) as a function of the age of product, being able to predict infantile, random of wear-out failures modes. Additionally Weibull analysis provides a reasonably accurate failure analysis and failure forecast with extremely small samples, with the possibility of increasing the accuracy of the results taking benefit of the effect of the non-failed units [2]. Another advantage of the Weibull analysis is that it provides a simple and useful graphical plot of the failure data, which may provide clues about the physics of the failures [1].

Apart from the aviation industry, due to its versatility, Weibull statistical analysis has been used in a wide range of analysis which may comprise warranty analysis, spare parts forecasting, maintenance planning and cost effective replacement strategies, wind distributions, signal scatter analysis, intangible assets life estimation, and other many analysis where time to failures is relevant [2].

2. Three and Two Parameter Weibull Distribution

The 3-parameter Weibull distribution function is defined through the following Cumulative Distribution Function (CDF), which represent the probability of failure prior to age t [1]:

$$F_{\beta,\eta,\alpha}(t) = 1 - exp\left[-\left(\frac{t-\alpha}{\eta}\right)^{\beta}\right] \quad \text{for} \quad t \geq \alpha \quad \text{and} \quad F_{\beta,\eta,\alpha}(t) = 0 \quad \text{for} \quad t < \alpha \qquad \textbf{(1)}$$

Where:

- β: Shape Parameter (Slope of the Weibull fitting line), which provides a scale of variation of the failure rate with age, in fact, if:
    - $\beta \leq 1$ Distribution shows that the failure rate decreases with age, and therefore it is associated with significant “infant mortality”
    - $\beta = 1$ Distribution shows that the failure rate remains constant with age, and therefore it is associated with a random success (exponential distribution)

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- $\beta \geq 1$ Distribution shows that the failure rate increases with time, and therefore it is associated with an ageing process, meaning that the items are more likely to fails as the times increases.

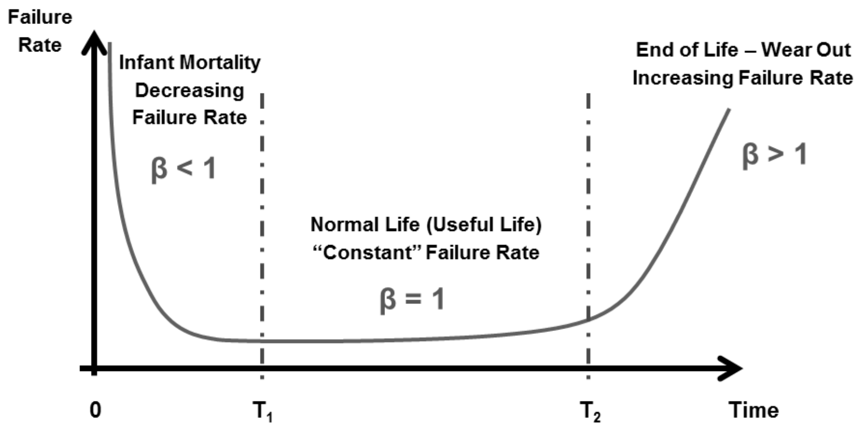

Figure 1: Relationship between the shape parameter (β) and failure rate throughout the life of a component

- η: Scale Parameter (Characteristic life), which represents, for those cases in which the Location Parameter (α) is null, the time at which the 63,2% of the units will fail, independently of the shape parameter (β) value. In the case in which the Location Parameter (α) is different from 0, then the time at which the 63.2% of the units will fail shall be measured from the time shift introduced.
- α: Location Parameter. Location parameter is used to introduce a time shift in the distribution.

For the case in which a positive location parameter is introduced, this indicates that the probability for that particular distribution is null up to that time, meaning that a failure cannot occur before this time. For this reason this parameters is generally set to 0, leading to the usual 2-Parameter Weibull distribution, which is the most widely used distribution for life data analysis. For this case, the Cumulative Distribution Function (CDF) is represented by the following equation [1]:

$$F_{\beta,\eta}(t) = 1 - exp\left[-\left(\frac{t}{\eta}\right)^{\beta}\right] \quad \text{for} \quad t \geq 0 \quad \text{and} \quad F_{\beta,\eta}(t) = 0 \quad \text{for} \quad t < 0 \qquad \textbf{(2)}$$

From this Weibull CDF, also the following Weibull characteristics functions may be defined [4]:

- Probability density Function (PDF), defined as the derivative of the CDF:

$$\mathrm{f}_{\beta,\eta}(\mathrm{t}) = \mathrm{F'}_{\beta,\eta}(\mathrm{t}) = \frac{\mathrm{d}}{\mathrm{dt}}\mathrm{F}_{\beta,\eta}(\mathrm{t})$$

$$f_{\beta,\eta}(t) = \frac{\beta}{\eta}\left(\frac{t}{\eta}\right)^{\beta-1} exp\left[-\left(\frac{t}{\eta}\right)^{\beta}\right] \quad \text{for} \quad t \geq 0 \quad \text{and} \quad f_{\beta,\eta}(t) = 0 \quad \text{for} \quad t < 0 \qquad \textbf{(3)}$$

- Reliability Function, defined as $1 - F_{\beta,\eta}(t)$:

$$R_{\beta,\eta}(t) = exp\left[-\left(\frac{t}{\eta}\right)^{\beta}\right] \quad \text{for} \quad t \geq 0 \quad \text{and} \quad R_{\beta,\eta}(t) = 0 \quad \text{for} \quad t < 0 \qquad \textbf{(4)}$$

- Failure Rate (or Hazard Function), defined as the probability that a lifetime comes to an end within the next small time increment given that the lifetime has exceeded t so far:

$$\lambda_{\beta,\eta}(t) = h_{\beta,\eta}(t) = \frac{\beta}{\eta}\left(\frac{t}{\eta}\right)^{\beta-1} \quad \text{for} \quad t \geq 0 \quad \text{and} \quad \lambda_{\beta,\eta}(t) = 0 \quad \text{for} \quad t < 0 \qquad \textbf{(5)}$$

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3. Weibull parameter estimation

Several methods exist to estimate the parameters that will fit a lifetime distribution to a particular data set. Some available parameter estimation methods include probability plotting through the use of Median Rank Regressions (MRR) and Maximum Likelihood Estimation (MLE). The most suitable methodology to be considered will vary depending on the data set, and in certain cases, on the life distribution.

