

Mathematical modeling of transient processes during start-up of main liquid propellant engine under hot test conditions

*Dmytro Koptilyy, Roman Marchan, Sergey Dolgoplov and Olexiy Nikolayev
FireFly Aerospace Ukraine, 103A, Gagarina Avenue, Dnipro 49050, Ukraine
dmitriy.koptilyy@fireflyaerospace.com, roman.marchan@fireflyaerospace.com,
sergey.dolgoplov@fireflyaerospace.com, aleksey.nikolaev@fireflyaerospace.*

Abstract

Predicting parameters of transient processes before static fire tests of liquid rocket engines (LRE), issuing recommendations to ensure the engine performance, stability and dynamic strength of its structure, monitoring and analyzing technical condition of the engine during tests, are important to be addressed before conducting static fire tests of main LRE. An advanced approach to the mathematical modeling of transient processes in aggregates and systems of the main rocket engine (similar to 2000 kN RD 191, liquid oxygen and kerosene) with oxidizer staged combustion is proposed. The development of a mathematical model of nonlinear dynamics of rocket engines is based on the results of long-term research of low-frequency dynamics of cavitating pumps, injection of oxidizer gas products into the flow of liquid oxygen, the Turbo Pump Unit (TPU) dynamic interaction with oscillatory processes in gas manifolds, and dynamic processes in flow regulators and others. Applied to the LRE bench tests conditions the analysis of transient processes in the bench hydraulic feed system during engine start-up has been performed. The calculations of the parameters of cavitation flow in the LRE pumps on the basis of the empirical model are carried out. It is shown that the natural frequencies of the propellants in the LRE bench feed system are significantly reduced if cavitation phenomena in engine pumps are taken into account during computations, and for reduced flowrate operation modes LRE low-frequency instability may develop as a result of dynamic interaction between the bench feed system and LRE. The analysis of the dynamic interaction of the TPU and gas paths of the engine is performed. It is shown that the development of low-frequency (5-20 Hz) instability of the "TPU-gas generator" dynamic system may occur during LRE fire tests at throttled mode conditions (with an increased residence time of combustion products in the engine gas generator).

1. Introduction

At the stage of static fire tests, the developers of liquid rocket engines (LRE) solve the following main tasks [1, 2]: check the correspondence of the experimental and design LRE parameters and confirm the performance of the engine systems during their joint work in the operational and critical modes; determine experimentally engine transient characteristics at start-up, shutdown, throttling and engine forcing; verify elements' lifetime of the propellant feed rocket system when simulating the start-up sequence of the full-scale rocket engine; assess tightness and durability of the engine feed system elements, reliability of the engine attachment points, taking into account vibration and acoustic loads; predict and identify potentially dangerous dynamic phenomena that can adversely affect the performance of the LRE systems.

Engine start-up is one of the most important and complex operation modes of LRE. The loss of the main engine performance during this operation period may lead to the destruction of not only the engine, but also a test stand [3]. Modern powerful computing capabilities, advanced modeling technologies (application software) and modern mathematical models of dynamic processes in the LRE systems [for example, 4 and 5], combined with traditional testing of the performance of engine systems during cold tests of its systems, can ensure reliability of engine transients predictions. Forecasting the parameters of LRE dynamic processes and issuing recommendations to ensure the required characteristics of start-up, shutdown of a liquid rocket engine, stability and dynamic strength of its structure, monitoring and analyzing the technical condition of the engine during testing are the most important tasks solved prior to conducting bench tests of main LREs by mathematical modeling.

Now mathematical modeling of working processes in LRE aggregates and systems of a main rocket engine is carried out using various applied software [4, 5, 6]. They are based on the thermodynamic characteristics of combustion products in the gas generator and in the combustion chamber obtained as a computation results of specialized computer

software (such as, for example, ASTRA [7]). Steady-state characteristics of the LRE aggregates for mathematical modeling of LRE low-frequency dynamics, as a rule, are determined using experimental data from different cold flow tests.

However, these well-known techniques reliably describe only a limited range of dynamic phenomena and processes developing in the LRE systems during its fire stand tests and during launch vehicle flight. In a number of complex cases, these techniques do not allow obtaining not only quantitative, but also qualitative coherence of the results of transients experimental studies in engine systems and mathematical modeling of these processes that occur during liquid rocket engine start-ups. Mathematical models presented in [8] allow to take into account the kinetics of propellant ignition and burnout in the combustion chamber and various thermophysical processes occurring in the propellant and gas paths at the initial period of LRE start-up. These processes include heating and partial gasification of cryogenic components, heat removal from the reacting mixture into LRE structural elements, two-phase flows, etc. The mathematical model of low-frequency processes in flow regulators, presented in [8, 9], can be used to perform an LRE stability analysis with respect to regulatory oscillations. In monograph [10], the methods of mathematical modeling of high-frequency oscillations of combustion products in the LRE gas paths were developed. However, in modern mathematical models of the LRE low-frequency dynamics there are no consideration of such important phenomena as cavitation auto-oscillations in the LRE feed system or the dynamic interaction of the tested rocket propulsion system with stand hydraulic systems.

The purpose of this paper is to develop an advanced approach to the mathematical modeling of transient processes of liquid propellant rocket engine during its starts-up under static fire test conditions. Using this approach, mathematical modeling of such phenomena as cavitation self-oscillations in the LRE feed system and self-oscillations of engine parameters caused by instability of the “Turbo Pump Unit - Gas generator - Gas manifold” subsystem can be carried out. The potential danger of such instabilities always exists for oxidizer-rich staged combustion engines. The implementation of this approach will make it possible to generalize and supplement the mathematical models of LRE low-frequency dynamics developed in leading rocket research centers and design bureaus [6, 8].

The developed approach is demonstrated by the example of a numerical study of transients during the start-up of the liquid propellant rocket engine with thrust of 2000 kN and oxidizer staged combustion like the well-known RD 191 [11] (see the simplified LRE flow schematic in Fig. 1). Liquid oxygen and kerosene are used as propellants. Fig.2 contains the 3D layout of the main units and systems of the engine under study. The mathematical modeling of the LRE start-ups was carried out for the conditions of its static fire tests in the engine operational mode with 60% of thrust.

