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Optimal multi-impulse space rendezvous considering limited impulse using a discretized Lambert problem combined with evolutionary algorithms

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Abstract

In this paper, a direct approach is presented to tackle the multi-impulse rendezvous problem considering the impulse limit. Particularly, the standard Lambert problem is extended toward several consequential orbit transfers for the rendezvous problem. A number of different evolutionary algorithms are taken into consideration. It is shown that the proposed approach can lead to the optimal multi-impulse transfer maneuver that has the minimum amount of fuel similar to the traditional two-impulse transfer without violating the impulse limitation. Results also indicate that the approach is efficient even when the number of stages increases due to lower impulse limitations.

1. Introduction

Long-range rendezvous is the early phase of the space rendezvous, in which the spacecraft is expected to have an orbital maneuver that transfers the space vehicle from the initial orbit to a final orbit. When some specific considerations such as the impulse limit are applied, the orbit transfer becomes more challenging. Because of that, the problems usually can not be solved analytically and therefore, semi-analytical and numerical approaches have been taken into consideration by researchers more, and analytical solutions are obtained only for specific missions and assumptions.

In recent years, many approaches have been developed to face the impulsive maneuvers in different problems with various conditions, limitations and considerations. Such considerations may be impulse limit, time limit, orbital perturbations or uncertainties in some design parameters. For instance, in research by Kitamura et al.[13], an analytical solution is obtained for minimum energy orbit transfers. The time-averaged Hamiltonian derivation was utilized in this research and good results were achieved. However, the method is only applicable to coplanar transfers when the transfer time is fixed. Such analytical approaches commonly use optimal control theory and the averaging method[16] under heavy assumptions either regarding the space mission or the thrust vector. As another example, an explicit form of the transfer trajectory under the assumption that the thruster direction is expressed as a combination of sinusoidal functions, is obtained by Asai et al.[2]. In research by Koblick and Xu[14], a semi-analytic approach is developed to determine the minimum velocity increment for a two-impulse rendezvous. Although this approach will work for both coplanar and non-coplanar 3D geometries for any orbit type, the impulse limit is not considered. Research by Xie et al.[25] is another example which is dedicated to impulsive orbital rendezvous with respect to a constraint on the transfer trajectory for coplanar transfers. However, only two-impulse transfers are considered in this research. Similarly, constraints considered in research by Zhang et al.[29] consist of a lower bound for perigee altitude and an upper bound for apogee altitude, but no observation is applied through the magnitude of impulses. Research by Santos et al.[9] is dedicated to a four-impulse transfer. This research aims to analyze rendezvous maneuvers between two coplanar circular orbits, seeking to perform this transfer with the lowest possible fuel consumption, assuming that this problem is time-free and uses four burns during the process. The assumption of four burns is used to represent a constraint posed by a real mission. In this research and other similar papers,[21] the Lambert problem (sometimes referred to as Lambert method), which is actually an orbit determination method based on two position radii and a transfer time, is mostly involved in impulse orbital maneuvers. Handling the terminal condition and involving fewer variables in optimization are the main reasons for using the Lambert method in optimal impulsive maneuvers. As for instance,

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research by Zhang et al.[28] focused on the two-cooperative-spacecraft-rendezvous problem with the same direction of terminal velocities, i.e., the same arrival flight-path angle based on the Lambert method. Also, Shen and Tsiotras[20] developed a method for determining the optimal two-impulse solution. However, Shen and Tsiotras's method is only for coplanar circular orbits. A closed form solution to the minimum-fuel Lambert problem between two assigned positions in two distinct orbits is presented by Avendano and Mortari[3]. In research by Zhang[30] an analytical linear covariance prediction is formulated for Lambert's boundary value problem with navigation errors and in research by Carter and Humi[4], a new approach is presented for the problem of optimal impulsive rendezvous of a spacecraft in an inertial frame near a circular orbit. Other research also exists[5,6,12,26], which is dedicated to the rendezvous problem with various assumptions and considerations.

Having an overview of the research in recent years shows that the problem of facing the fuel-optimal multi-impulse non-coplanar transfer has not received much attention in the literature. This paper presents an approach to confront this problem by introducing a new method based on the extension of the Lambert problem. The main focus in this research is to find the best multi-impulse orbit transfer, which satisfies the impulse limit for a given space mission while minimizing the fuel consumption. The proposed approach considers the entire orbital maneuver as several consecutive Lambert problems. In this approach, every two sequential impulses form a unique Lambert problem and the transfer trajectories are connected to each other through some orbits, called the intermediate orbits. The optimization problem is created and the orbital elements of the intermediate orbits and the variables associated with the Lambert problems are considered as the decision variables. The objective function includes total velocity increment and the penalty term associated with the impulse violations. Several evolutionary algorithms are used and compared for dealing with the problem. Their convergence and the probability of reaching the solution with minimized fuel without violating the impulse limit is investigated.

The paper is organized as follows. The developed approach is presented in Section 2, where the concept of multi-impulse orbit rendezvous based on the extension of Lambert problem is explained. Section 3 briefly describes the evolutionary algorithms employed in this research. Results from the numerical simulations are presented in Section 4. Finally, Section 5 concludes the research.

2. The Approach

This section is dedicated to describing the approach developed in this work. Since the approach is generally based on the standard Lambert problem, the application of the Lambert problem in the two-impulse orbit transfer is explained first. Next, the extension of the two-impulse orbit transfer based on the Lambert method into multi-impulse orbit transfer is introduced. The variables involved in the approach are presented, and the formulation of the objective function is discussed.

2.1 Two-impulse rendezvous

In a general long-range space rendezvous mission, all of the orbital elements suffer changes. The typical way to accomplish this mission is to use a two-impulse transfer based on the Lambert method. This transfer is a trajectory that has one intersection with the initial orbit and one with the final orbit, and the impulses act on these intersections. A schematic diagram of this concept is illustrated in Fig. 1.