- Probability Plotting MRR [7]: This method represents, on a specific probability plotting paper, the unreliability of each failure as a function of the failure time of each item. As the determination of the unreliability is a complex process, the median rank for each failure as function of the failure time of each item is considered as a good approximation for the unreliability representation. Once that the points have been represented, a least square methodology in X or in Y is used to find the best straight line that fits such set of data, allowing the determination of the shape and the scale parameters:
  - Shape parameter (β) is the slope of the fitted straight line.
  - Scale parameter (η) is the time at which the 62,3% of the units will fail.
- Maximum Likelihood Estimator MLE [8][9]: This method is based in the development of a likelihood function, taking into account the available data. Estimation of the parameter is based on the search of the values of (β, η) that maximized this likelihood function. This maximization process can be achieved through the partial derivation of the likelihood equations and setting the resulting equations equal to zero. Finding solution for these equations requires numerical methods, which should not be an issue considering current state of the art of the computing processes.

4. Suspension times

Normally in the industry is not usual to have a complete set of information in which it is known at the same time the failure time date for each one of the failed components and the other components have ended their operation time without being failed. Real study cases usually only offers a reduced set of items failed and some other components that are installed and operating without being failed (suspension times)

Performing the analysis considering only the failed items will lead to a conservative approach as all the information that the suspension data can provide are not being considered in the analysis [1][5].

For such purpose it is strongly recommended to consider, when available, the suspension times. Both MRR and MLE methods allows considering suspension times for the evaluation of the Weibull parameters [1][7][8][9].

5. Analysis of a Data Set with Unknown Suspension Times

Omission of suspension times can lead to a misleading analysis of the failed data. Some analyses show the relevancy of considering the suspension times. In [5] the authors performed a comparative between the different results obtained for the Reliability vs. Time curve, starting from a given data set of failed items, and considering the following five scenarios for the Weibull Parameters Estimation:

1. Only Considering the failed items without taking into account any suspension data
2. Assuming all the Suspension data with a life of 1 hour
3. Assuming all the Suspension data with a life higher than the latest failed item life
4. Assuming that all the suspensions are distributed within the failure time range (based on a Monte Carlo Analysis):
   - Case A: Weibull parameters corresponding to average value of η
   - Case B: Weibull parameters corresponding to maximum value of β

The results have been plotted in Figure 1, where it can be observed that assuming all the suspension data with a life on 1 hour provides a Reliability vs. Time curve almost identical to the one obtained through the consideration of only the failed data, being this a pessimistic approach of Reliability curve. In the other hand, the curve obtained for the case in which all the suspension data have a higher life than the latest failed item overestimates the Reliability prediction, being this approximation the most optimistic one. Additionally, and as it could be expected, the curve obtained with a distributed suspension along the failure time range (based on a Monte Carlo Analysis) is within the two predictions, the most pessimistic and the most optimistic.

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Despite the results of this analysis are very representatives, the main purpose of current paper is to complete the study performed in [5], providing an alternative methodology to compare and select the best approach for the case of unknown suspension items, by means of tracking the variation of the two Weibull Parameters as a function of the positon of the suspension times, comparing the results with the standards procedures and providing a methodology to evaluate directly the boundaries of such Weibull Parameters.

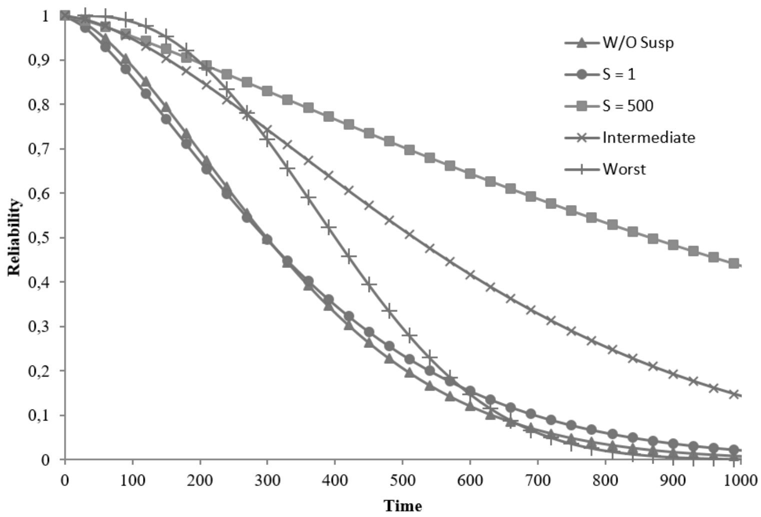

Figure 2: Reliability Vs Time plot

6. Alternative methodology for data set analysis with unknown suspension times

6.1 In field data

Comparison of the different methodologies will be performed based on a real case, analyzed as a consequence of a failure detected during the operation of a given in-serve aircraft fleet.

