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A Study on the Design of GNSS Using 2-D Lattice Flower Constellation Pattern

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Abstract

In the era of the 4th Industrial Revolution, it is expected that the satellite navigation system and technology development competition will intensify in each country. This study deals with designing a satellite group for a new GNSS. In this research, the first step of optimal constellation design is selecting 2-D Lattice Flower Constellation pattern recently developed. The next step is to obtain a reasonably reduced problem dimensionality without cutting out potentially useful solutions. The performance index is designed to minimize DOP to evaluate the positional accuracy. After optimization, the performances of the proposed constellations for global navigation are presented.

1. Introduction

The satellite navigation system provides precise position, velocity, and time information to users by using radio waves from multiple navigation satellites. The satellite navigation system can be classified into a Global Navigation Satellite System (GNSS) and a Regional Navigation Satellite System (RNSS) based on service area. GNSSs such as United States' Global Positioning System (GPS), Russia's GLONASS, European Union's Galileo and China's Compass provide global coverage while RNSSs such as India's IRNSS and Japan's JRANS target only to neighbors of those countries. In Republic of Korea, GPS developed by the United States government and operated by the United States Air Force is mainly used. However, Korean satellite navigation system as a RNSS is under construction for the purpose of improving the security and accuracy of the surrounding area.

In this research, designing a satellite group for a new GNSS that should be needed after RNSS construction is addressed. There are many degrees of freedom in the design parameters for constellations because each orbit of each satellite constituting the satellite constellation has six parameters called classical orbital elements. Specifying a constellation by defining all of the orbit elements for each satellite is complex, inconvenient, and overwhelming in its range of options. Therefore, an efficient way to design to a constellation is adopting some orbital elements with common values and some other derived by algorithms and various algorithms have been proposed. In other words, it is necessary to use a satellite group pattern such as Walker constellation and Streets-of-Coverage constellation to use common orbit parameters. In this research, 2-D Lattice Flower Constellation pattern is used. . . Flower constellation, developed by Mortari et al.,[11,12,22] was first used for Global Navigation Satellite System (GNSS) constellation with a series of trials by Park, who proposed a new GNSS, a combination of two Harmonic Flower Constellations.[14,15] Tonetti used a Genetic Algorithm (GA) to improve the Park's results.[20] Both of these Flower Constellations were designed for 30 satellites and employed large numbers of orbital planes (15 and 30 respectively), which is unattractive from a launch and operational standpoint. Alternatively, Bruccoleri found a Harmonic Flower Constellation with 24 satellites that showed improved performance over the GPS constellation.[2] Jeremy present a case study designing a GNSS using Elliptical Flower Constellations with comparison to the Galileo constellation.[4]

For global coverage missions, fitness functions for constellation design are computed at globally distributed points. Most of the grid data sets are provided with a fixed step in latitude and longitude. Therefore, conventionally computed points are distributed with a fixed step in latitude and longitude. Since these are certainly not uniform distributions of points on the Earth, mainly due to the increase of point density at high latitude regions, converting these data into an equivalent distribution of points is needed. A quasi-equal subdivision algorithm is applied to distribute uniform points on three dimensions in order to efficiently perform simulation for users located on the global surface. Optimization is performed using Genetic Algorithms to estimate the navigation performance of the constellation. The

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performance index is designed to minimize dilution of precision (DOP) in order to evaluate the positional accuracy. The minimum distance between the satellites was used as a constraint in the optimization process in order to prevent collision between the satellites

This paper is organized as follows. The first section of this paper briefly describes the GDOP. Then, a summary on the 2-D LFC theory is provided. We then discuss design considerations and parameters for LFC and optimize the design of the constellations. Finally, the performance of selected constellations for GNSS mission are presented.

