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Imperfection sensitivity analysis for Buckling of thin cylindrical shell submitted to axial harmonic compression

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Abstract

Depending on the launcher configurations, two, three or four boosters can be attached on the Lower Liquid Propulsion Module. The axial compression induced by the boosters loads on the liquid tank is then not uniform but can be represented by a modal or harmonic function (sine or a cosine function). The paper concerns buckling of thin-walled cylindrical aluminium shell structures induced by a local axial compression load, or by harmonic or modal compression load. The problem is numerically solved using ABAQUS/Implicit finite element code. First, linear and geometrical nonlinear analyses are conducted to determine the critical stress for a perfect shell for different boundary condition. Then, sensitivity of geometric imperfections has been performed by considering different initial defect shapes which were introduced in the numerical models. Especially, local defects have been simulated as local inward bump or axisymmetric mode or the first buckling mode issued from linear buckling analysis. This study shows that the common practice which assumes for the design an equivalent axisymmetric load equal to the maximum value which occurs only at some azimuths in the reality (real local or modal load) is an approach which is too conservative for bucking analysis of cylindrical shells. Then, suitable values of Knock-down factors KDF are not only proposed for the design in the case of a modal load but also compared to the KDF given by NASA rules for axisymmetric compression. A substantial gain was noticed.

1. Introduction

The aim of the present study is to give another method to analyse the buckling of the Ariane 6 Lower Liquid Propulsion Module (LLPM) Oxygen tank. The method of today is mainly based on the NASA SP8007 rule [1], established in the late 1960s. The Lower Liquid Propulsion Module (LLPM) is connected to four or two boosters depending on the Ariane 6 launcher configuration. During the flight, the launcher is submitted to several sources of loadings. This corresponds mainly to a combination of axisymmetric compression, bending load, shear load and non-axisymmetric load induced by the boosters. The compression induced by the boosters can be approximated by a harmonic function $q\cos(n\alpha)$, where n=2 or 4. For the design, the common practice used the simple and conservative approach which considers an axisymmetric load $Q=q_{max}$ instead of the harmonic or modal load $Q=q_{max}\cos(n\theta)$.

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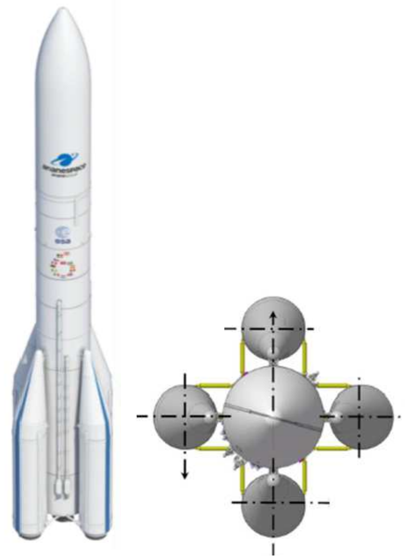

Figure 1. Ariane 6 launcher with 4 boosters configuration

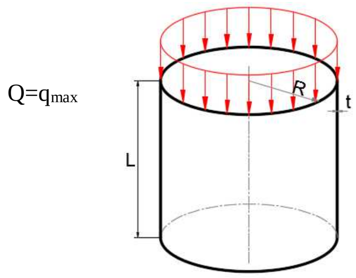

Figure 2. Axisymmetric loading

2. Theoretical critical stress and NASA SP8007

The theoretical buckling stress associated to the elastic bifurcation load of thin cylindrical shell under uniform axial compression ([2],[3],[4]) is given below:

$$q_{CL} = \frac{E}{\sqrt{3(1-\nu^2)}}\frac{t^2}{R} \quad \textbf{(1)}$$

The results of many experiments in the literature showed that the buckling of cylindrical shells under uniform axial compression is very sensitive to geometric imperfections. So, the critical buckling stress of a real cylinder is always lower than the theoretical critical stress and very scattered (Figure 3) [5] [6].

