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Assessment of 4D-trajectories predictability and reliability

Álvaro Rodríguez-Sanz [a, 1], Cecilia Claramunt Puchol [a, 2], Fernando Gómez Comendador [a, 3], Javier Pérez-Castán [a, 4], Rosa Arnaldo Valdés [a, 5], and Mar Najar Godoy [b, 6]

[a] Universidad Politécnica de Madrid (UPM), Plaza Cardenal Cisneros N3, Madrid 28040, Spain.
[b] ISDEFE, Calle Beatriz de Bobadilla, N3, Madrid 28040, Spain.
[1] alvaro.rodriguez.sanz@upm.es; [2] c.claramuntp@alumnos.upm.es; [3] fernando.gcomendador@upm.es;
[4] javier.perez.castan@upm.es; [5] rosamaria.arnaldo@upm.es; [6] mnajar@e-externas.enaire.es

Abstract

The current Air Traffic Management functional approach is changing: 'time' is now integrated as an additional fourth dimension on trajectories. This concept will impose on aircraft the compliance of accurately arrival times over designated checkpoints, called Time Windows (TWs). We characterise these TWs with a stochastic approach (Monte Carlo simulations), and manage the uncertainty associated to the evolution of 4D-trajectories with a causal and predictability analysis, using a Bayesian Networks methodology. Then, we analyse the 4D-trajectory reliability through Multi State Systems and Markov Chains models, which may allow stakeholders to establish performance indicators and compliance metrics.

1. Introduction and motivation

The recent increase in air traffic demand provides a challenging operational situation for the current European Air Traffic Management (ATM) system [1]. In this way, SESAR (Single European Sky Air Traffic Management Research) and NextGen (Next Generation Air Transportation System) are changing the ATM framework [2], [3]. Achieving accurate prediction of trajectories is a fundamental condition for reliable detection and resolution of conflicts. SESAR and NextGen support the 4D-trajectory operational concept, in order to improve efficiency, reliability, sustainability and cost-effectiveness of aircraft operations [2], [4]. The future ATM system relies on the Trajectory Based Operations (TBO) concept. This will require aircraft to follow an assigned 4D-trajectory (time-constrained trajectory) with high precision. TBO involves separating aircraft through a definition of a strategic trajectory (long-term), rather than the currently practicing tactical (short-term) conflict resolution [5]. The main goal is to increase air traffic capacity by reducing the controllers' workload. Nevertheless, real time measures, (over the trajectory will be required to improve reliability and react to unplanned conditions; thereby, maintaining expected capacity [2].This approach will require aircraft to follow an assigned 4D trajectory with high precision.

The 4D trajectory concept is based on the integration of time into the 3D aircraft trajectory. Each point is defined by position (latitude, longitude and flight level) and time. Certain restrictions are currently associated with flight levels. In the future operational framework, it is foreseeable that there will be restrictions with respect to time [6], [3]. 4D-trajectories will enable a flight to follow a practically unrestricted, optimum trajectory for as long as possible. In exchange, the aircraft will be obliged to meet very accurately an arrival time over a designated point. In the context of TBO, airspace users will agree a preferred trajectory with Air Navigation Service Providers (ANSPs) and airport operators (AOs). Aircraft and ground systems will exchange information on the trajectory and the expected airspace capacity to ensure that flights comply with the assigned Controlled Time of Arrival (CTA) [6], [3].

4D trajectory management is expected to improve air traffic operations, in particular to increase the overall predictability of traffic, with several benefits to airlines and air traffic management [7]. These benefits [1], [4], [8], [9], include: (a) the improvement of air traffic operations and their reliability by increasing the overall predictability; (b) an information-rich environment with real-time data, as well as prediction data system trends that allow the optimization of services to airspace users; (c) optimal operations for airlines (routes and preferred levels); (d) absorption of delay; (e) increased safety; (f) improved flight paths; (g) reducing costs and emissions; (h) easier to handle traffic for the controllers.

However, real measures (over the trajectory) will be required to improve reliability, react to unplanned conditions and thus maintain the expect capacity. Trajectories are degraded due to environmental and operational uncertainties [10], [11]. Sharing, updating and coordinating changes in trajectory becomes necessary to ensure reliability. Therefore, these measurements will ensure that the necessary adjustments are made to correct trajectory degradation, a degradation that increases over time if appropriate actions are not taken.

To exploit these benefits and avoid potential conflicts, aircraft must be kept within very small volumes around their agreed reference trajectory [12]. The main objective is to ensure compliance with the stated separation standards.

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Therefore, it is necessary for 4D-trajectories to be monitored with a high degree of accuracy, precision, data integrity and safety [2]. Given the required level of demand, new tools are essential to allow ATM planners, air traffic controllers, route developers and airspace users to obtain the TWs in which aircrafts are expected to be found at each checkpoint. Accurate TW and trajectory predictions are required for highly efficient Air Traffic Management procedures [13]. However, this process is strongly influenced by uncertainties [14], [11], such as actual performance of aircraft, operation along the trajectory, and weather/atmospheric conditions affecting the flight. Improved predictability and reliability is one of the eleven target KPAs set in ICAO's Doc 9854 for the Global ATM Operational Concept [5]. It is also one of the requirements of SESAR and NextGen to optimise 4D-trajectory synchronisation and conflict detection/resolution.

The purpose of this study is to develop a model to deal with uncertainty and increase predictability, while providing a methodology to evaluate the robustness of 4D-trajectories, by quantifying its perturbations. This can be applied in a predictive way, thus being able to anticipate the trajectory degradation in order to apply corrective actions. These models improve traffic synchronization and potentially ease conflict resolution in 4D-trajectories, which are cornerstones in future airspace operational environments.

The paper is structured as follow: first, we state the problem and review how past studies have approached this issue; then, we introduce the methodology (Scenario characterization and 4D-trajectory modelling, identification of influence parameters, Time Window (TW) estimation and causal analysis); and finally, we study the predictability and the reliability of 4D trajectories with which we will obtain the results and their conclusions.

2. Background and problem statement

Some of the different objectives of SESAR for the future ATM operational concept are to improve efficiency, reliability, sustainability and cost-effectiveness of aircraft operations [2], [4]. The future trajectory management approach is based on a four dimensional framework, composed by the three spatial dimensions and a time constraint [15]. This constrains are called time windows (TWs) and require the capability of developing accurate and reliable trajectory predictions [16]. These predictions need to consider external disturbances to the aircraft and internal uncertainty sources [10], [11], [17]. These disturbances and uncertainties may cause a degradation in the trajectory. Therefore, trajectory prediction, degradation, and uncertainty management are key elements in the new operation concept of air traffic.

Multiple studies have dealt with the prediction of trajectories in different phases of flight [18]–[21]. The prediction of trajectories is a fundamental tool in conflict detection and resolution [22]–[24], traffic load forecasting and weather impact assessment [25]. Several studies have analysed sensitivity in trajectory prediction in ATM [26]. In this article, we propose a causal model to understand the relationships between the parameters that influence the prediction of trajectories. Most of the methods used to predict trajectories problem can be categorised as either deterministic or probabilistic [27]. The traditional approach is deterministic and treats the issue as a mathematical problem that describes aircraft motion. This approach is strongly and inherently limited by the accuracy of the models that represent actual aircraft behaviour and by the quality of the inputs [28]. Furthermore, the modelling assumptions may introduce potential prediction errors; i.e., sources of uncertainty that are usually not explicitly considered by such deterministic approaches. If some external forces or parameters, e.g. aircraft performance, weather conditions, accuracy of navigation systems, and traffic regulation), are unknown or cannot be precisely evaluated, the probabilistic approach transforms the deterministic problem into a stochastic one [11], [14].