2. Position Accuracy and Dilution of Precision (DOP)

The performance of GNSS are primarily affected by the satellite geometry and the ranging errors. Geometric factors can be calculated for any instantaneous satellite configuration with respect to the user receiver's position.[16] The basic pseudorange models between a user receiver and j-th satellite are given by.[8]

$$\begin{aligned} \rho^j &= d^j + B \\ &= \hat{\mathbf{e}}^j \cdot (\mathbf{R}^j - \mathbf{R}_u) + B, \qquad j = 1, \cdots, m \end{aligned} \tag{1}$$

where

ρ^j : pseudorange measurement from j-th satellite
d^j : geometric range between user and j-th satellite
B : range equivalent of unknown user receiver clock offset
$\hat{\mathbf{e}}^j$: line of sight vector from user to the j-th satellite
$\mathbf{R}^j$: known position vector of j-th satellite
$\mathbf{R}_u$: user's unknown position vector
m : the number of the included satellites

Eq. (1) is rearranged so that all of the unknown quantities such that $\mathbf{R}_u$ and B appear on the left. This rearrangement yields:

$$\hat{\mathbf{e}}^j \cdot \mathbf{R}_u - B = \hat{\mathbf{e}}^j \cdot \mathbf{R}^j - \rho^j \tag{2}$$

Eq. (2) can be written in matrix form as follows.

$$\mathbf{H}\mathbf{x} = \mathbf{z} \tag{3}$$

where,

$$\mathbf{H} = \begin{bmatrix} \hat{\mathbf{e}}^{1^\mathrm{T}} & -1 \\ \hat{\mathbf{e}}^{2^\mathrm{T}} & -1 \\ \vdots & \\ \hat{\mathbf{e}}^{m^\mathrm{T}} & -1 \end{bmatrix}$$

$$\mathbf{x} = [e \quad n \quad u \quad B]^\mathrm{T}$$

$$\mathbf{z} = \begin{bmatrix} \hat{\mathbf{e}}^1 \cdot \mathbf{R}^1 - \rho^1 \\ \hat{\mathbf{e}}^2 \cdot \mathbf{R}^2 - \rho^2 \\ \vdots \\ \hat{\mathbf{e}}^m \cdot \mathbf{R}^m - \rho^m \end{bmatrix}$$

and e, n and u are the three components of $\mathbf{R}_u$.

When more than four satellites are available, user position can be calculated by the least square method as follows.[7]

$$\hat{\mathbf{x}} = (\mathbf{H}^\mathrm{T}\mathbf{H})^{-1}\mathbf{H}^\mathrm{T}\mathbf{z} \tag{4}$$

The covariance matrix of the error is given by

$$\mathrm{cov}[\delta\mathbf{x}] = (\mathbf{H}^\mathrm{T}\mathbf{H})^{-1}\mathbf{H}^\mathrm{T}\mathrm{cov}[\delta\mathbf{z}]\mathbf{H}(\mathbf{H}^\mathrm{T}\mathbf{H})^{-1} \tag{5}$$

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With the additional assumption that the components of $\delta\mathbf{z}$ are identically distributed and mutually independent, we obtain.[8]

$$\mathrm{cov}[\delta\mathbf{z}] = \mathbf{I}_{m\times m}\sigma_\rho^2 \tag{6}$$

where $\mathbf{I}_{m\times m}$ is the $m \times m$ identity matrix and σ_ρ is a variance of pseudorange errors.

Substituting Eq. (6) into Eq. (5) yields

$$\mathrm{cov}[\delta\mathbf{x}] = \sigma_\rho^2(\mathbf{H}^{\mathrm{T}}\mathbf{H})^{-1} \tag{7}$$

If the state is parameterized so that $\delta\mathbf{x} = [\delta e \quad \delta n \quad \delta u \quad \delta B]^{\mathrm{T}}$, where δe, δn and δu, are east, north, and up position errors, respectively; and B is range equivalent of the clock offset error.