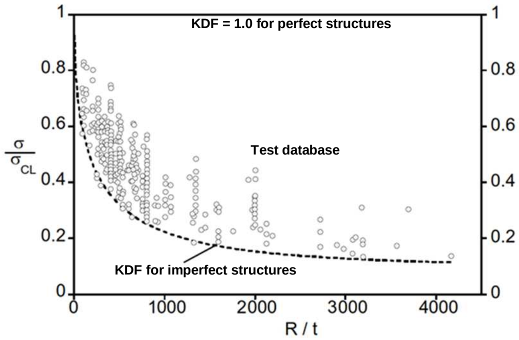

Figure 3. Experimental buckling stress comparatively to the theoretical one for cylindrical shells

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For the design, imperfection sensitivity is taken into account via a reduction factor, or knock-down factor, applied to the theoretical stress. Hence, the NASA SP8007 [1] rule define the buckling load under uniform compression by:

$$q_{cr} = KDF.q_{CL}$$
$$with\ \ KDF = 1 - 0.901(1 - e^{-\frac{1}{16}\sqrt{\frac{R}{t}}}) \qquad \textbf{(2)}$$

It is questioned here if this KDF proposed by the NASA rule for uniform axial compression is suitable for the harmonic load.

3. Numerical modelling

Numerical modelling was conducted using the commercial finite element program ABAQUS [6]. Figure 4 shows the 3D numerical model for the perfect cylinder of ratio R/t=470 and ratio L/R=1,1. The cylinder is clamped at the bottom, and uniform axial compression or harmonic (modal) compression is applied at the top edge of the shell. The radial displacement is blocked at the top edge, and the bottom edge of the shell is fully clamped. The meshing was performed via S4R 4-noded linear curved shell elements (six degrees of freedom per node) with reduced integration and hourglass control.

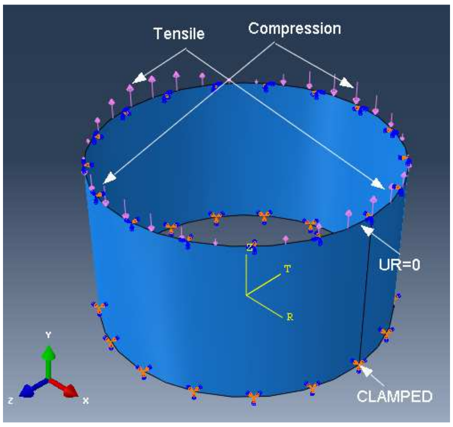
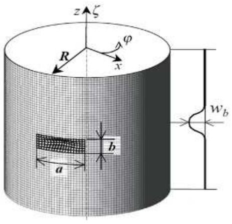


Figure 4. Scheme of the numerical model with harmonic (n=2) load application

Linear bifurcation analysis and geometrical nonlinear analysis with the modified Riks algorithm were conducted, without imperfection (LBA and GNA) or with imperfection (LBIA and GNIA). In this study, to evaluate imperfection sensitivity, three forms of defects with a varying amplitude $0 \leq A/t \leq 3$ were used:

1) The generalized axisymmetric defect (DE), called also Euler axisymetric defect or Koiter defect [7].This imperfection corresponds to the theoretical first buckling mode of analytical linear bifurcation analysis (LBA) in case of uniform compression.

2) Localized inward defect (DL) proposed by Wullscheledger[8]. This defect corresponds to an inward local bump or dimple imperfection. The defect has the width a=2πR/20 and the height b=2λo where $\lambda_o = 1.728\sqrt{Rt}$ is the classical axisymmetric buckle half-wavelength.



$$\Delta R(\varphi,\zeta) = \frac{w_b}{4}\left\{1-\cos\left(\frac{2\pi R}{a}\varphi\right)\right\}\left\{1-\cos\left(\frac{2\pi}{b}\zeta\right)\right\}$$
with: $0 \leq \varphi \leq a/R$ and $0 \leq \zeta \leq b$

Figure 5. Localized inward defect (DL)

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3) The inward axisymmetric triangular defect (DTRI) proposed by Limam [9] [10].