In this figure, r_i and r_f represent two vectors corresponding to each intersection and t denotes the transfer time. Finding the best transfer trajectory that has the least fuel consumption is an optimization problem. The unknown variables in this problem are the transfer time and the points on the initial and final orbits, where the spacecraft ascend and descend between two impulses. These points can be defined by their true anomalies θ_i and θ_f as the orbital elements of the initial and final orbits are known (obtaining state vectors from orbital elements is provided in Appendix I[7]). These variables act as the decision variables in the optimization problem and define a Lambert problem. Using the Lambert method based on the standard Gauss approach, the orbital elements of the transfer trajectory along with the magnitude of impulses at each intersection $\Delta v_1, \Delta v_2$ are obtained (details are provided in Appendix II[7]). Following these calculations, the overall fuel consumption can be obtained as the objective function of the optimization problem as follows.

$$J = |\Delta v_1| + |\Delta v_2| \tag{1}$$

Considering this objective function, the optimization problem for the two-impulse long-range rendezvous has only three unknown variables (θ_i, θ_f, t). Therefore, it can be easily approximated with the help of an evolutionary algorithm (EA) or a non-linear programming (NLP) method with small a number of iterations. However, if a propulsion system with low impulse is about to be utilized in the mission, the obtained impulses may exceed the limitation. Such

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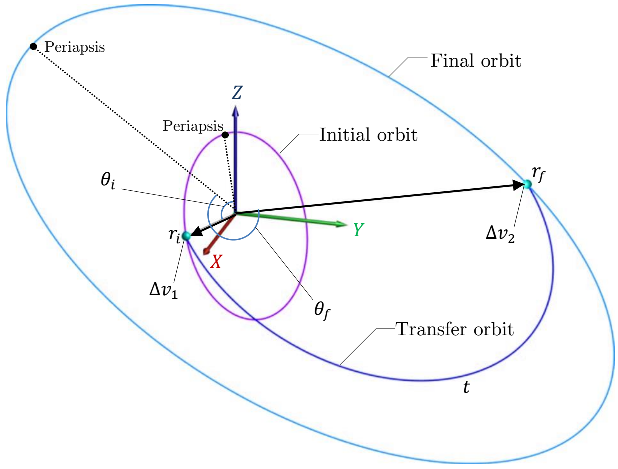

Figure 1: Scheme of a two-impulse rendezvous.

a condition dictates an impulse limit, which should be considered when minimizing the overall fuel consumption. Depending on the limitation, the problem might not be solved with a two-impulse transfer and multiple impulses are needed to satisfy the constraint on the impulse limit. Therefore, a new approach is needed, in which the multi-impulse orbital maneuver is considered for long-range space rendezvous. In this approach, the impulse limit of the propulsion system is taken into consideration along with minimizing the fuel consumption.

2.2 Multi-impulse rendezvous

Consider a multi-impulse orbit transfer as illustrated in Fig. 2. This figure shows an orbital maneuver from the initial orbit, with orbital elements denoted as $a_0, e_0, i_0, \Omega_0, \omega_0$, to a final orbit with orbital elements denoted as $a_f, e_f, i_f, \Omega_f, \omega_f$.

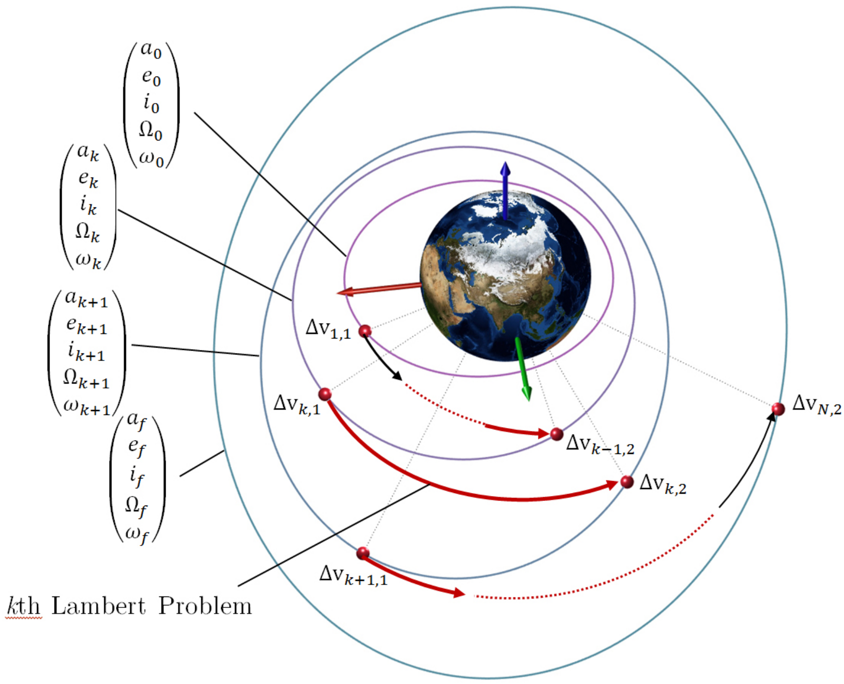

Figure 2: Multi-impulse long-range space rendezvous.

The whole maneuver is divided into N stages and each stage represents a unique Lambert problem. Having N stages will generate $N-1$ intermediate orbits, represented by a_k, e_k, i_k, Ω_k and ω_k (k = 1 to $N-1$), along with N jumps. Every jump is denoted by initial and final anomalies, denoted as $\theta_{i,1}$ and $\theta_{i,2}$, which correspond to the initial and final

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state vectors and the transfer time t_i in each stage. By considering the orbital elements of the stages and the Lambert problem variables $(\theta_{k,1}, \theta_{k,2}, t_k)$ as the inputs, a complete multi-impulse orbit transfer with $2N$ impulses ($\Delta v_{k,1}$ and $\Delta v_{k,2}$) is defined. Therefore, the decision variables, denoted by the vector $\vec{X}$, will be formed as:

$$\vec{X} = \vec{X}(a_k, e_k, i_k, \Omega_k, \omega_k, \theta_{k,j}, t_k) \tag{2}$$

Following this approach, the orbit transfer problem with thrust limitation consists of $2N - 1$ sets of orbital elements. These sets contain $N - 1$ intermediate orbits, which are the inputs of the problem and N jumps, which are trajectories representing the solution from each stage corresponding to a minor Lambert problem.