$$\mathrm{cov}[\delta\mathbf{x}] = \begin{bmatrix} \sigma_{ee}^2 & \sigma_{en}^2 & \sigma_{eu}^2 & \sigma_{eB}^2 \\ \sigma_{ne}^2 & \sigma_{nn}^2 & \sigma_{nu}^2 & \sigma_{nB}^2 \\ \sigma_{ue}^2 & \sigma_{un}^2 & \sigma_{uu}^2 & \sigma_{uB}^2 \\ \sigma_{Be}^2 & \sigma_{Bn}^2 & \sigma_{Bu}^2 & \sigma_{BB}^2 \end{bmatrix} \tag{8}$$

This relationship indicates that $(\mathbf{H}^{\mathrm{T}}\mathbf{H})^{-1}$ allows to translate from the pseudorange errors into the position errors. It is also possible to define some DOP parameters as being the some components of five kinds of DOP are usually defined, depending on the type of data to deal with.

$$\begin{aligned} GDOP &= \frac{\sqrt{\sigma_{ee}^2 + \sigma_{nn}^2 + \sigma_{uu}^2 + \sigma_{BB}^2}}{\sigma_\rho} \\ PDOP &= \frac{\sqrt{\sigma_{ee}^2 + \sigma_{nn}^2 + \sigma_{uu}^2}}{\sigma_\rho} \\ HDOP &= \frac{\sqrt{\sigma_{ee}^2 + \sigma_{nn}^2}}{\sigma_\rho} \\ VDOP &= \frac{\sigma_{uu}}{\sigma_\rho} \\ TDOP &= \frac{\sigma_{BB}}{\sigma_\rho} \end{aligned} \tag{9}$$

where GDOP is the Geometrical DOP, PDOP is the Position DOP, HDOP is the Horizontal DOP, VDOP is the Vertical DOP, and TDOP is the Time DOP.[17] A higher DOP indicates poor satellite geometry and a less accuracy than a lower DOP as shown in Fig. 1.

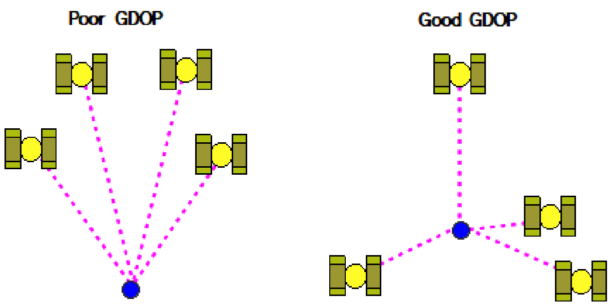

Figure 1: Spatial geometry for GDOP values.

3. LFC Optimization

3.1 2-D Lattice Flower Constellations Design

The 2-D Lattice Flower Constellations (LFC) design methodology has been introduced in Ref.[1] In general LFC are characterized by four continuous parameters (semi-major axis, eccentricity, inclination and argument of perigee) and three independent integer parameters establishing the constellation satellite distribution in the (M, Ω)-space. These

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integer parameters are: the number of orbital planes, N_o, the number of satellites per orbit, N_{so}, and the configuration number, N_c (phasing parameter). Using these integer parameters the satellites' right ascension of the ascending node (Ω_{ij}) and the initial mean anomaly (M_{ij}) are solutions of the following equation

$$\begin{bmatrix} N_o & 0 \\ N_c & N_{so} \end{bmatrix} \begin{Bmatrix} \Omega_{ij} \\ M_{ij} \end{Bmatrix} = 2\pi \begin{Bmatrix} i-1 \\ j-1 \end{Bmatrix} \tag{10}$$

where $i = 1, \cdots, N_o$, $j = 1, \cdots, N_{so}$, and $N_c \in [1, N_o]$. The "i-j" element is the j-th satellite on the i-th orbital plane. If repeating ground tracks are required, then the compatibility equation