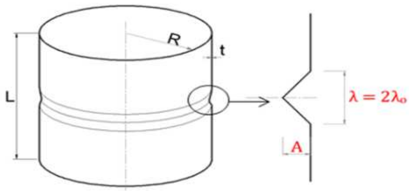
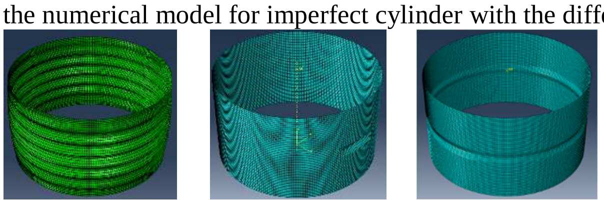


Figure 6. Inward axisymmetric triangular defect

The Figure 7 below represents the numerical model for imperfect cylinder with the different defects.



DE DL DTRI

Figure 7. The different studied defects

4. Numerical results and analysis

4.1 Linear bifurcation analysis

The classical buckling load is here used as a reference load. For the first case, we consider two boosters' configuration, then n=2. The *Figure 8* shows clearly that for all initial imperfections here considered, the axisymmetric compression conducts to lower bearing capacity comparatively to mode 2 loading. The gap is systematically (for all the configurations here studied) not negligible. Hence, considering axisymmetric loading instead of harmonic mode 2 loading can conduct to a very conservative approach.

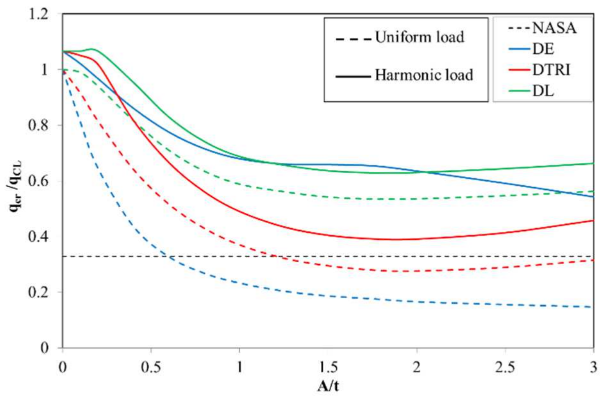
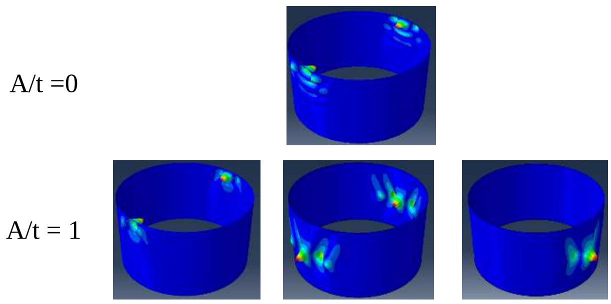


Figure 8. Imperfection sensitivity gauged trough LBA



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DE DTRI DL

Figure 9. Buckling modes obtained through LBIA

To quantify the effect of mode 'n' characterizing the load (loading in mode 2 when two boosters and in mode 4 when four boosters), a parametric study is carried out, where DTRI imperfection is considered as the initial geometrical imperfection.