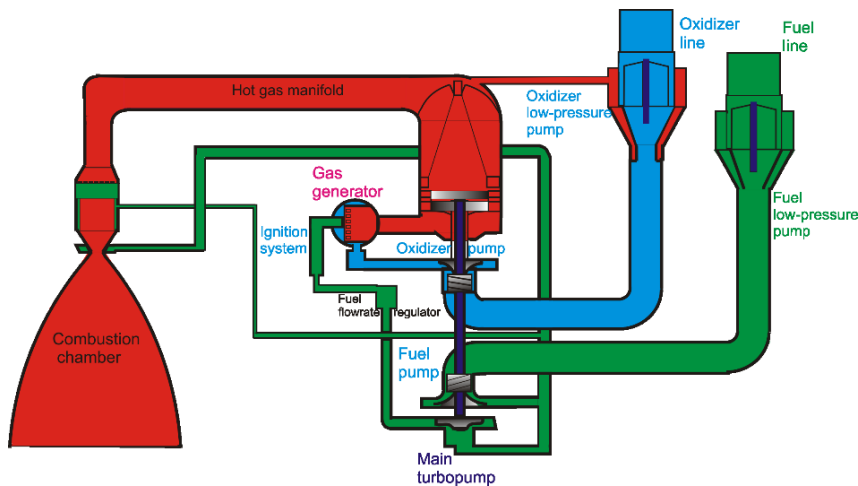


Figure 1: Simplified flow schematic of the LRE under research



Figure 2: 3D layout of the LRE under research

The advanced nonlinear mathematical model of the LRE start-up transients developed for these test conditions describes:

- the dynamics of the cavitating low-pressure pumps and main turbopumps (LOX and RP1);
- dynamic processes in the gas paths of the tested engine taking into account gas residence time;
- process of blowing gaseous oxygen into the stream of liquid oxygen;
- dynamic processes in the fuel flow rate regulator;
- dynamic processes in the stand hydraulic (feed lines) systems and etc.

2. Mathematical modeling of dynamic processes in the LRE cavitating pumps

Mathematical modeling of dynamic processes in LRE cavitating pumps was carried out on the basis of the principles of the theory of cavitation oscillations (traditional cavitation surge) in pump systems [12] developed by academician V. V. Pylypenko (1977). This theory makes it possible to predict the stability of the pump systems of the LRE with respect to cavitation oscillations and, in the event of pump system stability loss, to predict the frequencies and amplitudes of the cavitation oscillations.

Within the framework of this theory and on the basis of solving problem of unsteady cavitation flow around the pump inducer blades, the theoretical nonlinear hydrodynamic model of liquid-propellant cavitating pumps was developed by V.V. Pylypenko, which includes the equations of cavitation cavities dynamics in the flow part of the inducer, fluid continuity, and the equation for determining pressure at the outlet of the pump with inducer:

$$p_1 = p_{bd} + k^*(V_C, q) \cdot \rho \cdot W_1^2 / 2 + B_1 \cdot T_C \frac{dV_C}{dt}, \quad (1)$$

$$\rho g \cdot \frac{dV_C}{dt} = G_2 - G_1, \quad (2)$$

$$p_2 = p_1 + p_H \cdot \tilde{p}_H(\tilde{V}_K) - J_H \frac{dG_2}{dt}, \quad (3)$$

where p_1, G_1 are pressure and weight flow rate at the pump inlet; p_{bd} is pump breakdown pressure; t is current time; $k^*(V_C, q)$ is dependence of the number of cavitation on the cavitation volume V_C and the operational parameter q [13]; $\rho \cdot W_1^2 / 2$ is inducer dynamic pressure; B_1, T_C is cavitation elasticity and cavitation time constant; ρ is liquid density; p_2, G_2 are pressure and weight flow rate at the pump outlet; p_H is pump head; $\tilde{p}_H(\tilde{V}_K)$ is pump cavitation function of pump relative cavitation volume $\tilde{V}_K = V_C / V_C^{bd}$; $V_C^{bd} \approx 2,3 \cdot s \cdot (D_{ip}^2 - d_{hub}^2) / 4$ is the cavitation volume of the inducer flow part before the pump cavitation breakdown [13]; s is inducer pitch, J_H is coefficient of fluid inertial resistance in the pump flow part.

In [14] (1976) academician V.V. Pylypenko as well proposed an experimental and theoretical method for determining the cavitation elasticity and cavitation volume in inducer-centrifugal pumps. By means this method he developed an experimental-theoretical hydrodynamic model of LRE cavitating pumps. This model contains the equations (1) - (3), but the model coefficients were determined using the results of experimental studies of six inducer-centrifugal pumps at the cavitation auto-oscillation operational mode.

The experimental-theoretical hydrodynamic model of the LRE cavitating pumps (1998) was further developed in [15], and it summarized the results of dynamic tests of the 18 inducer-centrifugal pumps. In this model the empirical dependence of the intensity of reverse currents at the pump inlet [16] (1995), as well as the empirical dependence of the pump breakdown pressure [17] (2007) and the generalized dependencies of the relative cavitation characteristics of centrifugal pumps [18] (2008) on the operating parameters were used. The experimental-theoretical hydrodynamic model allowed us to satisfactorily coordinate and describe the results of dynamic tests of rocket engine pumps of various dimensions and performance.

Figure 3 shows the experimental values obtained from the results of dynamic tests of the inducer-centrifugal pumps, and the theoretical dependence [15] of the relative cavitation elasticity $\tilde{B}_1 = B_1 V_C^{bd} / (\rho \cdot W_1^2 / 2)$ on cavitation number $k^* = (p_1 - p_{BD}) / (\rho \cdot W_1^2 / 2)$ for operation mode parameter $q \approx 0,56$. This value is close to the values of the operational parameters for the pumps (LOX and fuel) of the LRE upon the study.

In the paper [19], to describe the dynamic interaction of cavitation oscillations (fluid fluctuations) and vibrations of the pipeline structure, the equation of dynamics of cavitation cavities (1) was obtained in differential form

$$\frac{dp_1}{dt} = \frac{G_1 - G_2}{C_C} + R_{K1} \frac{dG_1}{dt} + R_{K2} \frac{dG_2}{dt}, \quad (4)$$

where $R_{K2} = B_1 \cdot T_C / \rho g$ and $R_{K1} = B_2 - B_1 \cdot T_C / \rho g$ are coefficients; $C_C = -\rho g / B_1$ is inducer cavitation compliance; g is free fall acceleration; $B_2 = -(\partial V_C / \partial G_1) / (\partial V_C / \partial p_1)$ is negative cavitation resistance of pump inducer [12].

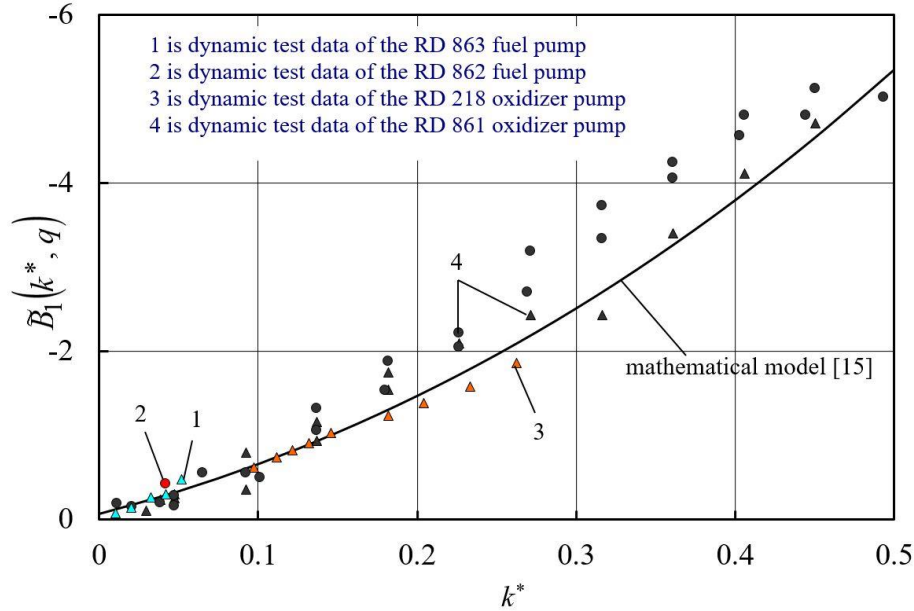


Figure 3: Relative cavitation elasticity $\tilde{B}_1(k^*, q)$ vs. the cavitation number k^* for operational mode parameter $q \approx 0.56$