After having three occurrences in the in-service A/C fleet due to the same failure mode of a given equipment, a detailed analysis carried out by the supplier revealed a quality escape during the forging process of a mechanical component installed in the failed item. This quality escape showed that there were potential inclusions in the raw material, not detected by the traditional inspection methodology, which could affect to the structural integrity of the component.

Based on the tracking data, all the items manufactured from the potentially affected raw material were identified, and were considered as suspicious of having a potential not detected inclusion within its structure. In parallel, the supplier perform a statistical analysis to evaluate that, from a conservative point of view, as a maximum of three inclusions could not have been detected in the raw material, and therefore only three components of all the non failed identified items shall be considered as suspension times, as the other components will not develop such failure mode along their operational life.

Therefore, the available data to perform a Statistical analysis can be reduced to the Component Failure Time Data set of the three items failed (Table 1) and the life of the components potentially affected by the quality scape, but not failed (Table 2), where only three of this components could be affected by the issue, but it is impossible to determine exactly the three items affected. Being this case a clear example of a Data Set with 3 items failed and 3 suspension where the life of the suspension items is unknown.

Table 1: Fielded component failure time data set

Component Failure Times [hours]		
150	300	500

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Table 2: Fielded component suspensions time data set

Component Suspension Times [hours]					
25	50	75	100	125	150
175	200	225	250	275	300
350	400	450	500	550	600
650	700	800	900	1000	1100

6.2 Weibull analysis without suspension data

As the suspension data is not available, the easiest, but less precise way, is the evaluation of the Weibull parameters considering only the Failure Times captured in Table 1, assuming that there is no Suspension Data. Results obtained Considering a Median Rank RRX and a MLE approach are captured in Table 3.

Table 3: Weibull parameters linked to failed components without considering suspension times

Model	β	η [hours]
RRX	1,60	376
MLE	2,42	359

6.3 Weibull analysis considering Dauser Shift approximation

Fred Dauser (Pratt & Whitney Aircraft) proposed a method to "adjust" the Weibull parameters obtained from the failed population, in order to approximate the Weibull parameter estimation to those that would have been obtained if the suspension times were know [1]. It is recognized that this methodology is not the best practice, and it should be considered as the last option. The steps to perform the Dauser Shift are [1]:

1. Plot the failure data on Weibull Probability sheet to estimate the value of β
2. Calculate the mean time to failure (MTTF) [the sum of the failure times divided by the number of failures]
3. Draw a vertical line on the plot at the MTTF
4. Calculate the proportion failed (%) [number of failures/sum of failures plus suspensions]
5. Draw an horizontal line at the cumulative %
6. At the intersection of the vertical and horizontal lines draw a line parallel to the original failure Weibull, being this line the estimation of the adjusted Weibull approach.

Based on this approach, Weibull parameters considering the Dauser Shift approximation are captured in Table 4:

Table 4: Weibull parameters considering Dauser Shift Approximation

Model	β	η [hours]
Dauser Shift	1,60	426

6.4 Median Rank Parametric Study of unknown suspension times

Even the rank adjustment is one of the most widely used method to perform analysis considering suspension times, it shall be noticed that this method takes into account the position where the failure occurred but not exact time-to-suspension [6]. This means that, for the Median Rank analysis point of view, it is only important to consider the relative position of the suspension component life compared with the time at failure of each component.

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Determination of the potential scenarios to be assessed:

Therefore, taking into account the original problem under study in which there are three items failed and three suspension times, all the cases to be analysed for the two Weibull Parameters (combination of β and η) will be reduced to the evaluation of a limited number of cases, where is it only relevant the relative position between the suspension times and the failed items. The number of scenarios to be evaluated can be derived from equation (6), which is equivalent to consider 3 suspension times from a total number of 4 different relative positions, with repetition:

$$Number\ of\ Scenarios = \binom{m+k-1}{k} = \frac{m+k-1!}{k!\cdot\ (m-1)!} = \frac{6!}{3!\cdot 3!} = 20 \quad \textbf{(6)}$$

Where:

- m: Number of failed and not failed items
- k: Number of failed items

Table 5 shows the 20 scenarios to be considered for the evaluation of all the combinations of β and η.

Table 5: Scenarios to be analysed considering the MRR model

ID	Scenario	ID	Scenario	ID	Scenario	ID	Scenario
1	S_1 S_1 S_1 $\mathbf{F_1}$ $\mathbf{F_2}$ $\mathbf{F_3}$	6	S_1 $\mathbf{F_1}$ S_2 $\mathbf{F_2}$ S_3 $\mathbf{F_3}$	11	$\mathbf{F_1}$ S_2 S_2 S_2 $\mathbf{F_2}$ $\mathbf{F_3}$	16	$\mathbf{F_1}$ S_2 $\mathbf{F_2}$ $\mathbf{F_3}$ S_4 S_4
2	S_1 S_1 $\mathbf{F_1}$ S_2 $\mathbf{F_2}$ $\mathbf{F_3}$	7	S_1 $\mathbf{F_1}$ S_2 $\mathbf{F_2}$ $\mathbf{F_3}$ S_4	12	$\mathbf{F_1}$ S_2 S_2 $\mathbf{F_2}$ S_3 $\mathbf{F_3}$	17	$\mathbf{F_1}$ $\mathbf{F_2}$ S_3 S_3 S_3 $\mathbf{F_3}$
3	S_1 S_1 $\mathbf{F_1}$ $\mathbf{F_2}$ S_3 $\mathbf{F_3}$	8	S_1 $\mathbf{F_1}$ $\mathbf{F_2}$ S_3 S_3 $\mathbf{F_3}$	13	$\mathbf{F_1}$ S_2 S_2 $\mathbf{F_2}$ $\mathbf{F_3}$ S_4	18	$\mathbf{F_1}$ $\mathbf{F_2}$ S_3 S_3 $\mathbf{F_3}$ S_4
4	S_1 S_1 $\mathbf{F_1}$ $\mathbf{F_2}$ $\mathbf{F_3}$ S_4	9	S_1 $\mathbf{F_1}$ $\mathbf{F_2}$ S_3 $\mathbf{F_3}$ S_4	14	$\mathbf{F_1}$ S_2 $\mathbf{F_2}$ S_3 S_3 $\mathbf{F_3}$	19	$\mathbf{F_1}$ $\mathbf{F_2}$ S_3 $\mathbf{F_3}$ S_4 S_4
5	S_1 $\mathbf{F_1}$ S_2 S_2 $\mathbf{F_2}$ $\mathbf{F_3}$	10	S_1 $\mathbf{F_1}$ $\mathbf{F_2}$ $\mathbf{F_3}$ S_4 S_4	15	$\mathbf{F_1}$ S_2 $\mathbf{F_2}$ S_3 $\mathbf{F_3}$ S_4	20	$\mathbf{F_1}$ $\mathbf{F_2}$ $\mathbf{F_3}$ S_4 S_4 S_4