This approach has several advantages over the traditional methods, which usually consider the impulses and their direction in three dimensions along with impulse timing as the inputs of the problem. The first advantage is that the total number of inputs is lower in comparison to the traditional approach, in which the direction, magnitude and time of impulses are considered as the decision variables. This is due to the fact that by defining multiple minor Lambert problems, the majority of the characteristics of the transfer trajectories will be revealed. In other words, the shape of the transfer trajectories is taken into account instead of the impulse directions in Cartesian coordinate system. Regarding this fact, this approach can be called an impulsive shape-based approach. The next advantage is handling the terminal conditions. The initial and final condition for point to point, point to orbit and orbit to orbit cases can be easily handled in the current approach by setting the Lambert problem variables in the first and last stages either free or fixed. However, satisfying the terminal condition in the traditional approach is an issue, which usually needs to be considered as an additional term in the objective function. Besides, since the shape of stages is defined via the actual orbital elements with physical meanings, the method benefits from rapid convergence as the orbital elements have known boundaries in real applications.

2.3 The objectives

As the boundary conditions are already satisfied by the proposed approach, two types of objectives are defined for the problem including fuel and impulse violation. Regarding the proposed approach, the overall fuel consumption in terms of Δv in every stage is denoted by J_f and is defined as:

$$J_f = \sum_{k=1}^{N} (\Delta v_{k,1} + \Delta v_{k,2}) \tag{3}$$

The impulse violation regarding a given impulse limit needs to be calculated for each Δv in every stage. As a result, the penalty denoted by the jth Δv in kth stage is calculated as

$$J_{k,j} = \frac{1 + sgn(\Delta v_{k,j} - \eta)}{2}(\Delta v_{k,j} - \eta) \tag{4}$$

where η is the given allowable impulse during the orbit transfer. Consequently, the overall magnitude of the penalty function due to the impulse violations in all stages is calculated as:

$$J_v = \sum_{k=1}^{N} (J_{k,1} + J_{k,2}) \tag{5}$$

Having the cost functions, the overall objective function can be written via scalarizing the two objectives as:

$$J = J_f + \zeta J_v \tag{6}$$

where ζ is the scalarization coefficient for impulse violation. The impact of the choice of the underlying scalarizing coefficient is still far from being well understood in the literature in space orbit design and optimization problems. Consequently, it is very important and crucial to choose these parameters according to the type of space transfer.

2.4 Simulation

Taking into account the optimization variables, the problem can be illustrated as in Fig. 3.

As shown in Fig. 3, each cost function evaluation consists of solving N number of Lambert problems. Each Lambert problem is formed for two sequential intermediate orbits and associates with three unknown variables. The number of stages (N) should be selected according to the given impulse limit (ζ). One option for obtaining an estimation for the necessary number of impulses is using the solution of two-impulse orbit transfer ($N = 1$). Having the total

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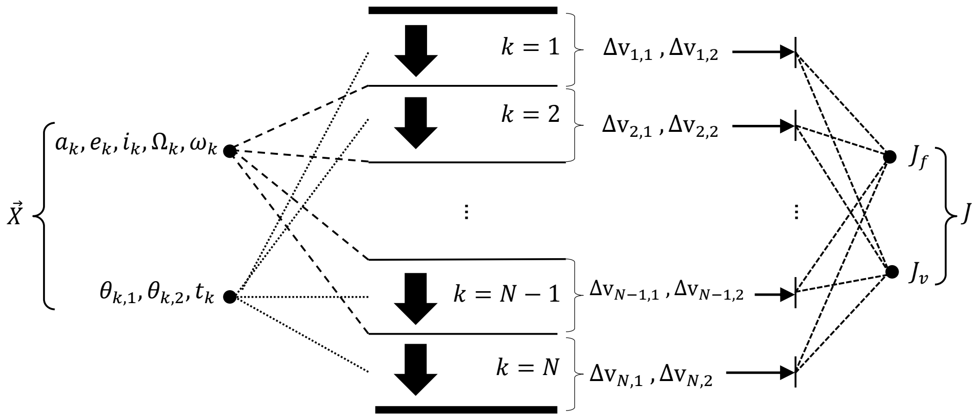

Figure 3: Scheme of the cost function evaluation.

velocity increment for this problem ($\Delta v_{N=1}$), an estimation of the number of necessary impulses for multi-impulse transfer ($N > 1$) can be obtained as:

$$\varphi = \lceil \frac{\Delta v_{N=1}}{\eta} \rceil \tag{7}$$

where φ is the minimum total velocity increment for the two-impulse orbit transfer and η is the impulse limit. Taking into account that each stage consists of two impulses in the proposed approach, one can estimate the number of stages as $N = \lceil \varphi/2 \rceil$. Obviously, if the algorithm finds a solution that includes one impulse almost equal to zero, it means that the optimal solution for that problem consists of an odd number of impulses.

Following the proposed approach, further aspects of the presented method can be analyzed. The one unique feature of the proposed approach is that by increasing the number of stages, the problem is converted to multiple two-impulse transfers that can be separately analyzed. For example, when $N = 2$, the rendezvous problem is a four-impulse transfer, including two sequential two-impulse transfers. If this problem is solved by the proposed approach and the obtained solution contains the overall fuel consumption similar to or less than the two-impulse transfer ($N = 1$) without violating the impulse limit, the process can be considered as a successful transition from $N = 1$ to $N = 2$. If we assume that the obtained solution for $N = 2$ is optimal, then the two sequential two-impulse transfers inside this solution are also optimal for this transfer. Now, each of these transfers can be seen as an isolated orbit transfer mission. They can individually be broken down into two other four-impulse transfers based on the amount of impulses that exist in every jumps. So, by performing this process in both problems and solving the two new four-impulse transfers, the obtained solution will be a solution for $N = 4$, which is an eight-impulse rendezvous. If only one of them is broken down to a new four-impulse transfer, then the overall solution will be a six-impulse rendezvous ($N = 3$). In conclusion, if a specific number of stages is required, there are two ways to obtain a solution regarding that number of impulses. The first one is considering the required number of stages directly and breaking down the two-impulse transfer into N stages and solving the one multi-impulse rendezvous problem as was done in a sample rendezvous mission in the previous subsection. The second option is to break down the two-impulse transfer ($N = 1$) into a four-impulse transfer ($N = 2$), solve the problem and then break down the obtained two-impulse transfers into another four-impulse transfers and solve the problems in each conversion and repeat this process over and over until the required number of stages is achieved. In this first case, the optimization algorithm is used only once in solving a big problem with a high number of variables. But in the second case, the number of optimization variables is always constant, which is associated with solving the problem with two stages. However, the optimization algorithm must be used several times in order to divide the stages as required. This recursive strategy is discussed with some examples in the empirical tests in this paper.