At the LRE start-ups the operation parameter q and cavitation number k^* of the pumps, as a rule, vary widely from pump cavitation mode to pump non-cavitation mode. In this regard, the hydrodynamic model of cavitating pumps [15-18] in the developed approach to modeling the dynamics of cavitating pumps is adapted to describe transient processes in pump systems with large cavitation numbers due to a change in the dependence of the relative elasticity of cavitation cavities on the operating parameters [20]:

$$\tilde{B}_1(k^*, q) = \frac{a(q) \cdot k^{*2} + b(q) \cdot k^*}{1 - (k^*/k_0^*)^2}, \quad (5)$$

where $a(q)$, $b(q)$ are dependences obtained on the basis of a generalization of the results of experimental studies of the eighteen LRE pumps with different inducer design [15]; k_0^* is cavitation number corresponding to cavitation nucleation at the inducer type pump [13].

3. Mathematical modeling of low-frequency dynamics of stand hydraulic feed system during static fire tests of the main engine under research

The mathematical modeling of transients at LRE start-ups was carried out in relation to the stand for conducting fire tests of LREs up to 1000 kH utilizing liquid oxygen/kerosene as propellant combination. The stand LOX feed system for LRE (see Fig. 5) consists of two LOX tanks with a common hydraulic feedline of 37 m length. The LOX receiver with an autonomous pressurization system is connected to the LRE feedline at a section located in a distance of approximately 3 m from the engine entrance.

The simplified LOX flow schematic of the stand oxidizer feed system from LOX tanks to the entrance into the Low Pressure Oxidizer pump is shown in Fig. 6. The schematic contains the designations of the LOX hydraulic system finite elements conventionally discretized at the mathematical modeling. For the stand-mounted hydraulic fuel feed

system (in the Figure 5 indicated by red) with the total length of 32 m, the results of low-frequency dynamics simulation are qualitatively similar, and here they are not shown. Mathematical modeling of low-frequency dynamics of fluid in pipelines of a stand-alone feed system of LRE as a system with distributed parameters is based on the equations of unsteady motion and continuity of fluid [12]:

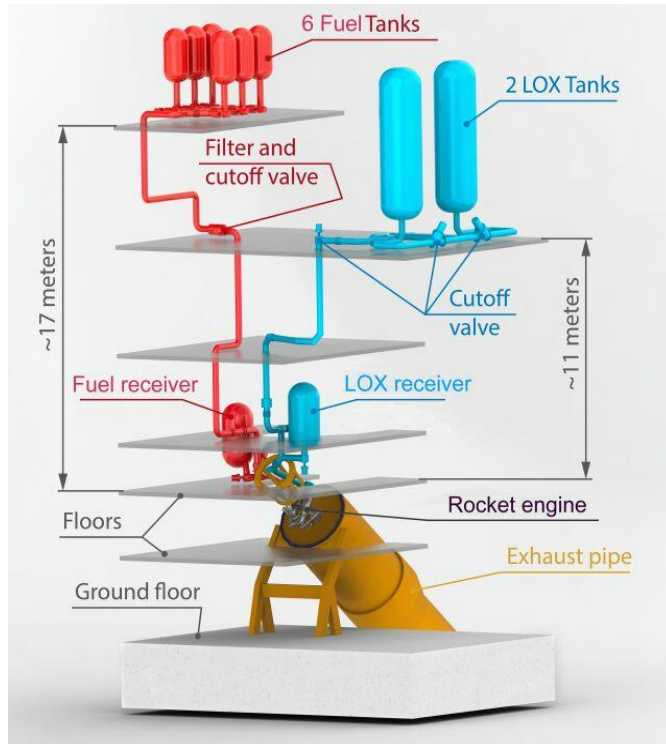


Figure 5: Propellant flow schematic of the LRE test stand

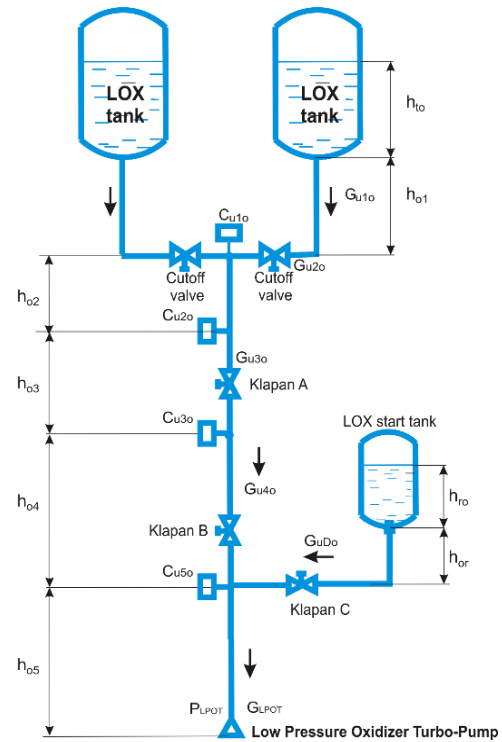


Figure 6: Simplified LOX flow schematic of the LRE test stand

$$\begin{cases} \frac{\partial p}{\partial z} + \frac{1}{g \cdot F} \cdot \frac{\partial G}{\partial t} + \frac{k}{g \cdot F} \cdot G = 0, \\ \frac{\partial G}{\partial z} + \frac{g \cdot F}{c^2} \cdot \frac{\partial p}{\partial t} = 0, \end{cases} \quad (6)$$

where p , G are pressure and weight flow rate of fluid; t is time; z is axial coordinate of the pipeline; F is area of the pipeline cross section; k is reduced coefficient of linear friction per pipeline length unit; Δp is hydraulic pressure loss; l is the pipeline length; \bar{G} is fluid weight flow rate at steady state conditions; g is gravity acceleration; c is the fluid sound speed in a pipeline with elastic walls.