Where:

- F_i: Failed items ordered progressively as a function of the time at which the failures were detected (F_1 = 150, F_2 = 300 and F_3 = 500)
- S_i: Suspension data. These values should not be necessary extracted from Table 2, but just arbitrary values which respect the relative position between failed items in accordance with the sequence identified in the Scenarios. For this case, the following suspension times were considered

$$S_1 = 50,\ S_2 = 200,\ S_3 = 400,\ S_4 = 800) \quad \textbf{(7)}$$

It shall be noticed that, for the purpose of the boundary analysis performed along this paper, it is considered the possibility of having repetition in the suspension times, regardless if it is aligned or not with the in-field data.

Evaluation of the Weibull parameters for each scenario:

Taking the scenario combinations identified in Table 5, the two Weibull parameters were estimated through a Median Rank Estimation, with a least square methodology in X. Results of these estimations have been compiled in Table 6.

Table 6: MRR Weibull parameters as a function of the relative position of the suspension times

ID	β	η	ID	β	η
1	1,46	383	11	2,05	422
2	1,68	404	12	1,93	462
3	1,59	442	13	1,69	528
4	1,34	517	14	1,89	488
5	1,87	416	15	1,60	579
6	1,77	455	16	1,47	644
7	1,52	526	17	1,87	506
8	1,73	481	18	1,55	618
9	1,44	578	19	1,39	705
10	1,31	649	20	1,30	775

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Visualization of the Weibull parameters results:

Data gathered in Table 6 can be represented in a η vs β chart (Figure 3), where it can be observed that:

- Weibull parameters have a dependency on the relative position between the time at suspension and the time at failure
- Even taking into account that the suspension times are not know, the whole spectra of values for the Weibull parameters are bounded within a characteristic shape
- Values corresponding to low scale parameters (η) are related with higher shape parameters (β), which are related with those cases in which the suspension times considered has as low life in comparison with the failed items
- In the opposite side, values corresponding to high scale parameters (η), are related with lower shape parameters (β), which are linked with those cases in which the suspension items considered has a higher life in comparison with the failed items.
- There is a particular relative position between the suspension times in relation with the failed items which maximizes the value of the shape parameter (β), noted as $\eta_{\beta max}$.

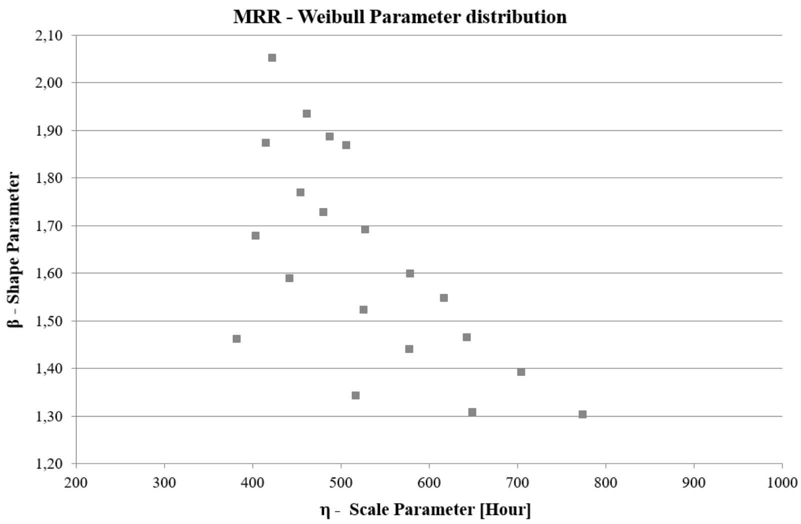

Figure 3: MRR Weibull parameter distribution

Weibull parameters boundary characterization:

Based on the information captured in Table 6 and the distribution of the Weibull parameters visualized in Figure 3, the boundaries of this cloud of points can be fixed based on the relative position between the suspension times and the failed items. Thus, the combination of suspension times and failed times that compounds the boundaries is defined in Table 7, where the last item of the left side upper boundary represent the combination of parameters that maximizes the value of the shape parameter (β). These boundaries have been represented in Figure 4.