Uncertainty management is a core component of many advanced ATM operational concepts, e.g. in conflict detection and resolution algorithms [29], air traffic synchronisation requirements [30] and automation functionalities for trajectory planning [31]. The presence of significant uncertainties in the prediction of trajectories may result in reductions in capacity, limited fuel efficiency and delays [1], [32]. Therefore, a detailed study of the variations in the parameters that influence flight operations is required. The parameters with the greatest influence on the evolution of trajectories evolution are meteorological conditions (wind, temperature), aircraft performance (flight phase, weight, speed), navigational constraints (holdings) and initial conditions [33]. Nevertheless, the effect of shear wind on optimum performance [34] has been recognized as one of the most relevant uncertainties in path deviation [35], [36]. These influential parameters may be added to the prediction model as variations in procedures, inaccuracy of navigation systems, or ATM interventions to reflect their influence [11], [37].

The CATS project introduced the concept of 4D target windows to be respected during flight execution as a way of managing uncertainty [12]. These target windows are formally agreed upon by the different actors involved in the execution of a flight and are located at the transfer of responsibility areas between them [38]. Han, Wong and Gauhrodger (2010) demonstrated that by defining target windows at intermediate locations along a 4D trajectory, and not just at sector boundaries, these can assist in managing en-route punctuality and uncertainty. Furthermore, target windows provide a useful balance between air traffic predictability and manoeuvrability [14]. Some studies consider target windows as circular cross-sections marked with arrival times [39]. Others model them as rectangles with time characteristics [37]. With respect to the 4D-trajectory operational framework, and when looking at safety, it is more

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reasonable to predict time intervals than exact aircraft positions [21]. Therefore, in this study we focus on time windows (TWs).
In the context of TBO and RBTs (Reference Business Trajectories), TWs should be sufficiently large to allow airspace users and ANSPs to respond flexibly to a variety of flight conditions but sufficiently small to increase certainty and improve capacity. An experiment conducted by EUROCONTROL in 2012, with an Airbus A320 test aircraft that flew from Toulouse to Stockholm, established a tolerance window of between -2 minutes and +3 minutes over the route and ±30 seconds for CTA [15]. In addition, pilots were exposed to situations where the aircraft was deviated from the planned path. The results showed that adherence to 4D-trajectories was feasible in cruise phase while the TW for CTA was more difficult to achieve and required increased coordination between pilots and controllers [15].
We propose to manage uncertainty with respect to time by establishing several intermediate locations (checkpoints) along a trajectory, where time uncertainty can be constrained by TWs. The methodology for defining TWs uses an aircraft performance model based on EUROCONTROL's Base of Aircraft Data (BADA) family 4.0 [40], [41] and probabilistic approaches to reflect the inherently stochastic nature of air traffic procedures [42]. The analysis focuses on the climb phase, the en-route flight phase in different scenarios (stabilised flight level and flight level changes - climb and descent) and the descent phase.
On the other hand, reliability analysis provides theoretical and practical tools to test the behaviour and performance of trajectories in a scenario of uncertainties. Traditional reliability models are binary [43], as they consider only two functional states (working state or fail). Multi-State System (MSS) reliability analysis introduces distinctive levels of efficiency, called performance rates. Hence, MSS reliability provides a more realistic approach to studying the behaviour of a system. Since the mid-70s, when the MSS reliability was introduced, this filed has experienced an intensive development and it has been applied in several fields [44]. In the field of aeronautics we can found examples as a Markovian type simulation model that is used to simulate operational uncertainties arising from aircraft turnaround operations [45]. The properties of Markov chains are used to model the interdependent effects between sequential procedures of aircraft turnarounds [45]. In Air Traffic Control (ATM), a MSS theory and a Markov model have been applied to measure the efficiency and reliability of an Air Traffic Automatic System (ATCAS) [46]. The estimation of potential benefits of new ATM tools has also been done using MSS reliability through Markov models [47]. Monte Carlo simulations were implemented to estimate the range of uncertainties in model parameters and technology performance accuracy. In the last years, with the increases in computer performance, the link between MSS reliability analysis and Monte Carlo models was strengthened. Monte Carlo simulation allows, with a simple computation procedure, modelling system operating scenarios to assess the reliability systems [48], [49].
The objectives of this study are to: (a) define the TW in which the aircraft will be located when it arrives at each checkpoint; (b) identify the parameters with the greatest influence on the evolution of the trajectory; (c) analyse trajectory deviations due to variations in the influential parameters; and (d) study the degree of degradation of the 4D-trajectories at each Waypoint, the probability that a degraded stat will be reached.
The main contribution of our study is the development of a causal model which provide key information that can be used in synchronising traffic and in strategic planning. It also improves the prediction of trajectories predictability which is useful in managing conflict and correcting deviations. Also, the reliability model can be used to evaluate the robustness of 4D trajectories and to deal with their perturbation, which is a cornerstone in traffic synchronization and conflict resolution.

3. Methodology

3.1 Scenario characterization

In the first part of the study, the aircraft performance and the 4D trajectory were modelled. The aircraft chosen to realize the flight was the Boeing 737-900ER, one of the most frequent aircraft in short- medium range flights used in Europe [50]. Next, we have created a model for the scenario in which the 4D trajectory will be found. For this, we use EUROCONTROL's BADA 4.0 methodology [40], which is built based on aircraft's kinetic and kinematic models and it allows us to construct a trajectory prediction model to define TWs along the aircraft flight path.
In order to develop the model, it was necessary to characterize 4D trajectories and identify their influence parameters, which are shown in Table 1(aircraft performances and variables related to the scenario). These functional relationships between the parameters are obtained directly from the BADA manual.
The BADA aircraft model is based on a mass-varying, kinetic approach to aircraft performance modelling [51]. It is structured in three parts: Aircraft Performance Model (APM), Airline Procedure Model (ARPM) and Aircraft Characteristics Model (ACM). The ACM comprehend the APM and the ARPM and these three elements, together

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with de Atmosphere model (AM) represent the Aircraft Dynamic Model (ADM), as shown in Figure 1. These models determine the interdependencies between the modelling parameters. Each aircraft model in BADA 4.0

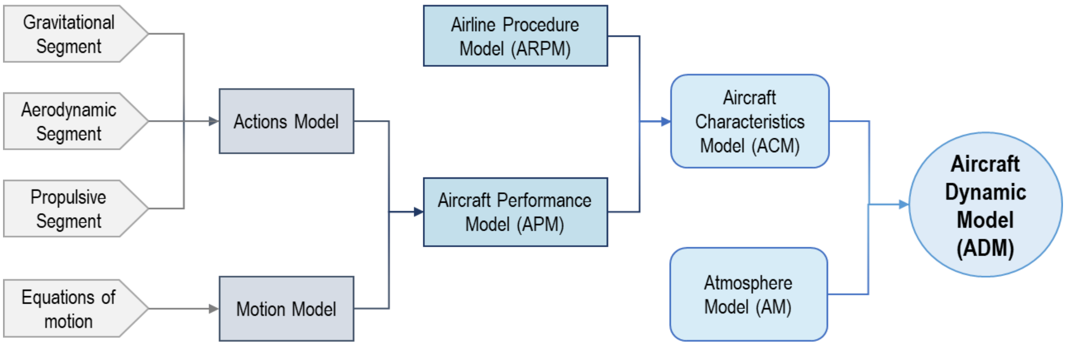

Figure 1: BADA 4.0 structure – models for generating trajectories

is characterized by a set of coefficients, called Aircraft Characteristics, which are used by the APM and ARPM [40]. These models allow us to estimate aerodynamic and propulsive variables from mass with functional relationships show in Table 1.