However, the use of equations (6) in partial derivatives even in linear mathematical models with a further approximate solution (for example, the method of characteristics [8]) is associated with cumbersome computations. Therefore, to calculate the frequency gains of the LRE feedlines, taking into account the distribution of parameters, the impedance method was used [12]. In this case, the solution of partial differential equations (6) can be represented, for example, by a passive quadrupole of the following form:

$$\begin{cases} \delta p_2 = b_{11} \cdot \delta p_1 + b_{12} \cdot \delta G_1, \\ \delta G_2 = b_{21} \cdot \delta p_1 + b_{22} \cdot \delta G_1, \end{cases} \quad (7)$$

where δp_1 , δG_1 , δp_2 , δG_2 are deviations of fluid pressure and weight flow rate from their values at steady state conditions at the inlet and outlet of the quadrupole; b_{11} , b_{12} , b_{21} and b_{22} are elements of the transfer matrix of the hydraulic pipeline with distributed parameters

$$b_{11} = ch(\gamma \cdot l), \quad b_{12} = -Z_B \cdot sh(\gamma \cdot l), \quad b_{21} = -\frac{sh(\gamma \cdot l)}{Z_B} \quad \text{and} \quad b_{22} = ch(\gamma \cdot l), \quad (8)$$

$\gamma = (Z^* \cdot Y^*)^{1/2}$ is complex "constant" of wave propagation per unit of pipeline length; $Z_B = (Z^* / Y^*)^{1/2}$ is wave resistance of the hydraulic pipeline; $Z^* = (s + k) \cdot (g \cdot F)^{-1}$ is hydraulic series impedance per unit length of pipeline; s is Laplace variable; $Y^* = g \cdot F \cdot s \cdot c^{-2}$ is parallel admittance of the pipeline length unit.

Using a passive quadrupole of the form (7), for each element of the hydraulic system with a previously known impedance $Z_2(j\omega)$ at the element output, the impedance $Z_1(j\omega)$ at the element input and the dynamic gain $W(j\omega)$ of the considered element of the hydraulic system can determine by formulas:

$$Z_1(j\omega) = \frac{\delta p_1(j\omega)}{\delta G_1(j\omega)} = \frac{b_{12} - b_{22} \cdot Z_2(j\omega)}{b_{21} \cdot Z_2(j\omega) - b_{11}}, \quad W(j\omega) = \frac{\delta p_2(j\omega)}{\delta p_1(j\omega)} = b_{11} + b_{12} \frac{1}{Z_1(j\omega)}. \quad (9)$$

Dynamic gain of the entire hydraulic system $W_\Sigma(j\omega)$ defined as the product of n the dynamic gain of each hydraulic element $W_\Sigma(j\omega) = W_1(j\omega) \cdot \dots \cdot W_n(j\omega)$.

The development of a non-linear mathematical model of low-frequency dynamics of complex branched pipelines for the calculation of transients in the LRE feed system is based on the approach outlined in [21]. In accordance with this approach, in the first stage, the dynamic gains of the pipeline are determined as systems with distributed parameters. Further, the dynamic gains of the same pipeline are determined as systems with lumped parameters. Moreover, each section of the pipeline can be represented as a quadrupole of the following form:

$$\begin{cases} \delta p_2 = \delta p_1 - (R + sJ) \cdot \delta G_1, \\ \delta G_2 = -Cs \cdot \delta p_1 + \delta G_1, \end{cases} \quad (10)$$

where R , J are concentrated linearized hydraulic and inertial resistance of the pipeline section; C is concentrated compliance of the pipeline section. The values of the parameters R and J are determined by the geometry of the pipeline, hydraulic pressure losses and fluid flow rate. The number of lumped compliances C and their values are selected from the necessary accuracy of matching in a given frequency range the considered pipeline dynamic gains, obtained as systems with distributed and lumped parameters.

Figure 7 shows the dynamic gains (by pressure) $\frac{\delta p_{1LPOT}}{\delta p_{TO}}(j\omega)$ of the oxidizer feed line from the engine inlet up to

the tank outlet of the LRE stand considered here as dynamic system with distributed parameters (here by the sign δ is marked deviation of LRE system parameter from the its steady-state value).

Due to the considerable length of the LOX stand feed system, its wave properties are noticeably manifested in the frequency range up to 50 Hz: resonance maxima corresponding to 5 modes of the natural frequencies (4.4 Hz, 13.4 Hz,

21.8 Hz, 31.4, 40.6 Hz) of the fluid oscillations in LOX feed line appear in the dynamic gain modulus $\frac{\delta p_{1LPOT}}{\delta p_{TO}}(j\omega)$

(see in Figure 7 the curve by red). Accounting for cavitation phenomena in the LRE pumps leads to a significant decrease in the LOX feed line natural frequencies. So, for example, for computation case of without taking into account cavitation phenomena the first natural oscillation mode is 6.2 Hz (the curve is green), but for the calculation case with taking into account cavitation phenomena this frequency is 4.4 Hz (the curve is red). The installation the receiver with a gas cavity volume of 0.6 m³ in the LOX feed line (see blue and violet curves in Figure 7) significantly changes the shape of the feed line dynamic gain $\frac{\delta p_{1LPOT}}{\delta p_{TO}}(j\omega)$. The LOX receiver manifests itself dynamically as a notch filter in

the frequency range from 0.1 Hz to 13 Hz, i.e. in this frequency range the fluid oscillations in the feed line are suppressed, and the modulus of the feed line dynamic gain in this case is close to zero.

Figure 8 shows the LOX feed line dynamic gain with allowance for cavitation in the LRE pumps and with the LOX receiver, calculated on the basis of mathematical models of the stand feed system as a system with distributed parameters and as a system with lumped parameters. From analysis of this figure, it follows

the dynamic gains $\frac{\delta p_{1LPOT}}{\delta p_{TO}}(j\omega)$ with distributed and concentrated parameters would differ greatly without taking

into account the walls compliance of the pipeline elements. The four lumped compliances C_{U10} , C_{U20} , C_{U30} , C_{U40} (as indicated in Figure 6) should be used in the mathematical model of the feed system low-frequency dynamics with lumped parameters for better matching of these dynamic gains in the frequency range up to 50 Hz.

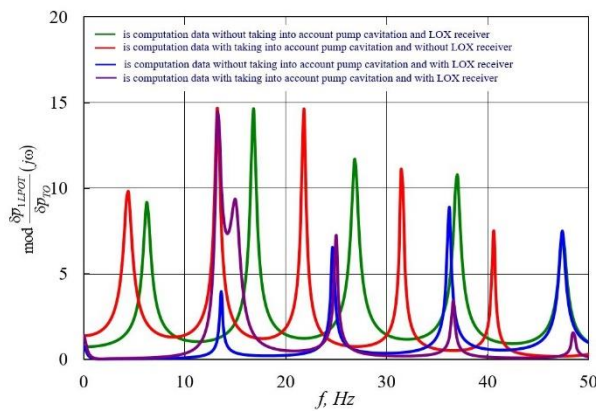


Figure 7: Modulus of dynamic gain $\frac{\delta p_{1LPOT}}{\delta p_{TO}}(j\omega)$ LOX feed line considered in modeling as system with distributed parameters

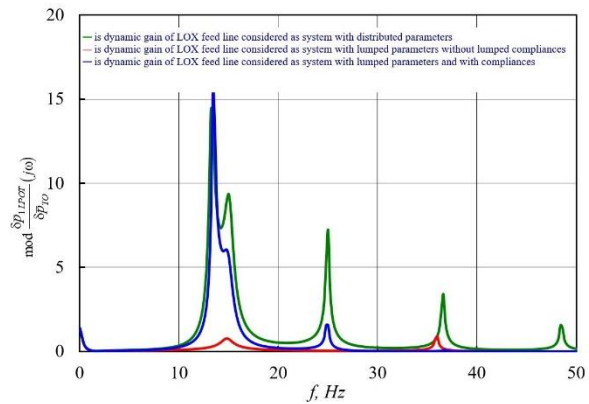


Figure 8: Modulus of dynamic gain $\frac{\delta p_{1LPOT}}{\delta p_{TO}}(j\omega)$ LOX feed line considered as system with distributed and lumped parameters