3. Evolutionary Algorithms

The proposed approach turns the multi-impulse orbit rendezvous mission into an optimization problem in continuous domain. Having N number of stages, the problem will have $n = 8N - 5$ decision variables, including $5(N - 1)$ unknown orbital elements for intermediate orbits plus $3N$ unknown Lambert problem variables in each transfer.

We considered optimizing this problem by applying a set of different evolutionary algorithms (EAs). EAs are general-purpose search procedures based on the mechanisms of natural selection and population genetics[8]. These

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algorithms are appealing to many users in different areas of engineering, computer science, and especially in spacecraft trajectory optimization[22], to name a few, due to their simplicity, ease of interfacing, and extensibility. EAs have attracted wide attention and found a growing number of applications, especially in the last decade.

By considering the scheme of the problem based on the proposed approach in Fig. 3, the decision variables are $N-1$ sets of the orbital elements of the intermediate orbits $(a_k, e_k, i_k, \Omega_k, \omega_k)$, and N sets of Lambert problem variables $(\theta_{k,1}, \theta_{k,2}, t_k)$. The objective function is defined as in Eq. 6, which consists of the overall fuel consumption and the penalty function for impulse violations. Five types of EAs are selected in this research to search for the best solution in this problem. They are briefly introduced in this section. However, the reader is urged to refer to the provided references for details.

The first EA considered in this research is the Genetic Algorithm (GA). GA, a heuristic search method used in artificial intelligence and computing,[11] is widely used for finding optimized solutions to spacecraft trajectory optimization problems.[22] It is based on the theory of natural selection and evolutionary biology and an excellent technique for searching through large and complex data sets. They are considered capable of finding reasonable solutions to complex issues[1] as they are highly adept at solving unconstrained and constrained optimization issues. The typical GA as described in[11] is used in this research. There are two basic parameters of GA, including crossover probability and mutation probability. The parameter related to crossover probability says how often crossover will be performed while the parameter related to mutation probability defines how often parts of chromosome will be mutated. Details are provided in[10].

Particle swarm optimization (PSO) is another population based stochastic optimization technique, inspired by social behavior of bird flocking or fish schooling. PSO shares many similarities with evolutionary computation techniques such as Genetic Algorithms (GA)[27]. The system is initialized with a population of random solutions and searches for optima by updating generations. However, unlike GA, PSO has no evolution operators such as crossover and mutation. In PSO, the potential solutions, called particles, fly through the problem space by following the current optimum particles. The process of PSO depends on some parameters, including two learning coefficients (global and personal) and a velocity limiter, which limits the velocity of the particles during the optimization process. In modern versions of PSO, the velocity is also dampened by a parameter called the damping ratio in order to improve the convergence[24].

Estimation of Distribution Algorithms (EDAs)[15] are stochastic heuristic search strategies that form part of the evolutionary computation approaches, where a number of solutions or individuals are created every generation, evolving once and again until a satisfactory solution is achieved. In EDAs, the problem specific interactions among the variables of individuals are taken into consideration. While in common evolutionary computations, the interactions are kept implicitly in the operators, in EDAs the interrelations are expressed explicitly through the joint probability distribution associated with the individuals of variables selected at each generation. The task of estimating the joint probability distribution associated with the database of the selected individuals from the previous generation constitutes the most difficult work to perform. This requires the adaptation of methods to learn models from data developed in the domain of probabilistic models. Various sampling and learning methods can be considered in different EDAs. Since the multi-impulse space rendezvous problem is an optimization problem in continuous domain, two types of EDAs are considered in this research including an EDA based on Multivariate Gaussian Distribution (EDA-MGD) and an EDA based on Univariate Gaussian Distribution (EDA-UGD).[17] These algorithms mainly differ in the class of probabilistic models employed and the learning and sampling methods they use. However, the selection and replacement strategies used can also determine important differences in the behavior of EDAs. Therefore, different techniques are chosen in this research for the two selected EDAs. Generally, it is difficult to decide which type of EDA is the best choice for a given problem. Therefore, comparing at least a short list of combinations of EDA operators in terms of their efficiency is necessary[19].

Differential Evolution (DE) is a stochastic direct search and optimization algorithm, and is an instance of an evolutionary algorithm from the field of evolutionary computation. It is related to sibling evolutionary algorithms such as the GA, and has some similarities with PSO[23]. DE includes three parameters including the scaling factor and the lower bound and upper bound of crossover weight. Experimental results have shown that their values have great effect on the convergence speed and solution quality[18]. For this research, the provided values have shown to have the best performance of DE in most instances of the problem.

4. Numerical Results

For the sake of validating the proposed multi-impulse approach, a variety of experiments have been conducted. Each experiment is tackled using the presented EAs. In this section, first the boundaries of the decision variables in the problems are provided, along with the settings of the EAs utilized in this research. Then, the experimental results are presented and discussed.

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4.1 Parameter settings of EAs

For all the algorithms in this research, the same population size and generations are considered as $n_{pop} = 10N$, and $n_{gen} = 20N$, where N is the number of stages considered in the multi-impulse orbit transfer based on the proposed approach. The boundaries of the decision variables are considered as shown in Table 1.