Table 7: MRR Weibull parameters as a function of the relative position of the suspension times

ID	Lower boundary	ID	Left Side Upper boundary	ID	Right Side Upper boundary
1	S_1 S_1 S_1 $\mathbf{F_1}$ $\mathbf{F_2}$ $\mathbf{F_3}$	1	S_1 S_1 S_1 $\mathbf{F_1}$ $\mathbf{F_2}$ $\mathbf{F_3}$	12	$\mathbf{F_1}$ S_2 S_2 $\mathbf{F_2}$ S_3 $\mathbf{F_3}$
4	S_1 S_1 $\mathbf{F_1}$ $\mathbf{F_2}$ $\mathbf{F_3}$ S_4	2	S_1 S_1 $\mathbf{F_1}$ S_2 $\mathbf{F_2}$ $\mathbf{F_3}$	14	$\mathbf{F_1}$ S_2 $\mathbf{F_2}$ S_3 S_3 $\mathbf{F_3}$
10	S_1 $\mathbf{F_1}$ $\mathbf{F_2}$ $\mathbf{F_3}$ S_4 S_4	5	S_1 $\mathbf{F_1}$ S_2 S_2 $\mathbf{F_2}$ $\mathbf{F_3}$	17	$\mathbf{F_1}$ $\mathbf{F_2}$ S_3 S_3 S_3 $\mathbf{F_3}$
20	$\mathbf{F_1}$ $\mathbf{F_2}$ $\mathbf{F_3}$ S_4 S_4 S_4	11	$\mathbf{F_1}$ S_2 S_2 S_2 $\mathbf{F_2}$ $\mathbf{F_3}$	18	$\mathbf{F_1}$ $\mathbf{F_2}$ S_3 S_3 $\mathbf{F_3}$ S_4
				19	$\mathbf{F_1}$ $\mathbf{F_2}$ S_3 $\mathbf{F_3}$ S_4 S_4
				20	$\mathbf{F_1}$ $\mathbf{F_2}$ $\mathbf{F_3}$ S_4 S_4 S_4

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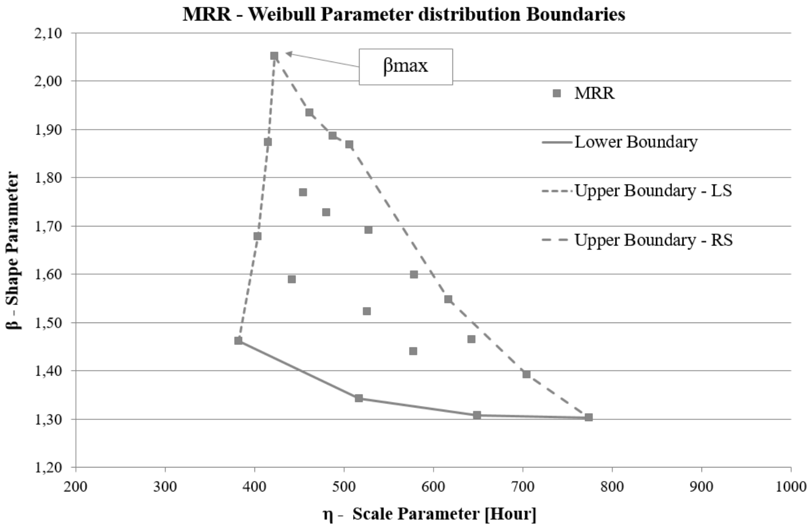

Figure 4: MRR Weibull parameter distribution Boundaries

In fact, the relative easy pattern that follows the combination of suspension times and failed times makes possible to generate an algorithm to quickly evaluate the boundaries of all the possible combinations of β and η parameters for the problem under study. These algorithms are described here below in pseudocode:

- MRR Lower boundary generation pseudocode algorithm:

```
START
    [L_FailedItems] = SET List with the Times of the Failed Items
    [MIN_ST] = SET Minimum Suspension time considered
    [MAX_ST] = SET Maximum Suspension time considered
    [L_SuspensionTimes] = SET List with the Suspension times equal, all of them, to MIN_ST
    [V_DATA] = COMBINE [L_FailedItems] and [L_SuspensionTimes]
    CALCULATE 2-Weibull parameters for [V_DATA]        'Evaluation of the first item
    FOR each suspension time in [V_DATA]
            REPLACE in [V_DATA] the [MIN_ST] suspension time considered in each FOR iteration by
                    [MAX_ST] value
            CALCULATE 2-Weibull parameters for [V_DATA]  'Evaluation of the additional boundary items
    ENDFOR
END
```

- MRR Upper boundary generation pseudocode algorithm:

```
START
    [L_FailedItems] = SET List with the Times of the Failed Items
    [MIN_ST] = SET Minimum Suspension time considered
    [L_SuspensionTimes] = SET List with the Suspension times equal, all of them, to MIN_ST
    [L_NonFailedItems] = Set List with the Times of the Non Failed Items    'As this methodology refers to
            MRR, list of non failed items can be reduced to arbitrary values that respect the relative position
            between failed items, not being necessary to consider all the intermediate values.
    [V_DATA] = COMBINE [L_FailedItems] and [L_SuspensionTimes]
    CALCULATE 2-Weibull parameters for [V_DATA]        'Evaluation of the first item
    FOR each time in [L_NonFailedItems] starting from the next value above [MIN_ST] [NFx]
            [NEW_S] = SET Time [NFx] considered in each FOR iteration
            FOR each suspension time in [V_DATA]
                    REPLACE in [V_DATA] the suspension time value considered in each FOR iteration by
                            [NEW_S] value
                    CALCULATE 2-Weibull parameters for [V_DATA]        'Evaluation of the
                            additional boundary items
            ENDFOR
    ENDFOR
END
```

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6.5 Maximum Likelihood Parametric Study of unknown suspension times

One of the main differences when comparing MLE and MRR methods is that MLE takes into account the exact suspension time for each not failed item when estimating the parameters. Therefore, taking into account proposed analysis described in section 6.1, to be able to consider all the potential scenarios, it is necessary to combine with the three failed items (Table 1) all the potential groups of three suspension times extracted from the list of non failed items (Table 2).