Taking into account all the above, the nonlinear mathematical model of the low-frequency dynamics of the stand oxidizer feed system was represented by the following system of equations in lumped parameters:

$$p_{TO} = p_{U10} + a_{U10}G_{U10}^2 - \rho_O \cdot g \cdot (H_{TO} + h_{O1}) + J_{U10} \frac{dG_{U10}}{dt}, \quad (11)$$

$$C_{U10} \frac{dp_{U10}}{dt} = 2G_{U10} - G_{U20}, \quad (12)$$

$$\bar{p}_{U10} = p_{U20} + a_{U20}G_{U20}^2 - \rho_O \cdot g \cdot h_{O2} + J_{U20} \frac{dG_{U20}}{dt}, \quad (13)$$

$$C_{U20} \frac{dp_{U20}}{dt} = G_{U20} - G_{U30}, \quad (14)$$

$$\bar{p}_{U20} = p_{U30} + a_{U30}G_{U30}^2 - \rho_O \cdot g \cdot h_{O3} + J_{U30} \frac{dG_{U30}}{dt}, \quad (15)$$

$$C_{U30} \frac{dp_{U30}}{dt} = G_{U30} - G_{U40}, \quad (16)$$

$$\bar{p}_{U30} = p_{U40} + a_{U40}G_{U40}^2 - \rho_O \cdot g \cdot h_{O4} + J_{U40} \frac{dG_{U40}}{dt}, \quad (17)$$

$$\bar{p}_{RO} = p_{U40} + a_{RO}G_{RO}^2 - \rho_O \cdot g \cdot h_{OR} + J_{RO} \frac{dG_{RO}}{dt}, \quad (18)$$

$$C_{U40} \frac{dp_{U40}}{dt} = G_{U40} + G_{RO} - G_{LPOT}, \quad (19)$$

$$\bar{p}_{U40} = p_{LPOT} + a_{U50}G_{LPOT}^2 - \rho_O \cdot g \cdot h_{O5} + J_{U50} \frac{dG_{LPOT}}{dt}, \quad (20)$$

where C_{U10} , C_{U20} , C_{U30} , C_{U40} are the feed line element lumped compliances; p_{U10} , G_{U10} , p_{U20} , G_{U20} , p_{U30} , G_{U30} , p_{U40} , p_{RO} , G_{U40} are pressures and weight flow rates in the final sections of the relevant sections of the stand line elements; a_{Ui0} , J_{Ui0} , h_{Oi} are respectively coefficient of hydraulic resistance, coefficient of inertial resistance and liquid height difference of i -th element of LOX feed system; \bar{p}_{TO} , p_{LPOT} , G_{LPOT} are respectively the pressure in the LOX tank outlet (the overline above the symbol means the stationary value of the parameter), pressure and weight flow rate at the inlet to the Low Pressure Oxidizer Pump.

4. Mathematical modeling of dynamic processes of the liquid rocket engine gas paths

For development of a mathematical model of low-frequency dynamic processes in the LRE gas paths (combustion chamber, gas generator and gas duct), the following simplifications are usually adopted [3, 8, 22, 23]. LRE gas paths considered as elements with lumped parameters; the processes occurring in them are assumed adiabatic. For modeling of dynamic processes in the LRE combustion chamber and the gas generator, the simplest combustion model is used. The combustion model based on the approximation of the burnout curve and the temperature transfer curve by the transportation lag, i.e. by unit step functions.

Taking into account the accepted simplifications, the non-stationary non-isothermal adiabatic gas movement in the LRE gas path elements is described in the low-frequency range by a differential-algebraic system of equations with lags (time delays). The structure of these equations in the developed system of equations of LRE nonlinear dynamics is the same for all LRE gas paths.

Below are the nonlinear equations describing the low-frequency dynamics of the gas generator of the LRE under study: equation pressure in the gas cavity; equation for calculating the gas flow rate at the outlet of the gas generator; equations describing the lagging process of hot gas combustion products; equations to determine the product of universal gas constant and temperature of combustion hot gas at the gas generator input and at the gas generator output:

$$\frac{dp_{gg}}{dt} = \frac{\kappa_{gg}(RT)_{gg2}}{V_{gg}} (G_{ggo}^* + G_{ggf}^* - G_T), \quad (21)$$

$$G_T = \mu_T F_T \sqrt{g \frac{2\kappa_{gg}}{\kappa_{gg} - 1} p_{gg}^2 / (RT)_{gg2} \left[\left(p_{gm} / p_{gg} \right)^{\frac{2}{\kappa_{gg}}} - \left(p_{gm} / p_{gg} \right)^{\frac{\kappa_{gg} + 1}{\kappa_{gg}}} \right]}, \quad (22)$$

$$G_{gg0}^* = G_{gg0} \left(t - \tau_f^{gg} \right), \quad G_{ggf}^* = G_{ggf} \left(t - \tau_f^{gg} \right) \quad (23)$$

$$(RT)_{gg1} = (RT) \left(k_{gg}^* \right), \quad k_{gg}^* = G_{gg0}^* / (G_{ggf}^*) \quad (24)$$

$$(RT)_{gg2} = (RT)_{gg1} \left(t - \tau_R^{gg} \right) \quad (25)$$

where p_{gg} , p_{gm} are pressure in the gas paths of the gas generator and gas pipeline; G_{gg0} , G_{ggf} are weight flow rates of liquid propellants at the gas generator inlet; G_{gg0}^* , G_{ggf}^* are weight flow rates of oxidizer and fuel at the entrance to the gas generator, taking into account the lag time τ_f^{gg} of gas formation of the propellant; k_{gg}^* is the ratio of the propellants in the gas generator; G_T is gas weight flow to turbine; $(RT)_{gg1}$, $(RT)_{gg2}$ is the products of the gas constant and the temperature of the hot gas at the inlet and outlet of the gas generator; $(RT) \left(k_{gg}^* \right)$ is dependence of the RT of hot gas on the gas generator propellant ratio k_{gg}^* ; κ_{gg} is adiabatic index of combustion products in the gas generator; V_{gg} is volume of the gas path of the gas generator; F_T , μ_T are area and flow rate coefficient of the nozzle array of the main turbine of the engine; τ_R^{gg} is gas residence time in the gas generator. The formulas presented in the works [3, 8] are used to estimate the lag times τ_f^{gg} and τ_R^{gg} .

Figure 9 shows presented in paper [23] the dynamic gain modulus (in the LRE LOX line pressure) of the main rocket engine main LRE with oxidizer-rich staged combustion, computed for different values of time lags in the low-frequency dynamics equations of the engine gas paths. The following notation is used here: δp_{LPOT} , δp_{CH} are the complex pressure deviation at the inlet to the Low Pressure Oxidizer Turbo-pump and in the LRE combustion chamber; j is imaginary unit; ω is the angular oscillation frequency. As follows this figure the time delays significantly affect the engine dynamic gain (compare curves by red and by black) and, therefore, the LRE dynamic processes. Herewith, the time lags in the equations of the dynamics of the gas generator and the gas manifold (see black and green curves) are decisive in the frequency range from 0 to 50 Hz, and the time delays in the equations of the dynamics of the combustion chamber can be neglected.