$$N_p\, T_p = N_p\, \frac{2\pi}{n + \dot{\omega}} = N_d\, T_d = N_d\, \frac{2\pi}{\omega_\odot - \dot{\Omega}}, \tag{11}$$

where T_p is the orbit nodal period and T_d is the Greenwich nodal period, provides the value of the orbit radius for each coprime integers, N_p and N_d. Equation (11) takes into account the main gravitational perturbation due to the Earth oblateness, known as the J_2 effect. The secular and persistent J_2 effect modifies the mean motion according to

$$n = n_0 \left[1 + \frac{3}{4} J_2 \left(\frac{R_\oplus}{p} \right)^2 (2 - 3\sin^2 i) \sqrt{1 - e^2} \right], \tag{12}$$

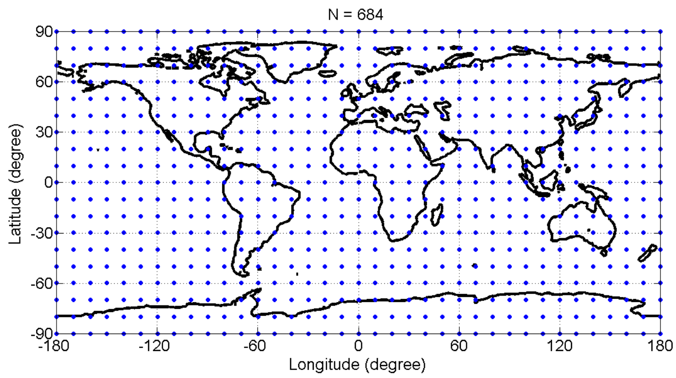
where $n_0 = \sqrt{\dfrac{\mu}{a^3}}$ is the unperturbed mean motion, linearly changes the right ascension of the ascending node,

$$\dot{\Omega} = -\frac{3}{2} J_2 \left(\frac{R_\oplus}{p} \right)^2 n \cos i, \tag{13}$$

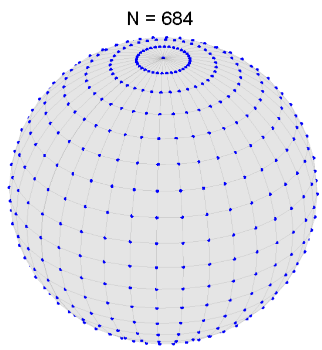
as well as the argument of perigee,

$$\dot{\omega} = \frac{3}{4} J_2 \left(\frac{R_\oplus}{p} \right)^2 n\,(5\cos^2 i - 1). \tag{14}$$

3.2 Uniform distribution of points on a sphere



(a)



(b)

Figure 2: Conventional scheme for distributing points (10 degree resolution)

In the case of a global mission, the fitness function for a constellation design is computed in globally distributed points.[5,13] Most of grid data sets are provided with a fixed step in latitude and longitude. Therefore, conventionally computed points are distributed with a fixed step in latitude and longitude.[19] Since this is certainly not a uniform distribution of points on the Earth, mainly due to the increase of point density at high latitude regions as illustrated in Fig. 2, uniform distribution of points on a sphere is required.

Various algorithms for uniform grid on a sphere have been developed.[?,9,19] This will dramatically decrease to small amount data sets and computational burden is then reduced. The creation of uniform distribution of points is generated by Quasi-equal area subdivision algorithm presented in Ref.[9] The algorithm start by applying cuts with an icosahedron (20 identical equilateral triangular faces) and performing 1 trisection and then 4 sequential bisections in