Table 1: Modelling parameters

Block	Parameter	Dependencies	Description
Atmosphere Model	Pressure	$p = f\,[T(h), \rho(h)]$	T (temperature), ρ (density) and h (altitude).
	Speed of sound	$a_0 = f\,[k, R, T, M]$	M (flight Mach), R (universal gas constant) and k (adiabatic air coefficient).
	Wind	$w = f\,[\varphi, \lambda, h]$	φ (latitude) and λ (longitude).
Aerodynamic Forces Model	Lift coefficient	$C_L = f\,[\delta, p_0, k, S, M, m, \varphi, g_0]$	δ (pressure ratio), p_0 (pressure at mean sea level), S (wing surface area), m (aircraft mass) and g_0 (acceleration of gravity at mean sea level).
	Lift	$L = f\,[\delta, p_0, k, S, M, C_L]$	-
	Drag coefficient	$C_D = f\,[C_L, \delta, d_1 \ldots d_{15}, M_{max}, p_0, k, S, M, m, \varphi, g_0]$	$d_1 \ldots d_{15}$ (characteristic parameters of aircraft).
	Drag	$D = f\,[\delta, p_0, k, S, M, C_D]$	-
Propulsive Forces Model	Thrust coefficient	$C_T = f\,[t_{i1} \ldots t_{i12}, a_1 \ldots a_{36}, M, \delta, \delta_T]$	$t_{i1} \ldots t_{i12}$ and $a_1 \ldots a_{36}$ (characteristic parameters of aircraft) and δ_T (throttle ratio).
	Thrust	$T_h = f\,[\delta, m_{ref}, W_{mref}, C_T]$	m_{ref}, W_{mref} (aircraft reference mass and weight).
	Fuel consumption coefficient	$C_F = f\,[\delta, \theta, M, f_{i1} \ldots f_{i9}, C_T]$	$f_{i1} \ldots f_{i9}$ (characteristic parameters of aircraft) and θ (temperature ratio).
	Fuel consumption	$F = f\,[\delta, \theta, m_{ref}, W_{mref}, a_0, L_{hv}, C_F]$	$f_{i1} \ldots f_{i9}$ (characteristic parameters of aircraft).

With all this, the 4D trajectory model was implemented and generated using the MATLAB software. To check its accuracy, it was performed a model validation by comparing the MATLAB simulated trajectory with real data flights extracted from the tool NEST of EUROCONTROL, using different intra-European routes (with similar characteristics to the scenario of study). The flights chosen for comparison were those that present similar characteristics. The test error regarding time and position presents an average value of 7%, reaching less than 5% during the stabilised flight level sections.

The 4D trajectory has been modelled in three phases. First, we generated the en-route flight phase considering level changes how it is shown in Figure 2. Then, we have generated the climb and descent phases, considering that these phases have 3 distinguished parts (when the aircraft passes the transition altitude and when it is in clean or non-clean configuration). The trajectory is viewed as a flight in two dimensions with the following characteristics (Figure 2): (a) The climb flight phase, since 400ft until the flight level 360 (1); (b) The cruise flight phase divided in 5 parts. We consider three stabilised horizontal flight sections (2, 4, 6), an en-route climb section (3) and a descent section (5), that represent a level change between FL360 and FL380. It is not considered any turning in this phase; and (c) The descent flight phase, since FL360 until 50ft (1). The total length of the trajectory is 560NM (~ 76 minutes).

To adjust the generic trajectory model to the characteristics of the chosen scenario, we make the following simplifications:

- The main assumptions are: aircraft is considered as a symmetric rigid solid; variation in mass is due to fuel consumption only; limited manoeuvrability; airborne stage only; symmetric and coordinated flight.
- The atmospheric variables are modelled according to the International Standard Atmosphere (ISA) [52].

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However, the temperature is estimated as a function of flight altitude and this value is used to calculate pressure and density. The temperature is corrected by introducing δT in the motion equations.

- The aircraft is considered as a point mass with three-degrees-of-freedom (3DoF). The flight is assumed to be symmetrical with all forces acting on the centre of mass and contained in the plane of symmetry. The rotational equations are decoupled, the angular speeds small, and the lifting surfaces do not affect the forces.
- In the cruise phase we consider a flight at constant Mach (M) since this is the most common way to operate the selected aircraft type to save fuel and improve efficiency [53]. The aircraft adjusts its speed (V) along the trajectory to achieve the agreed value for M (M=0.78); i.e., we drop the speed equation from the full model and use V as a control input. This is a reasonable approximation since the speed does not change much when cruising and when it does it responds rather quickly to the thrust commands [37]. We also assume that the aircraft moves with constant heading angle in each segment. This assumption is not restrictive since aircraft track laterally very well [37].
- The aircraft type chosen uses a turbofan engine. Idle rating configuration for descent phase and non-idle rating for the rest.
- Stabilised flight and changes in flight level have been achieved by modifying the position of the gas lever, which has a direct effect on thrust $T_h = T_h(V_{TAS}, h)$.
- It is assumed that the aircraft's initial mass is 10% lower than the aircraft's MTOW (Maximum Take-Off Weight). This hypothesis reflects the fact that aircraft do not always take-off with MTOW, and also considers the fuel consumption during take-off until 400ft (before initial climb).
- The prediction algorithm uses the aircraft's ground speed (V_{gs}), which is the horizontal speed of the aircraft relative to the ground. V_{gs} can be calculated, using vector addition, from wind speed (w), wind direction, heading angle ψ and the aircraft's true airspeed (V_{TAS}).
- We do not consider correction or deviation control measures along the trajectory.
- Finally, all variables, both input and output, must be within the operational and/or structural limits of the chosen aircraft model.

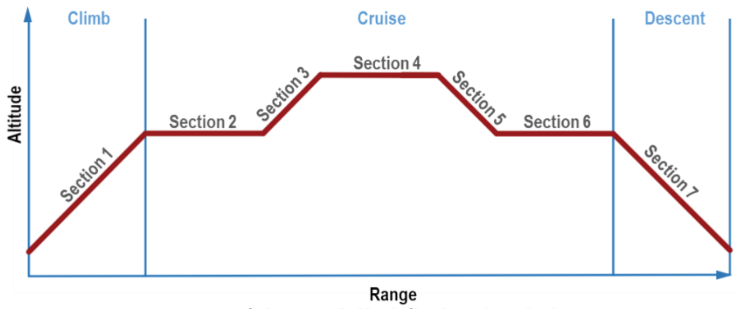

Figure 2: Diagram of the modelled flight (divided into sections)

3.2 Identification of the influence parameters

Once the model was developed and validated, the next step was to perform a variation of the influence parameters in order to appraise different potential situations. This is achieved through a Monte Carlo simulations approach [54], which approximate the model to reality, obtaining data that represent real situations by the sampled of input random variables (aircraft mass, temperature, pressure, density and wind).