Determination of the potential scenarios to be assessed:

For these cases, the number of scenarios to be analyzed can be derived from equation (8), which is equivalent to consider 3 suspension times from the whole set of 24 non failed items, without repetition:

$$Number\ of\ Scenarios = \binom{m}{k} = \frac{m!}{k! \cdot (m-k)!} = \frac{24!}{21! \cdot 3!} = 2024 \quad \textbf{(8)}$$

Where:

- m: Number of not failed items
- k: Number of items to be considered impacted by the deficiency

Comparing the results from equation (6) and (7), it can be observed that, despite the scenario under analysis is simple (three failed items and three potential suspension times from a list of 24 items), the number of potential scenarios has increased in a factor of two orders of magnitude (from 20 cases in MRR analysis up to around 2000 cases in MLE).

Evaluation and visualization of the Weibull parameters for each scenario:

Once that all the scenarios to be considered were identified, the two Weibull parameters were estimated through the Maximum Likelihood Estimator equations. Due to the long number of points to be evaluated, results have been only captured in a η vs. β chart (Figure 5), where it can be observed a similar pattern as the one identified during the MRR evaluation, described in section 6.4:

- It is demonstrated again that Weibull parameters have a dependency on the relative position between the times at suspension and the times at failure.
- The whole spectra of values for the Weibull parameters are bounded within a characteristic shape.
- Values corresponding to low scale parameters (η) are related with higher shape parameters (β).
- In the opposite side, values corresponding to high scale parameters (η), are related with lower shape parameters (β).
- Exists a set of suspension times that, in combination with the failed items, maximizes the value of the shape parameter (β).

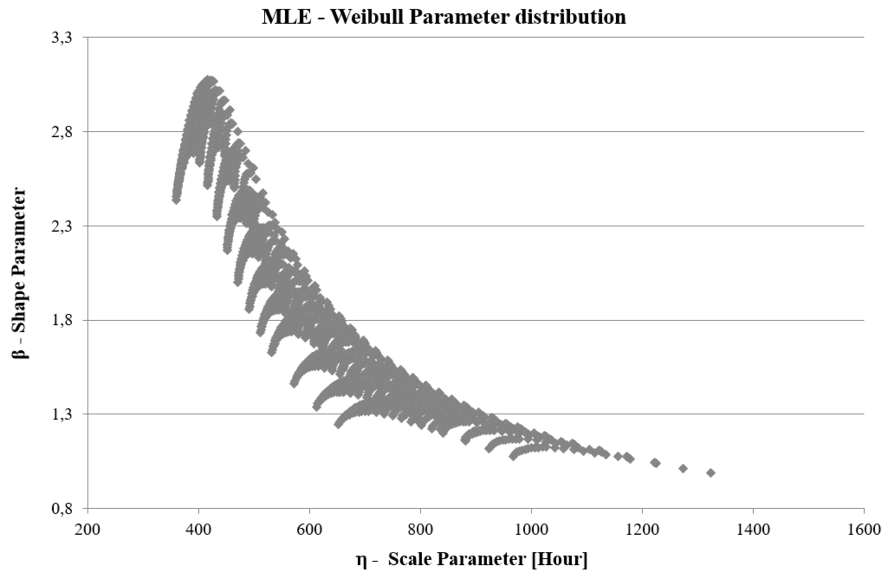

Figure 5: MLE Weibull parameter distribution

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Weibull parameters boundary characterization:

Similarly to the MRR case, combinations of Weibull parameters obtained through MLE methodology are bounded, and the boundaries of such cloud of points can be easily identified. Boundaries are represented in the Figure 6, and the algorithms for MLE boundaries evaluation are described here below in pseudocode.

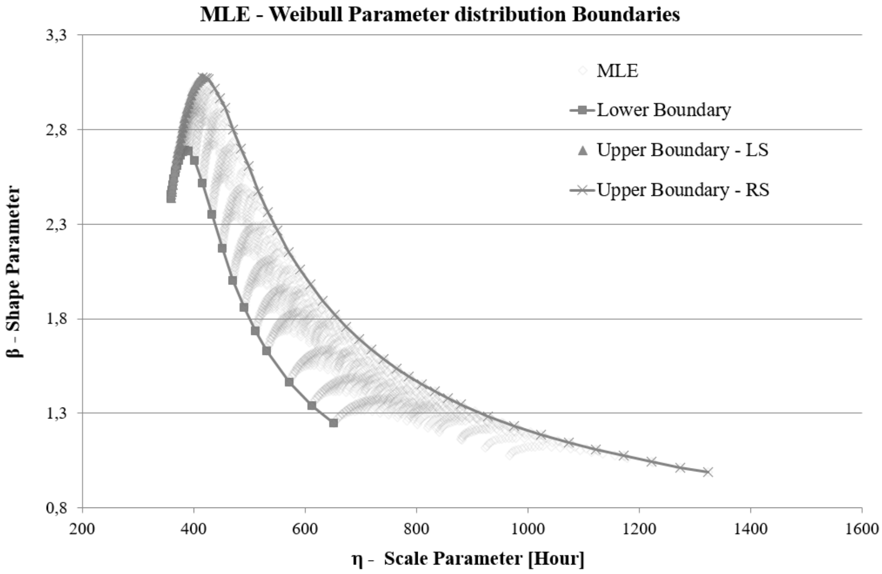

Figure 6: MLE Weibull parameter distribution Boundaries

- MLE Lower boundary generation pseudocode algorithm:

START
    [L_FailedItems] = SET List with the Times of the Failed Items
    [MIN_ST] = SET Minimum Suspension time considered
    [L_SuspensionTimes] = SET List with the Suspension times equal, all of them, to MIN_ST
    [L_NonFailedItems] = Set List with the Times of the Non Failed Items
    [V_DATA] = COMBINE [L_FailedItems] and [L_SuspensionTimes]
    CALCULATE 2-Weibull parameters for [V_DATA] *'Evaluation of the first item*
    FOR each time in [L_SuspensionTime] [S_x]
        [New_S] = SET Suspension time [S_x] considered in each For Iteration
        REPLACE in [V_DATA] the value of the first Suspension time by [New_S] value 'All the others Suspension times shall kept the original value of [MIN_ST]
        CALCULATE 2-Weibull parameters for [V_DATA] *'Evaluation of the additional boundary items*
    ENDFOR
END

- MLE Upper boundary generation pseudocode algorithm:

START
    [L_FailedItems] = SET List with the Times of the Failed Items
    [MIN_ST] = SET Minimum Suspension time considered
    [L_SuspensionTimes] = SET List with the Suspension times equal, all of them, to MIN_ST
    [L_NonFailedItems] = Set List with the Times of the Non Failed Items *'As this methodology refers to MLE, list of non failed items corresponds to those items that have not yet failed (i.e. Those captured in Table 2)*
    [V_DATA] = COMBINE [L_FailedItems] and [L_SuspensionTimes]
    CALCULATE 2-Weibull parameters for [V_DATA] *'Evaluation of the first item*
    FOR each time in [L_NonFailedItems] starting from the next value above [MIN_ST] [NF_x]
        [NEW_S] = SET Time [NF_x] considered in each FOR iteration
        FOR each suspension time in [V_DATA]
            REPLACE in [V_DATA] the suspension time value considered in each FOR iteration by [NEW_S] value

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CALCULATE 2-Weibull parameters for [V_DATA] *'Evaluation of the additional boundary items*
ENDFOR
ENDFOR
END

6.6 Comparative between all the methodologies

The best way to compare the results obtained through each Weibull Parameter prediction method is to present all the results in a η vs. β chart, to observe the relative position between all the potential pairs of (β, η) parameters.

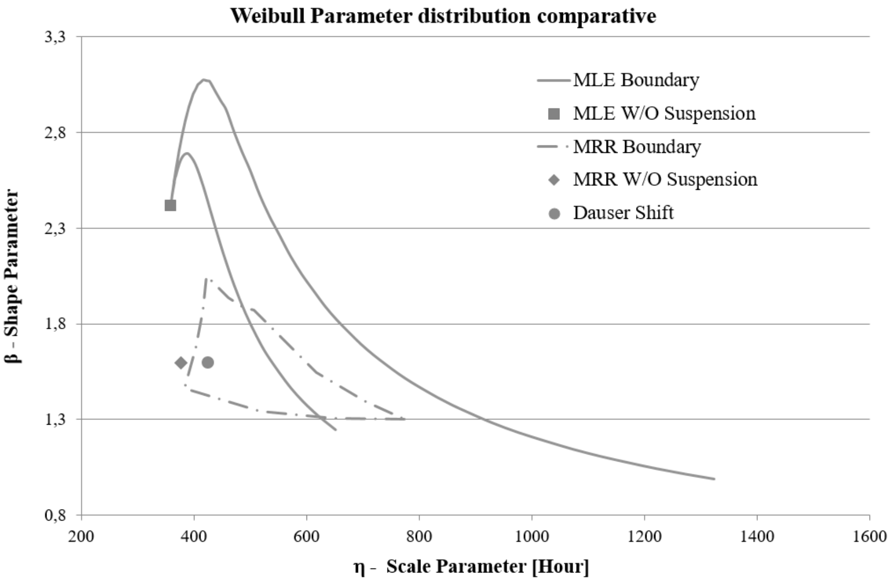

Figure 7: Comparative of all the pairs of Weibull parameters (η, β) for the different prediction methodologies

From the results plotted in Figure 7, the following conclusions can be extracted:

- Independently of the approach considered for estimating the 2-Parameters of the Weibull distribution, for a given case in which the life data for the failed items are known, the spectra of potential values of the 2-Parameters is bounded, independently of the life of the suspension items.
- Representation of the Weibull parameters in a (η, β) chart allows to select the most relevant combination of the 2-Parameters, depending of the purpose of the usage of the Weibull analysis. For example the following characteristics values can be highlighted:
  - Most Conservative Values:
    - 2-Parameters which maximizes the shape parameter (β)
    - 2-Parameters which minimize the scale parameter (η)
  - An mean value, that could be evaluated like the centroid of the area of the whole spectra of potential cases
- 2-Parameters obtained for MRR approach without suspension times are outside of the MRR boundary considering suspension times
- 2-Parameters obtained for Dauser Shift approach are part of the MRR spectra of values.
- Despite it is recognized that the Dauser Shift methodology does not have any mathematical fundamental, final value obtained is within the boundaries of the potential values of (β, η) parameters obtained for the MRR methodology, being therefore an valid approach to be considered in absence of additional information.
- 2-Parameters obtained for MLE approximation without suspension belong to the MLE spectra of values
- For the MLE and MRR region, there is a value of the scale parameter value ($\eta_{\beta max}$) which maximizes the value of the shape parameter (β). In both cases, these 2-Parameters correspond to the case in which the life of the three suspension data is close to the life of the second failed item.
- Taking into account that Shape parameter (β) allows to estimate the failure rate with age, MLE and MRR areas can be split in three groups, depending of the Shape parameter value:
  - β ≥ 1,25: Distribution of suspension times associated with a "infant mortality"
  - 1,25 ≥ β ≥ 0,85: Distribution of suspension times associated with a random success

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- 0,85 ≥ β: Distribution of suspension times associated with a wear-out process

This split allows to analysis the relative position between the suspension times and the failed items that makes the Weibull approximation switch from one model to the other.