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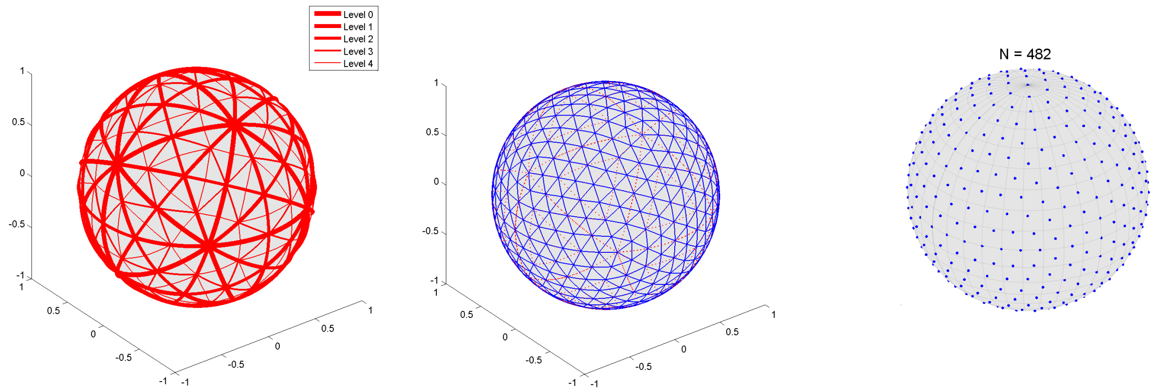
identical spherical triangles, $20 \cdot 3 \cdot 2^4 = 960$ triangles with quasi-identical spherical areas are obtained whose vertices approximate 482 uniformly distributed points on a sphere as shown in Fig. 3. . Before performing trisection, the initial icosahedron is rotated around specific axis, $[1, \quad 1, \quad 0]^T$ by 30° in order to avoid final distributed points having latitudinal and longitudinal symmetry.



(a) (b) (c)

Figure 3: Quasi-equal area subdivision algorithm

3.3 Genetic Algorithms

Genetic Algorithms are adaptive heuristic search algorithm that mimics the natural selection/mutation process. These are actually search processes and naturally useful for finding optimum solutions.[21] Although there is no guarantee that Genetic Algorithms will provide optimum solutions (and this is true for most optimization methods), Genetic Algorithms are most appropriated in highly nonlinear multi-parameter problems.[6] Since constellation design for GNSS is a highly nonlinear, Genetic Algorithms have been chosen to discover the optimum parameters for constellation. For the simulation, the population size is chosen to be 150 and the maximum number of generations as 100. The crossover rate and mutation rate are selected to be 0.6 and 0.2, respectively.

3.4 Fitness Function

The fitness function utilized to drive the optimization process is designed to minimize the mean GDOP. Fitness function has been implemented to average GDOP of uniformly distributed user points on the Earth. 1 period of GDOP values are considered in this research to find optimum parameters. The constellation was propagated with 5° steps in mean anomaly. Thus, the optimality is defined by the minimization of the following function

$$L = \frac{1}{N_t \cdot N_x} \sum_{t=1}^{N_t} \sum_{x=1}^{N_x} W_x GDOP \tag{15}$$

where N_t is the number of time steps, N_x is the number of users and W_x is a weight for each user grid point.

In the mathematical expression of the fitness function the parameters to be optimized are calculated in a set of N_t times N_x user grid points distributed on the Earth surface. This formula allows us to control the fitness function by giving more weight to interested user grid points. In this research, 3 different types of weight are used for interested regions. Interested areas and corresponding weight are as follows:

1. global : locationally fixed weight,
2. technographic : locationally fixed value weight in specific area, and
3. demographical : locationally varying weight (global population density) in land area, and
4. economic : locationally varying weight (gross domestic product) in land area.