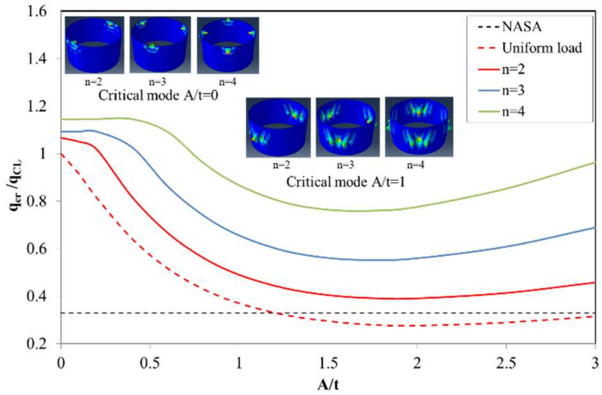

Figure 10. Effect of the wavenumber n of modal load

These calculations confirm that, for the perfect configuration, the critical stress for the modal load is higher than the classical theoretical stress. The defect here considered has no effect for low amplitude (A/t<0.2), knowing that the buckling mode appears mainly near the top boundary, and this zone is far from the imperfection positioned at the middle height of the shell. For high amplitude, the defect plays an important role and the buckling load decreases. It is clear from this curves that axisymmetric load conducts to lower buckling characteristic curve, imperfections are less sensitive for the modal load. Increasing 'n' conducts to decrease imperfection sensitivity. These main conclusions have to be confirmed with non-linear analysis, to be more confident when taken into account large displacement and large rotation effect which can be important near the boundary, the zone where bifurcation mode appears.

4.2 Nonlinear analysis

Non-linear calculations, taking into account the different proposed defects, are performed for uniform and harmonic (mode 2) loading.

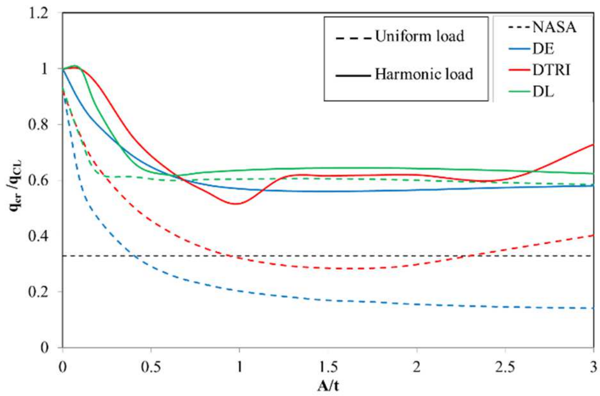

Figure 11. Non-linear bifurcation analysis

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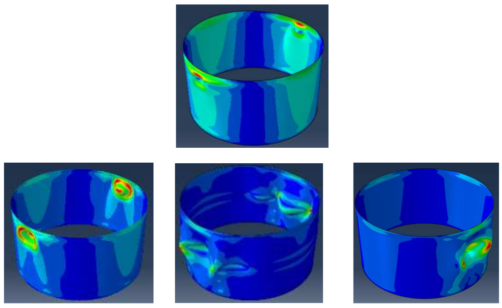

Figure 12. Obtained buckling modes according to GNIA approach

For the configuration here studied, a value of KDF equal 0,76 can be proposed in case of n=4 which corresponds to four boosters configuration, instead of 0,329 if axial compression uniform load was considered. For n=2, corresponding to two boosters configuration, the conducted calculations show that KDF=0,52 is still conservative for all the amplitudes of the most detrimental defects here studied. This leads to a great enhancement of the design on bearing capacity (+58%), knowing that the too much conservative hypothesis associated to axisymmetric equivalent load, conducts to take the NASA KDF of 0,329.

5. Conclusions

The buckling of thin-walled cylindrical shell subjected to mode 2 harmonic compression was numerically investigated in this research. To analyse the sensitivity of initial geometric imperfections in this particular case of loading, the most detrimental geometrical imperfections, according to the literature, for axial uniform compression, are introduced into the FE model. The results show that the common practice which assumes for the design an equivalent uniform load equal to the maximum peak value, instead of the real mode 2 harmonic load, is too much conservative. Considering mode 3 or mode 4 harmonic loading confirms that, for a perfect cylindrical shell, harmonic compressive loading conducts to a higher buckling stress comparatively to uniform compression loading. Furthermore, imperfection sensitivity for modal loads is lower than for axisymmetric load. An appropriate knockdown factor (KDF) is finally proposed for the design in case of mode 2 harmonic loading. The new methodology of design, which considers the real load (mode 2 loading), conducts to a gain of bearing capacity about 58%.

Abbreviations and acronyms

A : Amplitude of defect
R : cylinder mean radius
L : cylinder length
t, : cylinder wall thickness
E, ν : Young's modulus, Poisson's ratio of skin
n : wavenumber of applied compression load
q : load (N/mm)

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