All trajectory prediction models have sources of uncertainty. The specific sources of uncertainty depend on the model in question , however, in all cases the factors with the greatest influence are [11], [55], [56]: the initial conditions, operational uncertainties (commands, guidance modes and control strategies), weather forecast, inherent errors in the model, and technical flight errors (inaccuracies in flight control). These sources of uncertainty lead to differences between predicted and actual trajectories. These differences can be used to build a stochastic model based on the previous deterministic model.

The Monte Carlo approach is a statistical method used to model the probability of different outcomes in a process that cannot be easily predicted due to the intervention of random variables [57]. It establishes a set of runs (simulations) which depend on probabilistic inputs. These inputs are randomly generated from a probability distribution which characterises the uncertainty associated with the parameters in question. For each set of inputs, the deterministic problem is solved, generating a bunch of outputs that are aggregated to obtain the stochastic solution. This technique is capable of handling a large number of random variables, several types of statistical distributions and non-linear dynamic models. Unlike physical experimentation, Monte Carlo simulation performs a random sampling and facilitates the completion of a large number of numerical experiments [54]. The Monte Carlo method has been extensively used in air traffic control for conflict resolution [24], safety verification [58], and to estimate the impact of wind uncertainty

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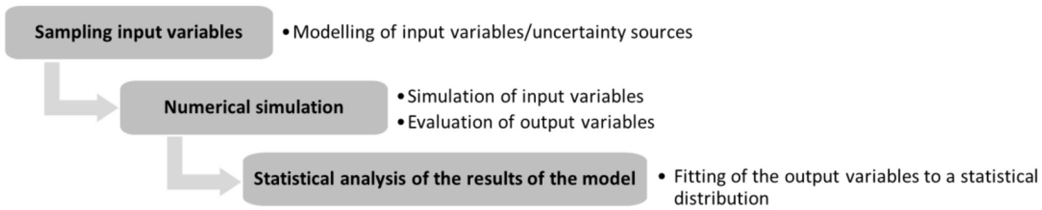
[59], [60].
In our study, the objective is to obtain a set of trajectories from the developed model by varying the input parameters, mentioned before, and applying the Monte Carlo method. The statistical characteristics of the experiments (model results) enable us to obtain conclusions with respect to potential deviations in trajectories (TW estimation). In each experiment, the possible values of the input random variables are sampled according to their distributions. A series of experiments performed in this way, provides us with a set of samples of the output variables for statistical analysis. This, in turn, allows us to estimate the characteristics of these variables.
Figure 3 shows the Monte Carlo simulation procedure that enables us to obtain a sample of trajectories to statistically



characterise the output variables.

Figure 3: Monte Carlo simulation procedure

The first step in the Monte Carlo simulation procedure is to sample the input variables. These variables are the parameters with the greatest influence on the aircraft's trajectory. Sensitivity analysis shows that the influential parameters with greatest impact on the evolution of the trajectory are: aircraft type, mass, fuel consumption, wind, temperature, pressure and navigation systems (positioning uncertainties). These inputs are modelled as the variable plus a precision error of the variable. Therefore, each variable follows a statistical distribution with respect to the nominal/ideal mean value along with a specific standard deviation for each variable. The errors of the variables evaluated are assumed to be statistically independent.
The stochastic approach used to predict trajectories introduces variability in the deterministic model. After doing 1.500 simulations of each flight phase, we obtain the results, such as those for the cruise phase shown in Figure 4. It is important to mention that the variance estimated by the Monte Carlo procedure converges to the inverse square root of the number of runs (N). Therefore, this method has an absolute error for the estimation that decreases like ${}^{1}/_{\sqrt{N}}$.

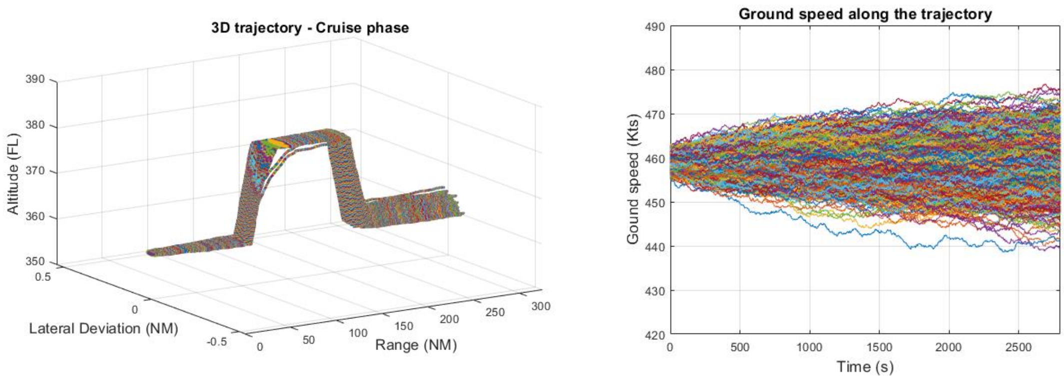


Figure 4: Results of the trajectory simulations: 3D-view (left) and Ground Speed (right)

Figure 4 (left) shows a 3D-view of the trajectory which relates longitudinal, lateral and vertical position. The aircraft begins the cruise at FL360, ascends to FL380 and descends back to FL360. The greatest variability occurs in the ascending and descending phases (sections 3 and 5), while in the stabilised phases of the flight (sections 2, 4 and 6) there is less fluctuation with respect to the ideal trajectory. This same reasoning could be applicable to the rest of the flight phases (climb and descent), where the uncertainty is greater. In the flight level change of the cruise, we define a constant climb/descent rate, therefore, the aircraft climbs and descends at a constant angle. Nevertheless, the climb/descent angle is different for each simulated trajectory due to the sources of uncertainty, whose parameters have been mentioned before and they have a greater impact in non-stabilised sections of the flight. Although the lateral deviations are relatively small, less than 0.1 NM, these increase towards the end of the trajectory, due to the impact of the sources of uncertainty. Figure 4 (Right) shows the evolution of ground speed over time. The variability in V_{gs} increases along the trajectory due to the cumulative effect of the sources of uncertainty too.

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3.3 Time-Window estimation

The aim of the 4D trajectory concept is to ensure flight on a practically unrestricted, optimum trajectory for as long as possible in exchange for the aircraft being obliged to meet very accurately an arrival time over a designated point. In this section, we evaluate these time constraints by defining TWs. The Monte Carlo method allows us to obtain a large number of simulations (potential trajectories) by varying the influential parameters. The results reflect the stochastic nature of the evolution of the flight and they make the deterministic model more realistic by including actual uncertainties in trajectory prediction. Once the simulations have been performed, we can define the expected TW at each checkpoint of the agreed trajectory, thereby, providing airspace users, ANSPs and AOs with a framework for traffic synchronization and conflict resolution. The size of a TW on a Reference Business Trajectory (RBT) should at least represent the time interval within which any aircraft arriving at the checkpoint can avoid collisions with other aircraft [39].

To estimate the TWs, we start by setting the position (x) of the checkpoints (CP), as shows Figure 5. Each aircraft must hit them within a specified time interval [14].