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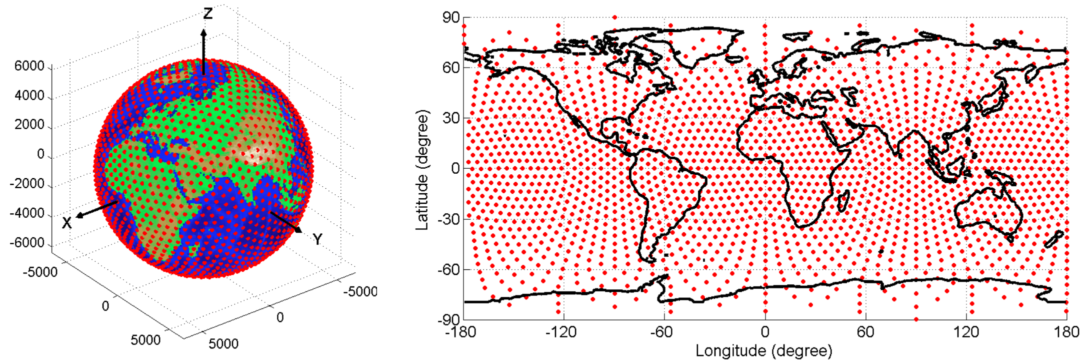
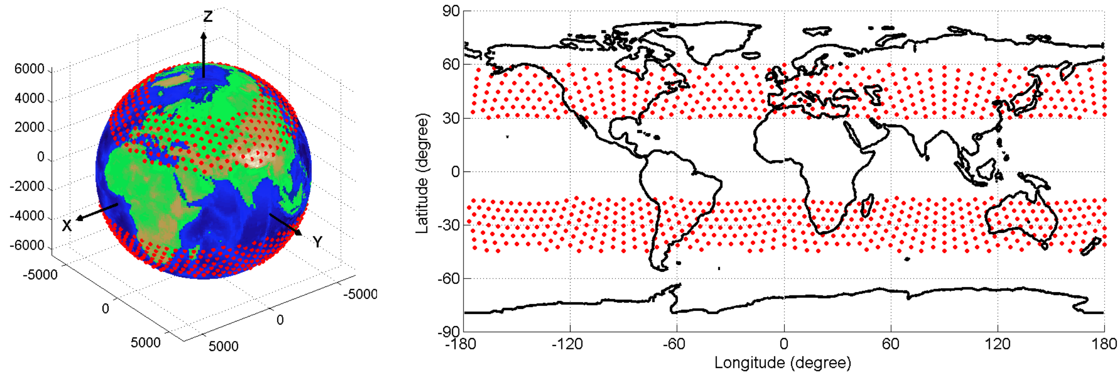


(a) (b)

Figure 4: Global weight point description (fixed values) : 1,922 user points

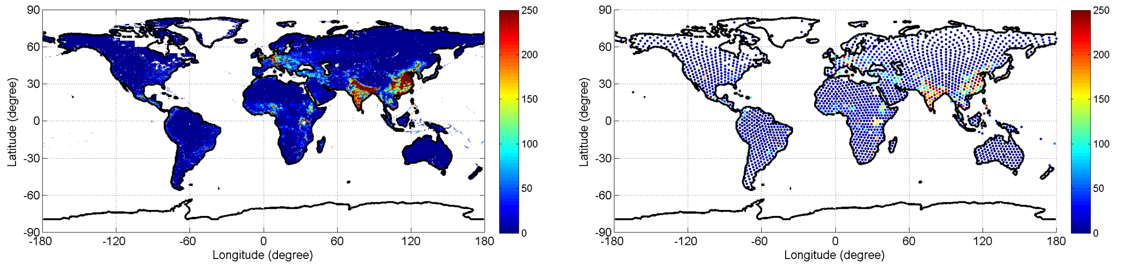


(a) (b)

Figure 5: Technographic weight point description (fixed values): 531 user points

3.5 Weight

Gridded Population of the World, Version 3 (GPWv3) consists of estimates of human population for the years 2000 by 2.5 arc-minute grid cells and associated data sets dated circa 2000. A proportional allocation gridding algorithm, utilizing more than 300,000 national and sub-national administrative units, is used to assign population values to grid cells. The population density grids are derived by dividing the population count grids by the land area grid and represent persons per square kilometer.[3]



(a) Population density (2.5 arc-minutes resolution): 29,652,480 user points

(b) Population density (Uniformly distributed) : 1,983 user points

Figure 6: Demographic weight point description (varying values)