Table 1: Boundaries for EAs

Semi-major axis	Eccentricity	Inclination	Right Ascension
$0 < a_k < 50000$	$0 < e_k < 1$	$0 < i_k < 2\pi$	$0 < \Omega_k < 2\pi$
Arg. of perigee	Initial true anomaly	Final true anomaly	Transfer time
$0 < \omega_k < 2\pi$	$0 < \theta_{k,1} < 2\pi$	$0 < \theta_{k,2} < 2\pi$	$0 < t_k < 24h$

To provide reproducibility for the results, the values of the parameters for each algorithm that were briefly described in the previous section are provided in Tables 2, 3, 4, 5, 6.

Table 2: Parameters for GA

Crossover percentage	0.6
Crossover range factor	0.3
Mutation percentage	0.4
Mutation range	0.2

Table 3: Parameters for PSO

Personal Learning Coefficient	1.8
Global Learning Coefficient	2
Damping Ratio	0.95

Table 4: Parameters for DE

Scaling factor	0.7
Minimum crossover weight	0.2
Maximum crossover weight	0.8

Table 5: Parameters for EDA-UGD

Sampling+Learning	Univariate
Selection method	Truncation
Selection parameter	0.1
Replacement method	Elitism
Replacement parameter	0.2

Table 6: Parameters for EDA-MGD

Sampling+Learning	Multivariate
Selection method	Exponential
Selection parameter	1.5
Replacement method	Elitism
Replacement parameter	0.3

Readers are urged to refer to the references regarding the details of the optimization process in each algorithm. Following the proposed approach, several rendezvous problems in different space missions are taken into account for simulation. Various aspects of the obtained solutions are analyzed and finally the focus of the research is to make a comparison between the performance of different EAs.

4.2 Coplanar Transfer

The first case that is selected for the simulation is a coplanar orbit transfer. This type of transfer is applicable when the initial and final orbits are in the same plane. In such a case, the two orbital elements inclination (i) and right ascension Ω are the same in the two orbits. As in this case, the coplanar transfer represented in Table 7 is considered.

Table 7: Orbital elements in coplanar transfer

	Semi-major axis (a)	Eccentricity (e)	Argument of Perigee (ω)
Initial orbit	9000 km	0.2	0°
Final orbit	20000 km	0.3	120°

The impulse limit of $\eta = 500m/s$ is considered for this mission. As the first step, the two-impulse transfer is considered to have an approximation for the required number of stages. The solution for the two-impulse transfer is obtained by means of a non-linear programming method (MATLAB *fmincon()* with default parameters is used considering a random initial). The best solution found is a transfer trajectory that starts on the initial orbit at the true anomaly of $\theta_i = 134.35°$ and ends on the final orbit at true anomaly of $\theta_f = 151.2°$. The transfer time is 8642 seconds and the overall velocity increment is $\Delta v_{N=1} = 2.0312km/s$. Considering this value and the given impulse limit, the

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number of stages is considered as $N = 3$ and the weighting coefficient of the penalty for the impulse violation is considered as $\zeta = 10$. The problem is formed according to the presented approach and the afformentioned EAs are utilized to reach the best solution. Each algorithm is run 10 times and the best convergence of the algorithms considered is shown in Fig. 4.

In this simulation, all EAs started with the same initial population. Regarding the obtained results, PSO and EDA-MGD have the best convergence in comparison to the other EAs as they found a solution with similar fuel consumption as the two-impulse transfer without violating the given impulse limit. The final solution obtained by each algorithm is shown in Fig. 5, in which the amount of impulse violation is separated from the overall velocity increment. Although all EAs tried to minimize a unique objective function, which includes total fuel consumption and the penalty for the impulse violation, not all of them managed to find feasible solutions in terms of impulse limit. Besides EDA-MGD, the solution found by PSO also satisfies the impulse limit. Solutions by GA, DE and EDA-UGD do not satisfy the impulse limit, even when the overall fuel consumption is near the one associated with the two-impulse transfer. The two-impulse transfer ($N = 1$) that was obtained initially is illustrated in Fig. 6 along with the best solution found so far between all of the obtained solutions for the six-impulse transfer ($N = 3$) in Fig. 7.

The obtained impulses satisfy the impulse limit considered for this space mission. According to Figs. 6 and 7, the overall fuel mass of the obtained six-impulse transfer is almost the same as the two-impulse transfer, which shows that the approach is effective in finding a transfer scenario without violating the impulse limit while having the same amount of fuel as in the two-impulse transfer. Detailed results regarding the obtained solution are provided in Table 8.

Table 8: Detailed results for multi-impulse coplanar rendezvous (optimization variables are in bold)

	Initial orbit	$k = 1$	Intermediate orbit	$k = 2$	Intermediate orbit	$k = 3$	Final orbit
a	9000	10465	**11093**	13413	**17541**	19965	20000
e	0.2	0.16936	**0.21609**	0.28747	**0.43598**	0.29942	0.3
ω	0	48.436	**52.741**	87.287	**104.7**	119.74	120
θ_i	**132.19**	83.754	**91.468**	56.922	**161.32**	146.28	-
θ_f	-	4.3051	**0**	53.236	**35.823**	72.087	**71.824**
t	-	**8824.7**	2157.7	**15353**	8037	**21752**	-

As shown in Table 8, by having the Lambert problem variables within a solution, the rest of the orbital characteristics of the transfer trajectories can be calculated along with the coast times (the time when the spacecraft stays in the intermediate orbit until reaching the next impulse) between two sequential Lambert problems.