We propose a flexible approach in which TWs can be distributed along the entire 4D trajectory of a flight in such a way Air Traffic Controllers (ATCOs) can manage the punctuality of aircraft as they transit between sectors and also along their entire trajectory, although, this will increase the amount of coordination required between pilots and controllers. To illustrate the method for calculating TWs we define five checkpoints along the three flight phases of the proposed scenario.

In Figure 5, we can see the location of every checkpoints. Checkpoint 1 and 5 are located in the middle of climb and descent phase, at 75 NM and 530 NM, respectively. In the cruise phase, we have three checkpoints (2, 3 and 4), located in the middle of each horizontal stabilised flights, at 200 NM, 303 NM and 421 NM, respectively. It is necessary to mention that these CP have been used for all the analysis done and each flight phase has been treated as independent when it comes to the calculations.

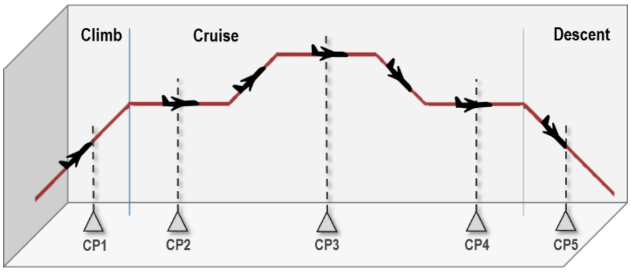

Figure 5: Defined checkpoints (Not to Scale)

After being obtained the arrival times at each CP by the 1500 Monte Carlo simulations, we can represent a histogram that can be modelled as a normal distribution for each checkpoint. For example, for Checkpoint 3, we obtained a normal distribution centred in $\mu = 1194$ seconds from the start of the segment and a standard deviation of $\sigma = 4.9$ seconds. Both a *K-S* test and χ^2 *goodness-of-fit* test were used to ensure the “power” of curve fitting [61]. Fitted probability curves for the arrival times at Checkpoints 3 and 4 are shown in Figure 6. These are given by the probability density function:

$$f_{NORMAL}\,(t, \mu, \sigma) = \frac{1}{\sqrt{2\pi} \cdot \sigma} \cdot e^{-\left(\frac{t-\mu}{2\sigma}\right)^2} \tag{1}$$

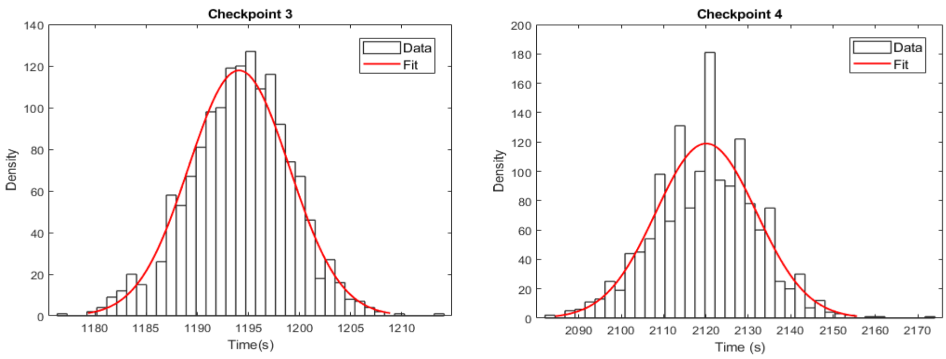

Figure 6: Statistical distribution of the aircraft’s arrival time in seconds at CP 3 (left) and 4 (right)

By setting the position (x) of the checkpoint and fitting time variation to a normal distribution we can define different TWs or time intervals depending on the accuracy required. Figure 9 gives the probability of an aircraft achieving the TW constraint as a function of a time interval centred at μ and depending on the multiple of σ chosen. The width of

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the TW is an indication of how predictable a flight is. Setting longer intervals increases predictability and reduces uncertainty but leads to less efficient time management.
Therefore, there is a 95.44% (±2σ) probability that an aircraft will be at Checkpoint 3 (303 NM) within a TW of ±10 seconds centred at 1194 seconds. Similarly, the arrival times at Checkpoint 4 (421 NM) can be fitted to a normal distribution with mean μ=2120 seconds and a standard deviation of σ=12 seconds. There is a 95.44% (±2σ) probability that an aircraft will reach CP 4 within a TW of ±24s. The values of σ and the Interquartile Range (IQR) are higher for CP 4 than for CP 3. This means that the further the distance the greater the uncertainty and data dispersion.
Table 2 gives the mean (μ) and the standard deviation (σ) of each TW (measured in seconds). The standard deviation is a measure of dispersion about the mean data that enable us to define the different tolerance windows at each point.

Table 2: TW values for CP

Checkpoint	μ (s)	σ (s)	Time Window (s)
1 (75 NM) - Climb	778	9.95	**±20**
2 (200 NM) - Cruise	393.8	0.9	**±2**
3 (303 NM) - Cruise	1194	4.9	**±10**
4 (421 NM) - Cruise	2120	11.9	**±24**
5 (530 NM) - Descent	478.63	2.67	**±6**

It is recalled that the three phases have been analysed separately, so the TWs are not cumulative from one flight phase to another.
Analysing the results, we can see that TWs increase along the trajectory, being greater in the climb phase and at the end of the cruise phase, after flight level change that comprehends a climb and descent 2000ft. That is, variability of the results increases with distance. If a feedback controller is included in the simulation to emulate the actions that ATCOs and pilots might take to limit deviations from the planned trajectory, we will obtain a significant reduction in variability and TWs sizes [14].
Our results suggest that TWs could be significantly reduced compared to the TWs reached in the CATS project (∼ 7 minutes [62]) without negatively impacting on the probability of satisfying the agreed tolerances. Nevertheless, the results are highly dependent on the sources of uncertainty evaluated. As such, an increase in the variability of the influential parameters lead to deviations in the size of TWs. To manage this variability, we carry out a causal analysis on the characterisation of the TW.

3.4 Causal and predictability analysis

We perform sensitivity analysis to measure how potential variations in parameter values can affect the results of the model [63] and to measure the influence of modelling parameters on flight time. Given uncertainty in the actual values of the input variables of the trajectory prediction model (introduced when performing the simulation), a study of the functional relationships between parameters enables us to understand the relative influence of each parameter on the final results. This causal analysis is performed using a Bayesian Network (BN) technique. A BN is a probabilistic graphical model that represents a set of random variables and their conditional dependencies via a directed acyclic graph (DAG). Formally, BNs are DAGs whose nodes represent random variables in the Bayesian sense: they may be observable quantities, latent variables, unknown parameters or hypotheses [64], [65]. Edges represent conditional dependencies; nodes that are not connected (there is no path from one of the variables to the other in the BN) represent variables that are conditionally independent of each other. Each node is associated with a probability function that takes, as input, a particular set of values for the node's parent variables and gives, as output, the probability (or probability distribution, if applicable) of the variable represented by the node [66]. Consequently, a BN is a pair (G, P), where G is DAG defined on a set of nodes x (the random variables), and P = {p ($x_1 \mid \pi_1$), …, p ($x_n \mid \pi_n$)} is a set of n conditional probability densities (CPD), one for each variable. π_i is the set of parents of node x_i in G. The set P defines the associated joint probability density of all nodes as (the chain rule for BN):

$$p(\boldsymbol{x}) = p\,(x_1, \dots, x_n) = \prod_{i=1}^{n} p\,(x_i|\pi(x_i)) \tag{2}$$