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3.6 Minimum Distance Constraint for Collision Avoidance

To avoid the design of constellations with satellites colliding the results provided by Ref.[18] is adopted. In that analytical study, the closest approach between the two satellites ($\rho_{\min}$) in two circular orbits with the same radius and inclination, is analytically expressed by the following equation

$$\rho_{\min} = 2\left|\sqrt{\frac{1+\cos^2 i + \sin^2 i \cos\Delta\Omega}{2}}\,\sin\left(\frac{\Delta F}{2}\right)\right| \tag{16}$$

where

$$\Delta F = \Delta M - 2\tan^{-1}\left[-\cos i \tan\left(\frac{\Delta\Omega}{2}\right)\right]$$

and where ΔM and $\Delta\Omega$ are the differences in mean anomaly and right ascension of ascending node, respectively. Note that $\rho_{\min}$ must be scaled by the orbit radius to find the actual value of the minimum approach distance. Due to the regular pattern (lattice) of the LFC, it is not necessary to evaluate the minimum distance using all pairs of satellites. It is sufficient to evaluate the minimum distance between the first satellite $[\Omega_{11}, M_{11},]$ with all the other satellites staying on different orbital planes. This greatly simplifies the effort in the optimization to avoid constellations affected by satellite conjunctions (sometime common for symmetric distribution). Constraint that no two satellites are ever closer than half the distance between two consecutive satellites in the same orbit has been used. For the specific case of $N_{so} = 1$ (just one satellite per orbit), the constraint adopted is that the minimum distance between any pair of satellites is greater than ten percent of optimal distance of constellation. N_s satellites are equally spaced if

$$S = \frac{4\pi}{N_s} = 2\pi(1-\cos\theta) \tag{17}$$

Therefore, the optimal distance of constellation is obtained by

$$\rho_{opt} = 2R\sin\theta \tag{18}$$

where R is orbit radius and θ is the Earth central angle.

4. Results

4.1 Designed LFC for $N_s = 27$

A mask angle for the reference points on ground have been chosen 10°. This mask angle is typical for usual GNSS applications and is assumed to represent an average mask angle for rural environments.[10] In Table 1, the simulation results with additional constraints are given. Design constellation with no additional constraints has nine orbital planes which are 56° inclined with reference to the equatorial plane. Three operational satellites are equally distributed by 120° in the orbital plane. Note that, in case of constraints $N_o = 3$ and $T = 14.08$, designed constellation is equivalent to the original Galileo constellation.

Table 1: Optimal parameters of 2-D LFC for various constraints

Parameters	**Additional constraints (N_o / T)**		
	– / –	3 / –	3 / 14.08
$[N_o, N_{so}]$	[9,3]	[3,9]	[3,9]
N_c	4	2	2
T (hr)	15.32	15.33	14.08
i (deg)	56.00	53.85	56.00
Mean GDOP	2.138	2.292	2.323
Max GDOP	3.857	3.685	3.772
Name	LFC9	LFC3	Galileo