4.3 Empirical Tests

Following the recursive strategy discussed in the previous section, it is advantageous to see whether or not the solution to the four-impulse rendezvous ($N = 2$) is always better or as good as the two-impulse rendezvous ($N = 1$) in terms of fuel consumption without violating the given impulse limit. A clear example for this investigation is the bi-elliptic Hohmann transfer,[7] which has a lower fuel than the two-impulse Hohmann transfer in some special cases. In order to

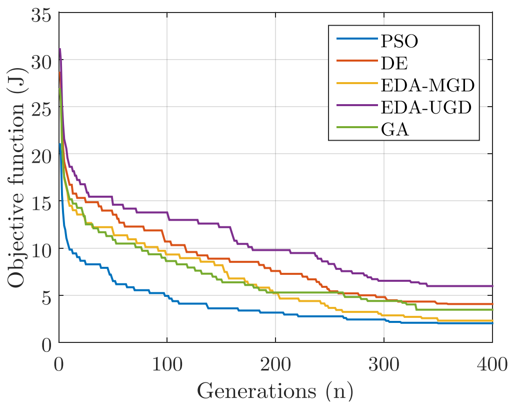

Figure 4: Best convergence of the EAs in multi-impulse coplanar rendezvous.

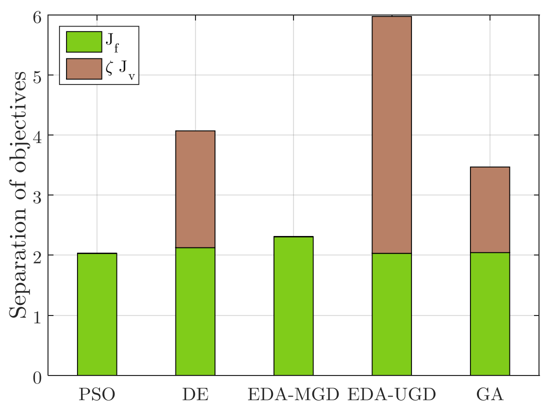

Figure 5: Objectives including the overall velocity increment and the impulse violation.

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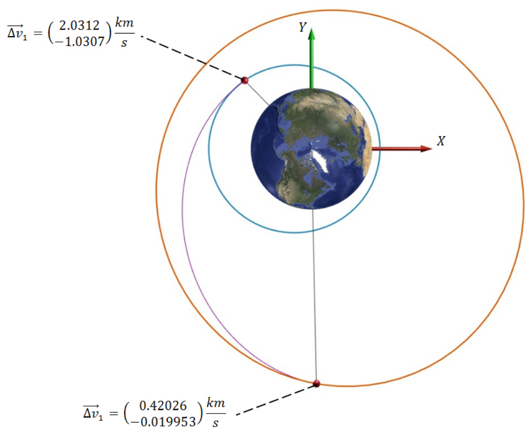

Figure 6: Two-impulse ($\Delta v = 2.0312 km/s, \eta = \infty$) long-range rendezvous.

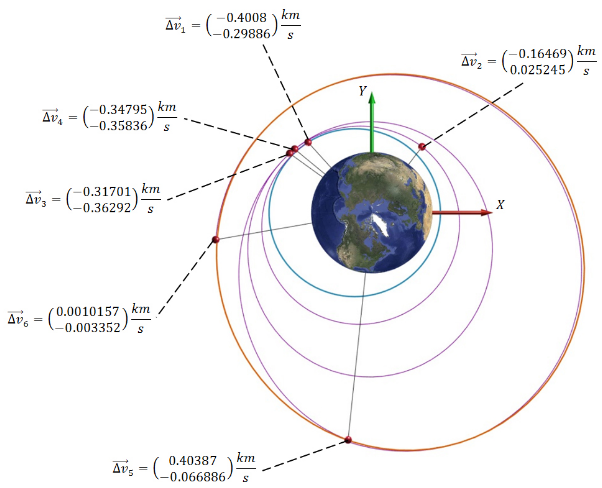

Figure 7: Six-impulse ($\Delta v = 2.0608 km/s, \eta = 500 m/s$) long-range rendezvous.

analyze this subject, a high number of long-range rendezvous missions (763 instances) is considered. In each instance, the orbital parameters of the initial and final orbits are randomly generated so that $|\Delta a| < 10000 km$, $|\Delta e| < 0.3$, $|\Delta i| < 60°$, $|\Delta\Omega| < 40°$ and $|\Delta\omega| < 40°$. For each instance, first, the two-impulse transfer ($N = 1$) is considered and solved by means of the mentioned EAs. Each algorithm is run 10 times and the best solution found by each algorithm is saved and assumed to be the best possible performance of that algorithm for that specific instance. Then, for each instance, the solutions obtained by all EAs are gathered and the best solution is extracted, denoted by $J_{N=1}$ and considered as the global optimal solution for that instance. Next, the algorithms are sorted and ranked according to the quality of the solutions they obtained in comparison to the global optimal solution for that instance. This process is repeated again with respect to the four-impulse transfer ($N = 2$). The impulse limit is considered as one forth of the two-impulse velocity increment, forcing the algorithms to search for the feasible solutions with respect to the impulse limit. Like before, each algorithm is run 10 times and their best performance is separated. Again, the algorithms are sorted and ranked and the best solution is extracted, denoted by $J_{N=2}$.

Having all of the solutions for the two-impulse ($N = 1$) and four-impulse ($N = 2$) transfers, it is possible to see the obtained results in Fig. 8. This figure shows the difference of the objective function related to the best solution found so far for the two-impulse and four-impulse transfer in each instances. The results are sorted based on the value of the difference. It can be seen that in nearly 33 percent of the instances, the four-impulse rendezvous is better than the two-impulse rendezvous in terms of fuel consumption, while in the majority of the instances, it is impossible to reach

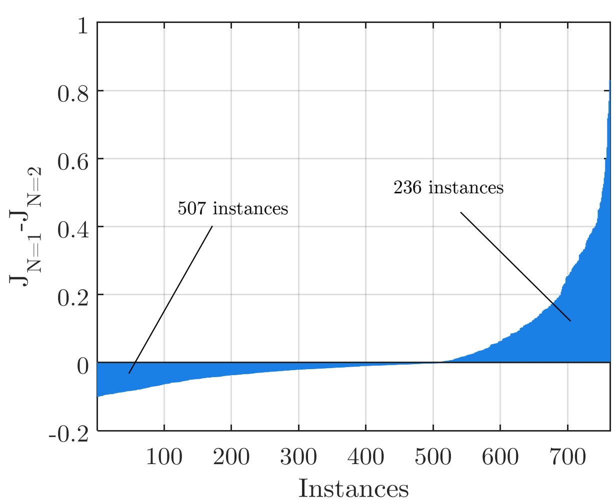

Figure 8: Difference of objectives in two-impulse and four-impulse rendezvous

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a solution with the same amount of fuel as in the two-impulse transfer by means of any EAs. Separating the instances in this figure according to the algorithms, which were capable of finding the best solution, can lead us to an insight regarding the performance of the aforementioned EAs. Fig. 9 shows their performances.