The graph G contains all the qualitative information about the relationships between variables, no matter which probability values are assigned to them. Additionally, the probabilities in P contain quantitative information, i.e., they complement the qualitative properties revealed by the graphical structure [67], [68].
A BN is created for every checkpoint along the trajectory to reflect specific information about the relationships between variables at each of these points. The first step in the BN construction process is to generate the correlation matrix for the variables, to assess the correlation between pairs (regression analysis). In our case, this matrix provides a large number of relationships between parameters; i.e., the influence network is complex, and we can obtain diverse results by varying a single parameter. A data-driven process is then used to build the BN, applying a Bayesian Search (BS)

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algorithm [69], [70]. This algorithm essentially follows a hill climbing procedure (guided by a scoring heuristic) with random restarts. The BS algorithm uses the BDeu (Bayesian-Dirichlet equivalent uniform) function as the network scoring function in its search for the optimal graph. This scoring function is a common tool for choosing between different statistical models and it represents the goodness of fit of the model to observed data [71]. The BN structure is then refined to represent the functional relationships of the trajectory model (equations) and the empirical knowledge of the problem. Therefore, our model is built through a combination of a data-driven process and practical adjustments, to reflect reality.

The BN for the trajectory prediction model consists of ten interrelated nodes (Figure 7 left). Each node corresponds to a parameter or influence factor in the evolution of the trajectory. The sample values for each parameter are distributed in different discrete states. The number of states at each node is adjusted to reflect the value of the specific parameter and its variability. Each state is assigned a probability whose value is defined by the number of times that the variable can be found in this state over the simulation experiments. We then obtain a different probability distribution for each variable.

The BNs for the different trajectory checkpoints have the same structure (Figure 7 left), but each of them has different probabilities and states distributions for the nodes, due to the increase in variability of parameters along the trajectory. Figure 7 (left) shows a generic BN for evaluating the TW at each checkpoint. Figure 7 (right) gives the specific design of the node states of the BN for CP 3 (checkpoint in the middle of the trajectory). A sub-sample of 90% of observations is selected to train/build and validate the model structure, i.e. establish the model's ability to explain variations in parameters. The remaining 10% of the data is used to test the accuracy of the predictions made by the model, i.e. test the model's predictive capacity. The training and test sub-samples are randomly selected from the complete dataset. The test error is 5% - 12% for the different variables. BN has been done for every Checkpoints defined before as show Figure 7 using the modelling tool GeNIe.

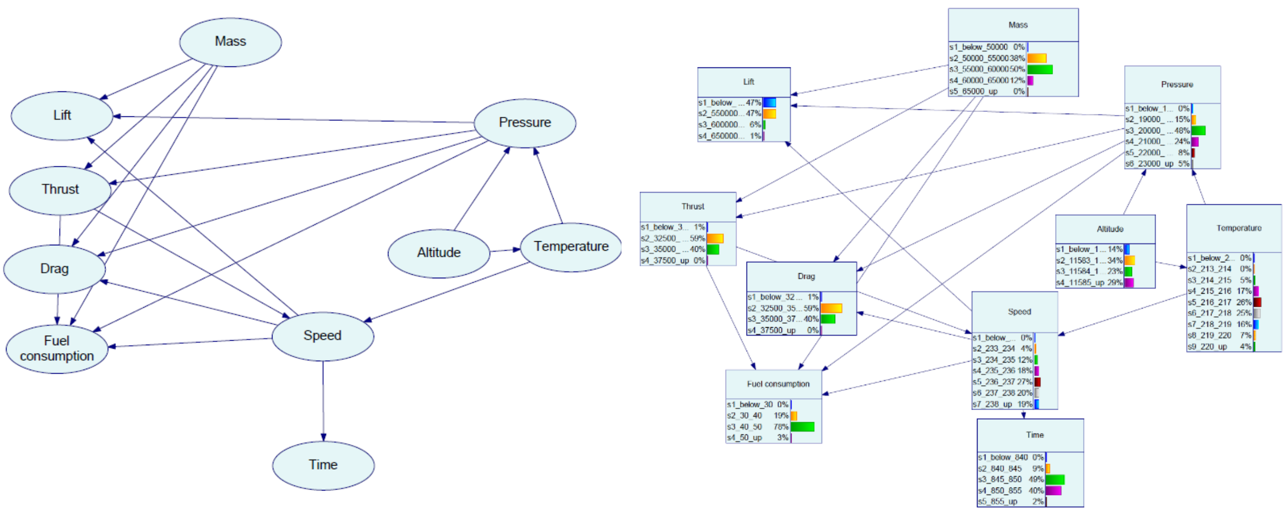

Figure 7: Generic BN structure for TWs (left) and Specific BN structure for CP3 (right)

Another BN obtained by GeNIe is the intensity network. Figure 8 shows the causal relationships between the influential parameters. The head of the arrow indicates the variable or parameter which "receives" the influence (the child node): e.g. as the "mass of the aircraft" directly influences the "lift", there is an arrow goes from the "mass" (parent node) to the "lift" node. At the end of the BN, we have the target node. In this case our target is to estimate the aircraft's time of arrival at each checkpoint (time constraints), so "time" node is placed at the "final" outcome of the BN. The dynamic forces (lift, thrust, drag) and fuel consumption are situated on the left-hand side of the network. The atmospheric variables (pressure and temperature) are located on the right-hand side of the structure. In this BN, due to the atmosphere modelling proposed by BADA 4.0, temperature determines pressure, since this relationship was set in the formulation. In addition, both parameters will vary depending on the altitude. The speed depends on the thrust, since both variables are related through the differential equation of motion. Finally, aircraft speed will determine the flight time needed to reach the defined checkpoint. We set out a statistical significance test on pairs of nodes connected by an arc in the BN: associations between the nodes are statistically significant at level 0.05 (p-value).

Figure 8 illustrates the intensity of the relationships between parameters and enables us to perform a sensitivity analysis. The greater the intensity of the red colour for each node, the higher the interdependence between this node and the target node. Moreover, the thickness of an arc represents the strength of influence between two directly connected nodes. We use two measures of distance between distributions to validate results: Euclidean and Hellinger [72]. As can be seen, the most influential parameters are speed, aircraft mass, temperature and thrust. These parameters

DOI: 10.13009/EUCASS2019-381

will be used in the reliability model. The aerodynamic forces (L and D) are shown in grey, what indicates that the

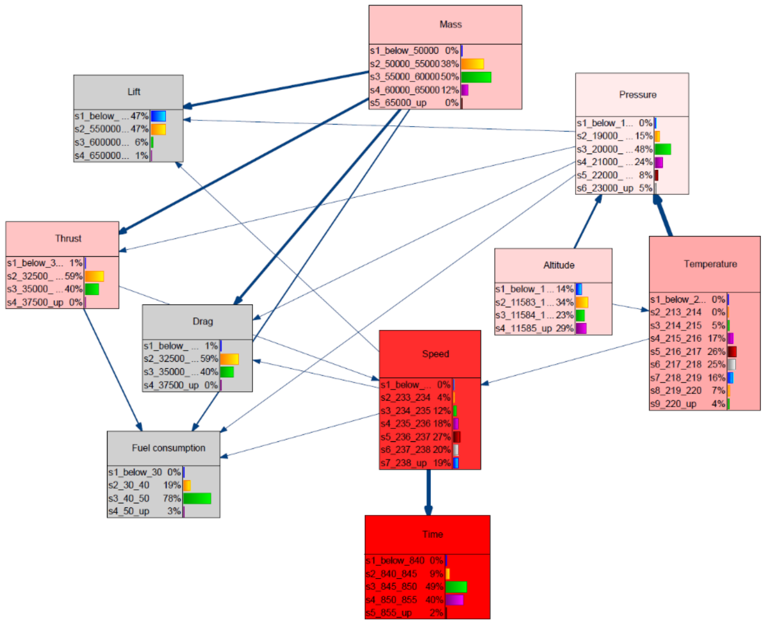

Figure 8: Causal relationships and sensitivity between influential parameters

dependence ratio is lower than whit other nodes.