In case of LRE transients mathematical modeling, it is needed to perform an approximate replacement of the delays in LRE dynamics equations by ordinary differential equations. To perform such a replacement, it has been proposed [22, 23] to use an approach based on approximation of the transfer function of the time delay link by fractional-rational functions $W_e(s\tau) = \exp(-s\tau)$. The following requirements are imposed to the approximating functions: the necessary accuracy of approximation of the LRE dynamic gains in a given low frequency range; the smallest order of fractional rational functions; stability of polynomial functions in its denominator. The following four fractional rational functions were analyzed: the 1st order Taylor series $W_e(s\tau) \approx T_{0,1}(s\tau) = 1/(1+s\tau)$, second order Padé series $W_e(s\tau) \approx P_{2,2}(s\tau) = (12 - 6s\tau + (s\tau)^2) / (12 + 6s\tau + (s\tau)^2)$, chain of four aperiodic links $W_e(s\tau) \approx [T_{0,1}(s\tau/4)]^4 = 1/(1+s\tau/4)^4$ and chain of two oscillatory links $W_e(s\tau) \approx [T_{0,2}(s\tau/2)]^2 = 1 / \left(1 + s\tau/2 + (s\tau/2)^2 \right)^2$.

It should be noted that the presented list of fractional and rational functions is not exhaustive, and the choice of the type of function for each case is individual.

The simplest approximation function is a function constructed using the first-order Taylor series. However, approximation by this function may not be sufficiently accurate if the delays are relatively large [22, 23]. In the present work, the transfer functions to account for the time lag approximated by $T_{0,1}(p\tau)$.

Figure 10 shows the modules of the theoretical dynamic pressure gain of the main LRE with oxidizer-rich staged combustion, computed in [23] by approximating the transfer function of the lag units introduced to account for the residence time of the combustion products in the gas generator and hot gas manifold by functions of $T_{0,1}(s\tau)$, $P_{2,2}(s\tau)$, $[T_{0,1}(s\tau/4)]^4$ and $[T_{0,2}(s\tau/2)]^2$. It can be seen from the figure the greatest accuracy is ensured by the use of a second-order Padé series and a chain of 2 oscillatory links. However, for the second-order Padé series the stability of polynomials in the numerator of transfer functions is not ensured. It can become a problem at the numerical study of the LRE start-ups. The approximating functions obtained using aperiodic chains and oscillatory links do not contain unstable polynomials. The approximation by a chain of two oscillatory links is more accurate for an equal order of the approximating system of ordinary differential equations. This approximation can be recommended for use. In view of the foregoing, the differential equations for the time lags in hot gas formation from the LOX and fuel (23) and the equations for determining the RT of the combustion hot gas at the outlet of the gas generator (25) have the form

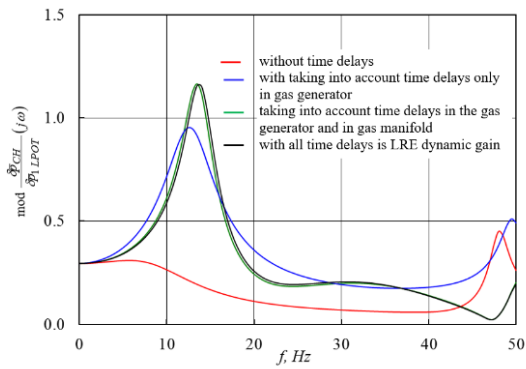


Figure 9: The effect of LRE time delays on the theoretical modulus of the engine dynamic gain (LOX line)

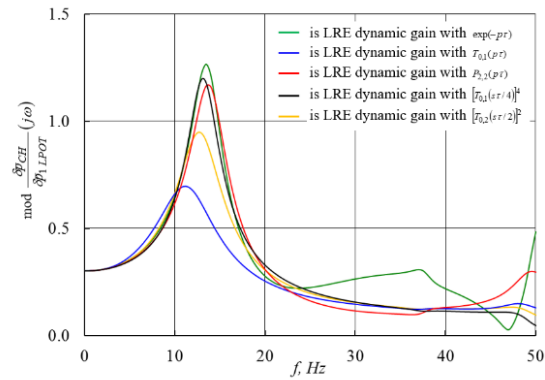


Figure 10: Theoretical modulus of LRE dynamic gain (LOX line) for different representation of the transfer functions of the residence time delay in the gas generator and hot gas manifold

$$\tau_f^{gg} \cdot \frac{dG_{ggo}^*}{dt} + G_{ggo}^* = G_{ggo}, \quad \tau_f^{gg} \cdot \frac{dG_{ggf}^*}{dt} + G_{ggf}^* = G_{ggf}, \quad (26)$$

$$0,125 \left(\tau_R^{gg} \right)^2 \frac{d^2 RT_{gg2}(t)}{dt^2} + 0,5 \tau_R^{gg} \frac{dRT_{gg2}(t)}{dt} + RT_{gg2}(t) = RT'_{gg1}(t), \quad (27)$$

$$0,125 \left(\tau_R^{gg} \right)^2 \frac{d^2 RT'_{gg1}(t)}{dt^2} + 0,5 \tau_R^{gg} \frac{dRT'_{gg1}(t)}{dt} + RT'_{gg1}(t) = RT_{gg1}(t). \quad (28)$$

5. The results of numerical modeling of transient processes in the 2000 kN engine during the start-ups for its static fire test

Based on the developed mathematical model of the main LRE start-ups, the transients at the engine start-up during its static fire test at 60% LRE thrust operational mode were determined – the LRE time responses of pressures, flow rates, propellant temperatures and combustion products, TPU shaft rotation speeds (see, for example, Fig. 11 and 12). The computed transients of the LRE are mainly typical for this type of LRE (with staged oxidizer-rich combustion), “reflect” the LRE start-up sequence and characterize the LRE transient process without pump cavitation disruption,

without overshoots of the engine pumps rotational speed and other negative phenomena. According to the results of the computations, there is a damping of the parameters of all dynamic processes occurring in the engine after reaching the 60% LRE thrust mode.

Analyzing the LRE transients, attention was paid to the development of oscillations of engine parameters (in particular, the main turbopump rotational speed) at the frequency of 8.8 Hz over a time interval from 1.6 s to 2.15 s. By numerical analysis established the cause of these oscillations is the instability of the engine oscillating circuit, consisting of the main turbopump and the gas generator. This oscillating circuit can be implemented in all engines with staged combustion cycle. This conditions for the studied LRE were realized under the LRE processes became unstable in this oscillatory circuit in the LRE start-up time t interval from 1.6 s to 2.15 s and are associated with higher values of the residence time τ_R^{gs} of the combustion products in the gas generator at a specified start-up period than the residence time τ_R^{gs} at the steady state conditions (for time t greater than 2.7 s). Computations made without taking into account the hot gas residence time in the gas generator (red curve is for $\tau_R^{gs} = 0$ in Fig. 11 and 12) show that there are no oscillations of these parameters in the time interval from 1.6 s to 2.15 s and, thus, confirm the above conclusions.