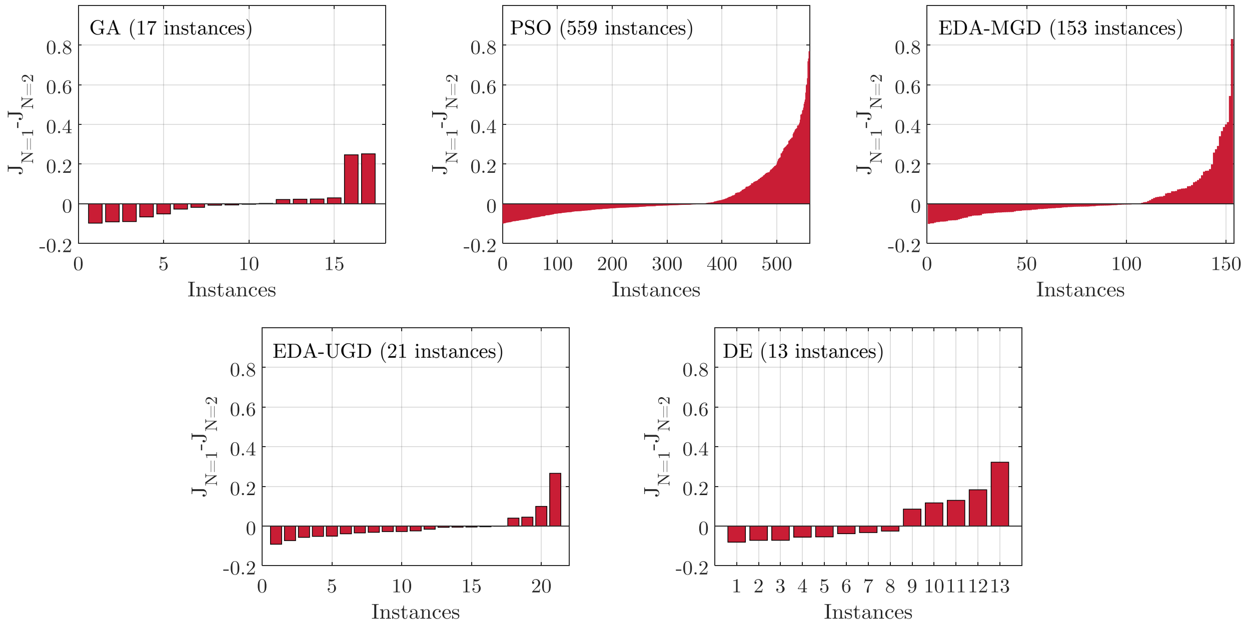

Figure 9: Performance of the algorithms in finding the best solution

In this figure, the instances with the best solution related to each algorithm are separated. As shown, PSO is capable of finding more of the best solutions between all other EAs, while DE has the worst performance. Between the two EDAs, EDA-MGD, which is associated with the multivariate Gaussian distribution, has quite an advantage over the EDA-UGD. An interesting point is that although PSO has a superior advantage over the other EAs in finding the global best solution, the global optimal solution of the space rendezvous mission related to the highest difference between the four-impulse and two-impulse transfers belongs to EDA-MGD, leading to the conclusion that EDA-MGD is a potential option and tuning its parameter or perhaps enhancing the algorithm may make this algorithm competitive to PSO.

Another investigation is to find out the relation between the characteristics of the space orbits in the rendezvous mission and the superiority of the multi-impulse transfer over the two impulse transfer in terms of fuel. This can be investigated by considering the absolute value of the total velocity increment of the two-impulse orbit transfer in each instance. It is clear that, as the differences between the orbital elements of the initial and final orbits increase, the required Δv for the space mission also increases. Plotting the difference between the velocity increment of the two-impulse and four-impulse rendezvous versus the absolute amount of velocity increment in the two-impulse rendezvous gives Fig. 10.

According to Fig. 10, it can be concluded that, as the two-impulse transfer requires more fuel, the chance of the fact that multi-impulse transfer becomes more fuel-optimal than the two-impulse transfer is higher. In other words, when the solution to the two-impulse transfer has a relatively small amount of Δv, possibly due to low differences between the orbital elements of the initial and final orbits, there is little chance for the multi-impulse transfer to have less fuel consumption. On the other hand, when the initial and final orbits have huge differences in their orbital elements, there is a high possibility that the multi-impulse transfer consumes less fuel than the two impulse transfer.

5. Conclusion

In this paper, a multi-impulse approach for optimizing long-range rendezvous considering the impulse limit is proposed. The presented approach divides the whole transfer into several minor Lambert problems. The main goal of this approach is to handle the impulse limit, which may be considered due to the requirement of the space mission. The objective function in this approach consists of the overall velocity increment, which is the summation of all impulses plus the penalty function for excessive impulses with respect to a given impulse limit. In order to validate the proposed approach, various types of EAs were utilized to find the optimal solution in different types of space missions. The EAs

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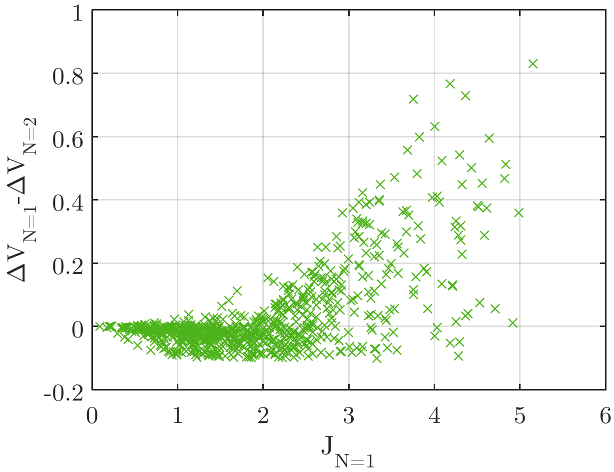

Figure 10: Superiority of the multi-impulse transfer over the two-impulse transfer

are compared and investigated regarding their convergence and practicality. The obtained results indicate that generally PSO outperforms other EAs in finding the best solution. However, the EDA based on the multivariate Gaussian distribution showed that it is potentially a good choice for some specific cases. Results also indicate that if the orbital elements of the initial and final orbits have huge differences, there is a high chance that multi-impulse transfer requires less fuel than the two-impulse transfer. Future research involves the expansion of this approach and development of a hybrid EA based on the extended Lambert problem.