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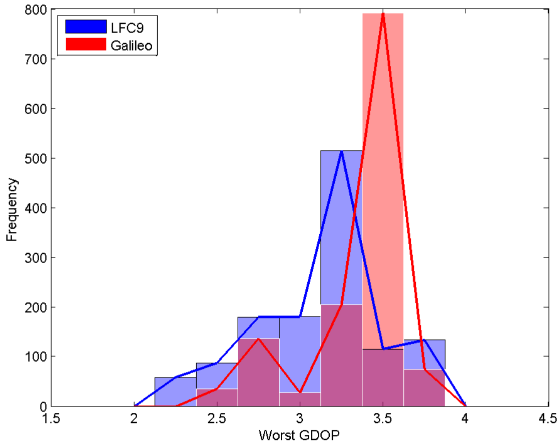
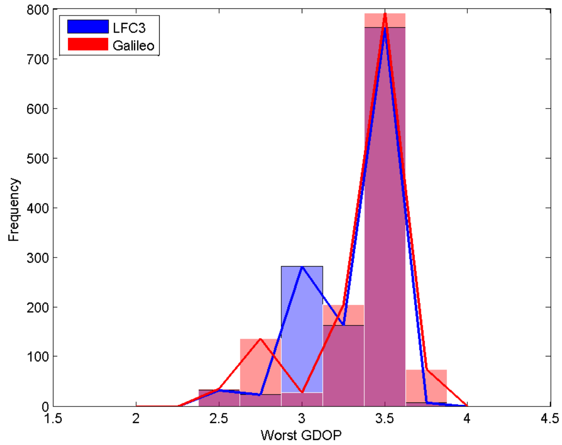
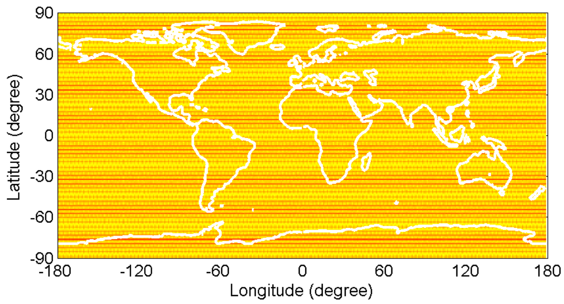


(a) LFC9 versus Galileo

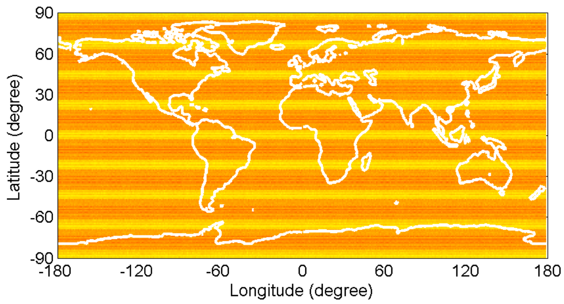


(b) LFC3 versus Galileo

Figure 7: Worst GDOP comparison of designed LFC and Galileo

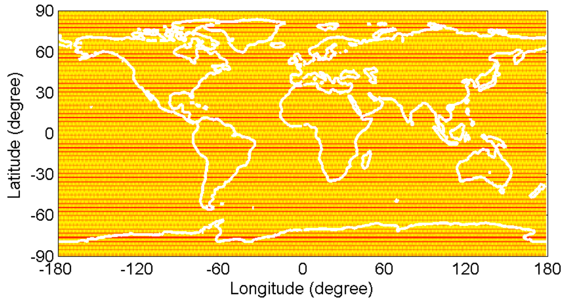


(a) LFC9

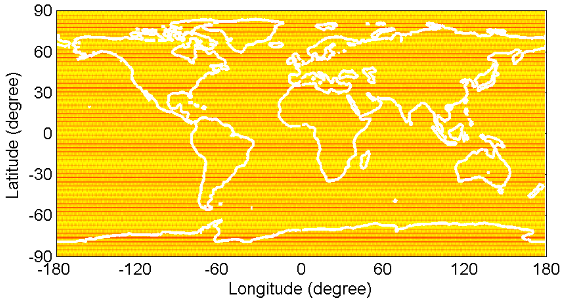


(b) Galileo

Figure 8: Mean GDOP over the global map



(a) Demographic



(b) Technographic

Figure 9: Mean GDOP over the global map

5. Conclusion

We applied the 2-D LFC theory as a satellite group pattern and designed the GNSS satellite group to minimize the GDOP using genetic algorithm. In GDOP calculation, the amount of computation is reduced by using equally distributed points using quasi-equal area subdivision algorithm, and it is confirmed that the designed satellite group shows superior performance in terms of GDOP compared to Galileo.

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