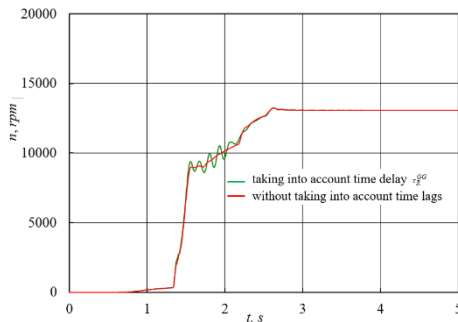


Figure 11: Time dependence of the rotation speed of the main Turbopump during the LRE start-ups

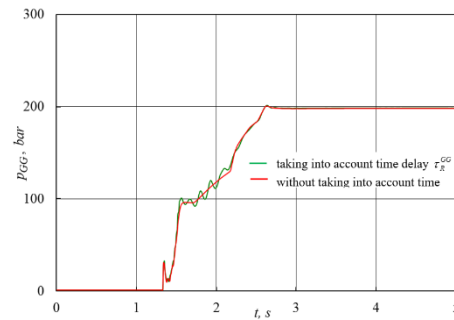


Figure 12: Time dependence of the gas generator pressure during the LRE start-ups

Using the results presented in Figures 13 and 14 as an example, the operation of the stand LOX feed system at the LRE start-ups is demonstrated (the operation of the fuel feed system at the LRE start-ups is similar). It is shown that with simultaneous feeding from both the LOX tanks and the LOX receiver in the initial start-up period, the main share of oxidizer feeding (the oxidizer flow rate) to the engine is carried out mainly from the LOX receiver. As the LOX receiver is emptied, the receiver pressure in its gas cushion decreases, as a result of that the LOX flow rate from the receiver decreases. After a certain time caused by the difference in the volumes of the gas cushions of the receiver and the LOX stand tanks, the engine feeding will completely switch to the feeding from the LOX tanks.

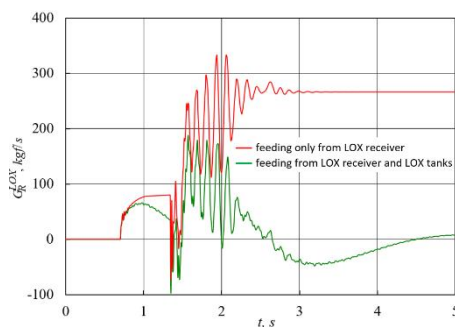


Figure 13: LRE start-up transients of weight flow rate from the LOX receiver

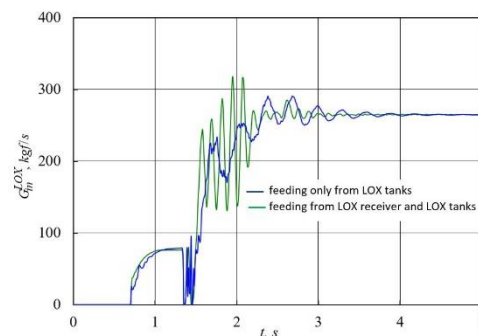


Figure 14: LRE start-up transients of weight flow rate at the inlet to the Low Pressure Oxidizer Pump

It is established that the receivers installed at the stand feed lines, significantly damp the fluctuations in pressure and flow rates of the oxidizer and fuel. There are damped oscillations in case of the "disabled" receivers in the feed lines of the oxidizer and fuel during the start-up period (see Fig. 14 for LRE start-up in case of LOX "disabled" receiver) with oscillation frequencies close to the natural frequencies of the oscillation first mode of the liquid in the stand lines: about 3.5 Hz for LOX feed line and about 6 Hz for fuel feed line.

6. Discussion

During LRE static fire tests, the study of the transient processes in the engine systems is planned in throttled mode corresponding to 60% of the engine thrust. The dynamic instability of the liquid propellant rocket engine during the transition to a deep throttled mode was noted at firing tests of this type of main engines with the oxidizer-rich staged combustion cycle. In particular, the low-frequency instability of the RD 191 (rocket engine with thrust 2000 kH developed by JSC 'NPO Energomash') with an oscillation frequency of 4 Hz was detected during engine fire tests on deep throttling modes up to 27 ... 30% of the engine nominal thrust. The engine instability leads to an unacceptably large increasing the range of pressure fluctuations in the elements of the gas path - up to 40% - 50% of their nominal values [25]. Figure 15 shows the experimental time dependences of the fuel pressure in front of the mixing head of the chamber during fire tests of the RD-191 rocket engine.

In work [25] (2013), it was assumed that the excitation of low-frequency oscillations with the frequency of 3.5 Hz - 4.5 Hz of the main parameters of the LRE of RD 191 during its fire tests is due to the large values of the gas residence time in the gas paths of the LRE operating in throttled mode. Note, at the 100% thrust mode of the above LRE RD 191, the total gas residence time in the gas volumes of the gas generator, gas manifold from the turbine to the combustion chamber and the combustion chamber itself of this engine is 0.034 sec, and for 30% thrust mode is 0.065 sec.

As shown in paper [24] (2007), the risk of engine dynamic instability increases due to the instability of the 'main turbopump - gas generator' circuit for such large values of the gas residence time τ (more than 0.035 s) in the LRE gas paths.

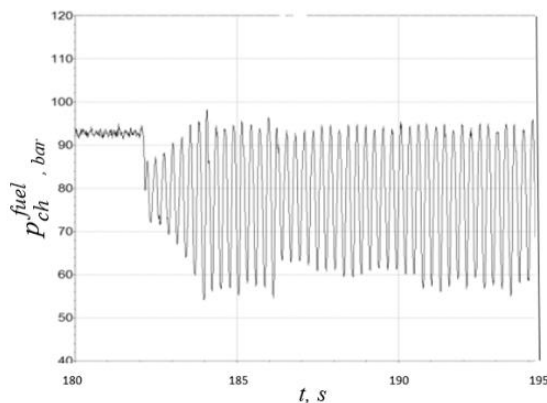


Figure 15: Time dependencies of fuel pressure in front of the mixing head of the RD 191 engine combustion chamber

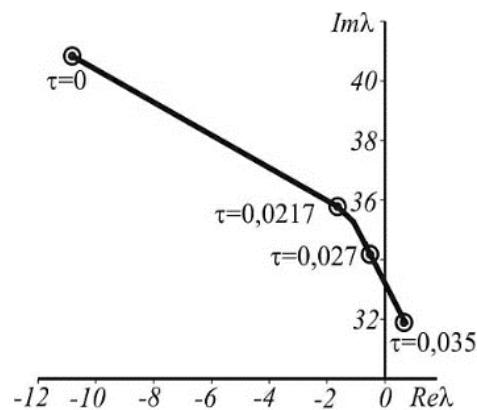


Figure 16: Dependence of the degree of LRE stability (real part of system matrix eigenvalue) of the dynamic system for the RD 120 engine on gas residence time τ

Figure 16 shows the hodograph of the eigenvalue matrix of the differential equations system describing the LRE low-frequency dynamics developed in this work. The hodograph obtained by changing the residence time τ of the gas in the gas generator of the RD 120 engine (the main engine of second stage of the Zenit launch vehicle) with rich-oxidizer combustion cycle. According to the results of mathematical modeling, an increase in the gas residence time τ in the gas generator provided such phase relations between the variables of the 'main turbopump - gas generator' oscillatory subsystem, under which the feedback efficiency in the turbopump rotation speed increased significantly. This circumstance in turn led to the appearance of low-frequency instability of the LRE under study at frequency (at $32/2\pi$ Hz in Fig. 16). The result obtained is important for the theory and practice of liquid rocket engines. The possibility of this type of dynamic instability of the engine (including LRE throttling modes) should be taken into account for designing LREs with staged combustion cycle.