Appendix I: Conversion of orbital elements to state vectors

Having the orbital elements a, e, i, Ω, ω representing semi-major axis, eccentricity, inclination, right ascension of ascending node and argument of perigee respectively along with true anomaly θ, the position vector $\vec{r}_\circ$ and velocity vector $\vec{v}_\circ$ relative to perifocal frame are calculated as:

$$\vec{r}_\circ = \frac{h^2}{\mu}\frac{1}{1+e\cos\theta}\begin{pmatrix}\cos\theta\\ \sin\theta\\ 0\end{pmatrix} \tag{8}$$

$$\vec{v}_\circ = \frac{\mu}{h}\begin{pmatrix}-\sin\theta\\ e+\cos\theta\\ 0\end{pmatrix} \tag{9}$$

where μ is the Earth's gravitational constant ($\mu = 398600 km^3/s^2$) and h is the angular momentum of the orbit, calculated by:

$$h = \sqrt{a\mu(1-e^2)} \tag{10}$$

The transformation matrix $[\boldsymbol{Q}]$ is calculated by:

$$[\boldsymbol{Q}] = [R_\omega] \times [R_i] \times [R_\Omega] \tag{11}$$

where the tree rotation matrices are obtained as:

$$[R_\omega] = \begin{bmatrix}\cos\omega & \sin\omega & 0\\ -\sin\omega & \cos\omega & 0\\ 0 & 0 & 1\end{bmatrix} \tag{12}$$

$$[R_i] = \begin{bmatrix}1 & 0 & 0\\ 0 & \cos i & \sin i\\ 0 & -\sin i & \cos i\end{bmatrix} \tag{13}$$

$$[R_\Omega] = \begin{bmatrix}\cos\Omega & \sin\Omega & 0\\ -\sin\Omega & \cos\Omega & 0\\ 0 & 0 & 1\end{bmatrix} \tag{14}$$

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The state vectors relative to the initial frame are calculated as:

$$\vec{r} = [\boldsymbol{Q}]' \vec{r}_\circ \tag{15}$$

$$\vec{v} = [\boldsymbol{Q}]' \vec{v}_\circ \tag{16}$$

Appendix II: Solution of Lambert's problem via Gauss Method

Having two radii $\vec{r_1}$, $\vec{r_2}$ and the transfer time between two positions (t), the angle between the two vectors (α) is computed as:

$$\cos\alpha = \frac{\vec{r_1}.\vec{r_2}}{|\vec{r_1}| \times |\vec{r_2}|} \tag{17}$$

Considering prograde trajectory, one can calculate:

$$\sin\alpha = \begin{cases} \sqrt{1-\cos^2\alpha} & (\vec{r_1} \times \vec{r_2})_Z \geq 0 \\ -\sqrt{1-\cos^2\alpha} & (\vec{r_1} \times \vec{r_2})_Z < 0 \end{cases} \tag{18}$$

Then, the parameter A, is calculated as:

$$A = \sin\alpha \sqrt{\frac{|\vec{r_1}| \times |\vec{r_2}|}{1-\cos\alpha}} \tag{19}$$

Having A, $\vec{r}_1$ and $\vec{r_2}$, the function $y(z)$ can be defined as:

$$y(z) = |\vec{r_1}| + |\vec{r_2}| + A\frac{zS(z)-1}{\sqrt{C(z)}} \tag{20}$$

where $S(z)$ and $C(z)$ are Stumpff functions defined as:

$$S(z) = \begin{cases} \dfrac{\sqrt{z} - \sin\sqrt{z}}{(\sqrt{z})^3} & z > 0 \\ \dfrac{\sinh\sqrt{-z} - \sqrt{-z}}{(\sqrt{-z})^3} & z < 0 \\ \dfrac{1}{6} & z = 0 \end{cases} \tag{21}$$

$$C(z) = \begin{cases} \dfrac{1-\cos\sqrt{z}}{z} & z > 0 \\ \dfrac{\cosh\sqrt{-z} - 1}{-z} & z < 0 \\ \dfrac{1}{2} & z = 0 \end{cases} \tag{22}$$

Having the functions defined, the function $F(z)$ is formed as:

$$F(z) = [\frac{y(z)}{C(z)}]^{\frac{3}{2}} S(z) + A\sqrt{y(z)} - \sqrt{\mu}t \tag{23}$$

where μ is the Earth's gravitational constant ($\mu = 398600 km^3/s^2$). Solving for $F(z^*) = 0$ using an iterative Newton's method, the solution z^* is obtained. Following the obtained parameter, the following Lagrange coefficients are calculated:

$$f = 1 - \frac{y(z^*)}{|\vec{r}_1|} \tag{24}$$

$$g = A\sqrt{\frac{y(z^*)}{\mu}} \tag{25}$$

$$\dot{g} = 1 - \frac{y(z^*)}{|\vec{r}_2|} \tag{26}$$

Having the Lagrange coefficients, the velocity vectors correspond to $\vec{r}_1$ and $\vec{r}_2$ can be obtained as:

$$\vec{v}_1 = \frac{1}{g}(\vec{r}_2 - f\vec{r}_1) \tag{27}$$

$$\vec{v}_2 = \frac{1}{g}(\dot{g}\vec{r}_2 - \vec{r}_1) \tag{28}$$

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