7. Impact of the number of suspension times

Along Section 6, an overview of the effect over the final Weibull parameter prediction of the suspension times has been provided, In fact, a quick algorithm has been provided to evaluate the boundaries of the Weibull parameter prediction for both MRR and MLE approach. These algorithms are valid, not only for this case under study, but for those cases in which the number of failed items and the number of predicted suspension times are different that the ones analysed along this study. In fact, these algorithms allows to evaluate the impact over the boundary shape and the maximum β values predicted for this method in case that, keeping the number of failed items, the number of potential suspension times increases. Which such purposes, Figure 8 and Figure 9 captures the evolution of the MRR and MLE Weibull parameters boundaries in case that, instead of three potential suspension times, 6 and 10 suspension times are considered.

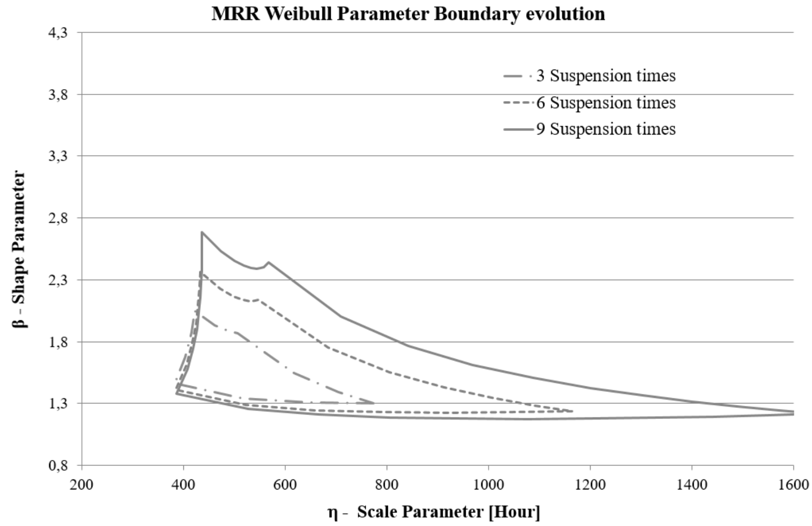

Figure 8: Evolution of the MRR Weibull Parameter distribution boundaries as a function of the suspension times

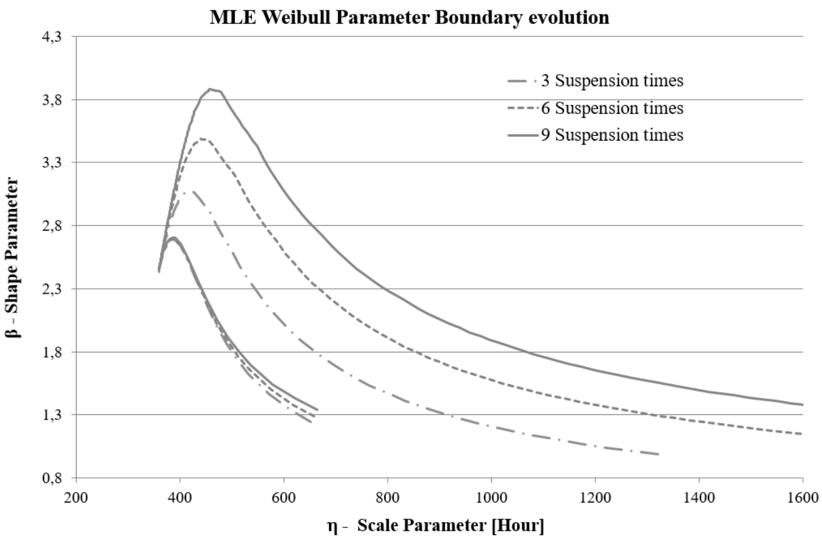

Figure 9: Evolution of the MLE Weibull Parameter distribution boundaries as a function of the suspension times

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8. Conclusions

In the real life of the industry evaluations, the suspensions are very common, leading to a situation that, in some particular cases the failure modes under analysis may be present in only a sub-set of the population under study, but the life of the population of unfailed units are unknown. Along this study two main approaches have been considered for the analysis of the impact of the suspension items of the predicted values of the Weibull parameters (β, η), Median Rank Regression and Maximum Likelihood Estimator. These two methods drawn a well bounded area of potential values for the 2-P Weibull parameters, were the cases in which suspensions are not taken into account represents a particular point of all the spectra of parameters obtained for each method. In fact, it can be observed that despite it is recognized that the Dauser Shift methodology does not have any mathematical fundamental, final value obtained is within the boundaries of the potential values of (β, η) parameters obtained for the MRR methodology, being therefore a valid approach to be considered in absence of additional information. Additionally, the alternative representation of all the potential values of the 2-parameters in a η vs. β chart, provides to the analyst the opportunity to select the most coherent pair of (β, η) value to perform the subsequent analysis that shall be carried out. As a last step, a rough view about the sensitivity of the analysis in front of the number of suspension times to be considered is provided, offering to the analyst some tools to evaluate the risks derived from the uncertainties associated to the estimation of the potential non failed items affected by the particular failure mode under study.

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