As show in section 5 the start-ups of the studied LRE over the time period from 1.6 s to 2.15 s have oscillations of parameters with a frequency of 8.8 Hz due to the 'main turbopump - gas generator' instability. After the engine goes

to 60% operational mode, these oscillations subside. Figures 17 and 18 show the results of computations of transient processes at the start-ups of the studied liquid propellant rocket engine with 50% increase in the residence times of the combustion products in the gas generator τ_R^{gg} and gas manifold τ_R^{gm} .

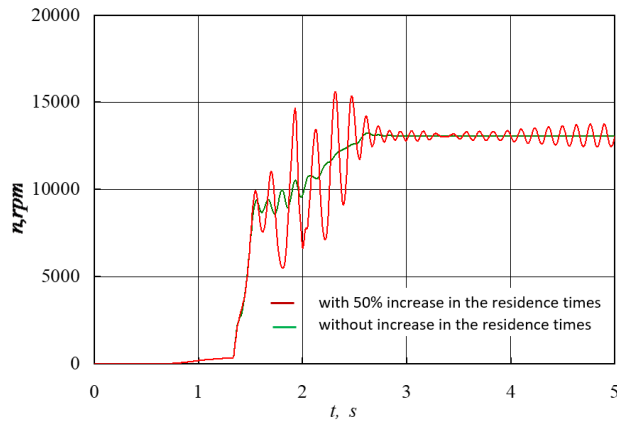


Figure 17: Time dependence of the rotation speed of the main turbopump at LRE start-up

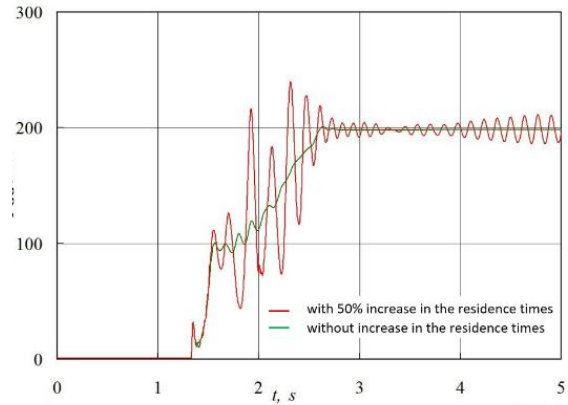


Figure 18: Time dependence of the gas generator pressure at LRE startup

Such increase in τ_R^{gg} and τ_R^{gm} leads to the existence of oscillations in the LRE parameters, not only at transients but at LRE stationary operation modes. Figures 17 and 18 demonstrate the principal possibility of the presented approach to mathematical modeling of rocket engine start-ups transients and to obtain by computation analysis the LRE low-frequency dynamics to oscillations due to the dynamic instability of the ‘main turbopump - gas generator’ contour like obtained during static fire tests [25] of RD 191 rocket engine.

The approach presented in this work to the mathematical modeling of the LRE start-ups transients also can predict another type of LRE dynamic instability well known as cavitation surge auto-oscillations [12]. Figures 19 and 20 show the principal possibility of cavitation surge auto-oscillations in the stand oxidizer feed system. Figures presents the results of computation of start-up transients of the engine in case of the propellant is fed only from the LOX receiver, reduced pressure in the gas cavity of the receiver and reduced pressure losses in the oxidizer line from the receiver to the entrance into the engine.

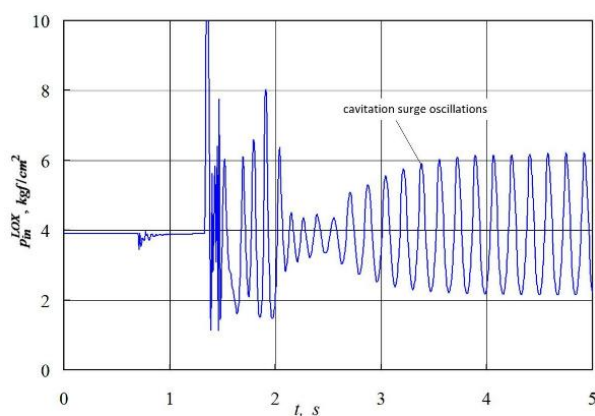


Figure 19: Time dependence of the pressure at the engine LOX inlet during at the LRE start-up

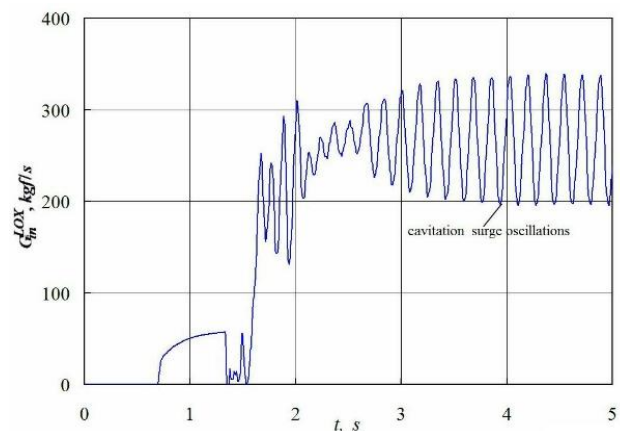


Figure 20: Time dependence of the oxidizer weight flow rate at the engine inlet during LRE start-up

7. Conclusion

The approach to mathematical modeling of main rocket engine transient processes at engine start-ups, shutdowns, forcing or throttling in LRE aggregates and systems under hot test conditions has been developed. By use of the developed approach such dynamic phenomena as cavitation surge oscillations in the liquid propellant feed system and auto-oscillations caused by the instability of the subsystem ‘main turbopump - gas generator’ can be identified and studied.

On the basis of the presented approach, the parameters of transients are computed for the main rocket engine of 2000 kN thrust with oxidizer-rich combustion under of stand fire tests for the 60% LRE thrust operation. The conditions for the appearance of engine parameters oscillations of due to the instability of the ‘main turbopump - gas generator’ subsystem are determined, and numerically obtained by mathematical modeling at both transitional and steady-state conditions. The existence of such oscillations for the LRE RD 191 was noted during its fire tests on modes of deep throttling. The principal possibility of mathematical modeling of another type of instability of the LRE - cavitation surge auto-oscillations in the engine feed system is demonstrated.

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