The sensitivity analysis enables us to: (a) test the robustness of the results of the prediction model in the presence of uncertainty; (b) increase our understanding of the relationships between input and output variables in the causal model; and (c) Manage uncertainty by identifying inputs that cause significant variations in the output.

The analysis should, therefore, focus on these inputs to make the model more robust.

This methodology for constructing BN can be applied to different checkpoints, i.e. different distances, to analyse potential TWs along the trajectory. If we consider the information of the trajectory as a whole, we can define a holistic BN that connects successive checkpoints. In line with this idea, we evaluated the five checkpoints defined, as depicted in Figure 5. The result is a global BN that interconnects five partial BNs (one for each checkpoint). This structure allows us to understand the relationships between successive TWs. In other words, it enables us to generalise the analysis to the entire trajectory by setting different parameters at different checkpoints, we can reduce uncertainty and gather information about successive TWs (direct analysis) or by setting target TWs along the trajectory, the model gives us the most probable states of the influential parameters (backward analysis). Therefore, the BN allows us to individuate the relevant factors and also to understand how different combinations may affect the probability of avoiding deviations from the planned trajectory.

The BN approach allows us to graphically and quantitatively express the causal relationships between the parameters involved in the trajectory prediction model. This can be used to identify those variables with a significant, or negligible, influence on the global evolution of the trajectory. The BN can also manage uncertainty using forward/inter-causal analysis. In such cases, the model estimates an interval (TW) for the arrival time by setting the probability of having certain configuration, i.e. by setting one or more parent-input nodes at different checkpoints. Alternatively, knowing the causal relationships also allows us to do a backward analysis, i.e. by setting a target TW at a specific checkpoint, we can discover at which values (or states) it is most possible to find the input variables. The BN model evaluates the impact of uncertainty on the outputs and analyses the propagation of uncertainty along the trajectory.

3.5 Reliability analysis

Reliability is defined as the probability that a goods function properly during a determined period under specific operational conditions. In this case, we define the 4D trajectory as the goods or system to be studied The period will be the flight phase (ascent, cruise or descent) and the specific operational conditions are the uncertainties of the proposed scenario (aircraft mass, wind (speed), temperature, etc.). 4D trajectories can be understood as complex Multi-State Systems (MSS) that depend on environmental conditions and internal parameters.

In this system, its global performance will be given by the performance of the system elements, which in this case will be the different parameters that define the trajectory. Therefore, it is necessary to define states or rates of system performance and states or rates of performance for the elements, considering the interrelationships and cross-

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influences. In this phase, the system is considered (4D trajectory) as a system with a state vector, composed of three states: optimal, acceptable and degraded, with their performance rates associated:

$$\begin{bmatrix} Optimal \\ Acceptable \\ Degraded \end{bmatrix} \Rightarrow \begin{bmatrix} G_1 \\ G_2 \\ G_3 \end{bmatrix} = \begin{bmatrix} 100\% \\ 50\% \\ 0\% \end{bmatrix} \tag{3}$$

The study of Markov's chains can be reduced to the algebraic study of the properties of transition matrices. These transition matrices or probabilities of being and/or going from one state to another of the system, are calculated from the data obtained from the simulations.

Therefore, the proposed model is a methodology based on MSS and Markov Chains, two new concepts that will be explain next. The combined application of these concepts is very useful because it allows to divide the system in different sub-operations where each state will have its own rate of performance so that will be easier to study the uncertainty.

3.5.1 Multi-State Systems

Every system is designed to accomplish a determined task in a determined environment. Traditionally, systems have been modelled in a binary way, thus each element has only two possible states: perfect functioning or complete failure [73]–[75]. However, the majority of real systems can be in more than two states. The so-called multi-state systems are the systems that present a finite number of states. Usually, a multi-state system is composed of elements that can also be in different states [75].

In practice, any system consisting of different binary states units that have a cumulative effect on the entire system performance should be consider a MSS. In addition, when the performance rate of elements composing the system can vary because of their deterioration (fatigue, partial failure) or because of variable ambient conditions, the entire system may be considered a MSS [74].

Before studying a complete multi-state system, it is necessary to characterize the elements that constitute it. Any element j of the system can have different k_j states corresponding to the performance levels of the element:

$$g_j = \left\{g_{j1}, g_{j2}, \dots, g_{jk_j}\right\} \tag{4}$$

Where $\mathrm{g_{ji}}$ is the level of performance of the element j at state i ($\mathrm{i} \in \{1,2, \dots \mathrm{k\_j}\}$.

The level of performance $\mathrm{G_j}$ $(\mathrm{t})$of the element j for any instant $\mathrm{t} \geq 0$ is a random variable, which takes values from $\mathrm{g_j}$. Therefore, for the time interval $[0, \mathrm{T}]$ in which T is the moment of operation, the level of performance of the element j can be defined as a stochastic process. The probabilities associated with each state of the system element j for any instant t can be represented with the following set of equations:

$$\begin{gathered} \boldsymbol{P}_j(t) = \{p_{j1}(t), p_{j2}(t), \dots, p_{jkj}(t)\} \\ \boldsymbol{P}_j(t) = \{p_{j1}(t), p_{j2}(t), \dots, p_{jkj}(t)\} \end{gathered} \tag{5}$$

Where

$$p_{ji}(t) = \Pr\{G_j(t) = \ g_{ji}\} \tag{6}$$

Since the element states are a mutually exclusive group (which means that the element j can be in one of the states and only one), the following condition must be fulfilled:

$$\sum_{i=1}^{k_j} g_{ji}(t) = 1, \qquad \forall \ 0 \leq t \leq T \tag{7}$$

Eq. (5) defines the probability function of a discrete random variable $\mathrm{G_j}(\mathrm{t})$ at any instant t. The pairs $\mathrm{g_{ji}}, \mathrm{p_{ji}}$ (with $\mathrm{i} = 1,2, \dots \mathrm{k_j}$) completely determine the probability distribution of element j at any instant t.

If a multi-state system is composed of n elements, its performance level is determined in an unambiguous way by the performance levels of the elements that compose it. At each moment, the system elements have a level of performance that corresponds to their current state. The status of the entire system is determined from the states of its elements. Therefore, the definition of a multi-state model must include the stochastic process performance for each element of the system i: $\mathrm{G_i}(\mathrm{t})(i = 1, \dots n)$ and the structure of system operation that causes the stochastic process corresponding to the output of the entire multi-state system: $G(t) = \varphi\big(G_i\ (t), \dots, G_n\ (t)\big)$.

3.5.1 Markov Chains

Markov chains are a special type of discrete stochastic process in which the probability of an event only depends on the previous state of the system. This type of systems are memoryless thus satisfy the Markov property [74]:

$$\begin{gathered} P(X_{n+1} = x_{n+1} | X_n = x_n, X_{n-1} = x_{n-1}, \dots, X_2 = x_2, X_1 = x_1) \\ = P(X_{n+1} = x_{n+1} | X_n = x_n) \end{gathered} \tag{8}$$

DOI: 10.13009/EUCASS2019-381

The probability of transition between state $X_{n-1} = i$ and $X_n = j$ is given by $\gamma_{i,j}$, where n is the number of transitions. The matrix $\boldsymbol{P}$ collectively represents these transition probabilities [76]:

$$\boldsymbol{P} = \begin{pmatrix} \gamma_{1,1} & \gamma_{1,2} & \cdots & \gamma_{1,k} \\ \gamma_{2,1} & \gamma_{2,2} & \cdots & \gamma_{2,k} \\ \vdots & \vdots & \cdots & \vdots \\ \gamma_{k,1} & \gamma_{k,2} & \cdots & \gamma_{k,k} \end{pmatrix} \tag{9}$$

The vector π_n^T defines the probability of finding the system in a particular state on the n-th transition:

$$\pi_n^T = \begin{bmatrix} \pi_{1,n} & \pi_{2,n} & \cdots & \pi_{k,n} \end{bmatrix} \tag{10}$$

Where $\pi_{k,n}$ is the probability that the system is in state k on the n-th transition. The probabilities of the state for each transition are determined iteratively as follows:

$$\pi_n^T = \pi_{n-1}^T \boldsymbol{P} \tag{11}$$

The random evolution of a Markov chain is completely determined by its transition matrix P and its initial density distribution π_0^T. Therefore, with the transition matrix obtained, the study of Markov chains is reducible to the linear algebra study of its transition matrix and state vector (Eq. (11)).

Carrying on the reliability model, the choice of states is based on confidence intervals defined by Normal distributions of the parameters evaluated in Monte Carlo simulations. We used MATLAB software to develop Markov's model. This software allows to create and determine Markov chains and computes confidence bouds for P using a normal approximation to the distribution of the estimate:

$$\hat{\mu} + \hat{\sigma} q \tag{12}$$

Where q is the P-th quantile from a normal distribution with mean zero and standard deviation 1. The computed bounds provide the desired confidence level when μ and σ are estimated from large samples. Nevertheless, in smaller samples other methods of computing the confidence bounds may be more accurate [77].

The normal inverse function is defined in terms of the normal cumulative density function (cdf) as:

$$x = F^{-1}(p|\mu, \sigma) = \{x : F(x|\mu, \sigma) = p\} \tag{13}$$

Where

$$p = F(x|\mu, \sigma) = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{(t-\mu)^2}{2\sigma^2}} dt \tag{14}$$

The result, x, is the solution of the integral equation (14), where the desired probability, $\boldsymbol{p}$ is introduced as an input. Simulated 4D trajectories have been used to estimate μ and σ. μ arises from the deterministic model of the trajectory, obtaining a μ value for each instant of time and for each study parameter. As for the σ value, the standard deviation is considered at the initial instant. Therefore, the initial instant is taken as a reference point to study the degradation of the trajectory over time. Once we have calculated μ and σ, we define the intervals corresponding to each of the states into which the trajectory has been divided in Figure 9.

State	Interval
Optimal	68.3 %
Acceptable	27.3 %
Degraded	4.4 %

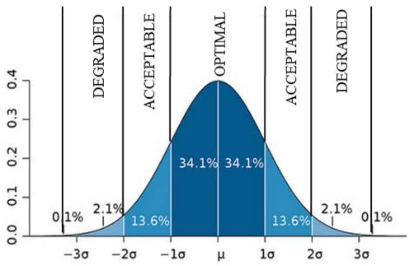

Figure 9: Intervals defining system states and the probability associated with each interval (centred at μ and indicating multiples of σ)

3.5.4 Transition matrix

Once the states are established at defined intervals, the next step is to obtain the transition matrix of the model. Firstly, we obtain the transition matrix of the partial parameters and the transition matrix of the global model. Transition matrices are calculated for all parameters included in the Monte Carlo simulation (mass, speed, temperature, thrust and range). The transition matrices for five of the variables are shown in Figure 10.

DOI: 10.13009/EUCASS2019-381

	Optimal	Acceptable	Degraded
Optimal	0.9740	0.0260	0
Acceptable	0.0319	0.9447	0.0234
Degraded	0	0.0147	0.9853

(a)

	Optimal	Acceptable	Degraded
Optimal	0.9738	0.0262	0
Acceptable	0.0324	0.9437	0.0239
Degraded	0	0.0152	0.9848

(b)

	Optimal	Acceptable	Degraded
Optimal	0.9891	0.0099	9.5871e-04
Acceptable	0.0376	0.9355	0.0269
Degraded	0	0.0161	0.9816

(c)

	Optimal	Acceptable	Degraded
Optimal	0.9651	0.0049	0
Acceptable	0.0021	0.9916	0.0062
Degraded	0	1.4620e-04	0.9999

(d)

	Optimal	Acceptable
Optimal	0.9853	0.0147
Acceptable	0.0015	0.9985

(e)

Figure 10: (a) Speed transition matrix; (b) Temperature transition matrix; (c) Thrust transition matrix; (d) Range transition matrix; (e) Mass transition matrix.

The global transition matrix (Figure 11) is generated with the five mentioned parameters. In addition, two parameter blocks are defined, fundamental and non-fundamental. With these two blocks we add a priority in the calculation of the global transition matrix. We chose speed and range as fundamental parameters because of their sensitivity and intensity of the relationship in the trajectory state. In non-fundamental parameters we put temperature, thrust and mass. If one of these two fundamental parameters are degraded, the trajectory is degraded. If not, the state is calculated taking into account the other parameters' state.

	Optimal	Acceptable	Degraded
Optimal	0.9822	0.0174	4.0418e-04
Acceptable	0.0182	0.9687	0.0131
Degraded	2.9920e-04	0.0086	0.9911

Figure 11: Trajectory global transition matrix

4. Results

A Markov chain is said to be irreducible if it is possible to get to any state from any state. A state i has period k if any return to state i must occur in multiples of k time steps. If k = 1, then the state is said to be aperiodic. Otherwise (k > 1), the state is said to be periodic with period k. A Markov chain is aperiodic if every state is aperiodic. An irreducible Markov chain only needs one aperiodic state to imply all states are aperiodic.
Furthermore, a Markov chain is called an ergodic chain if it is possible to go from every state to any other state (not necessarily in one move) so if it is both irreducible and aperiodic. This condition is equivalent to all nodes of the chain being ergodic (recurrent, aperiodic and positive) [76], [78], [79] That is the case of the transition matrix studied.
The stationary distribution π (steady-state or long-term, $n \to \infty$), does not change over time or:

$$\pi = \pi T \tag{15}$$

$$T^{\infty} = \lim_{n\to\infty} T^{n} \tag{16}$$

By the Perron-Frobenius Theorem [80], ergodic Markov chains have unique limiting distributions; i.e., they have unique stationary distributions to which every initial distribution converges. The result for the system is:

$$X = [0.2935\,,\ 0.2800\,,\ 0.4265]$$

If we define optimal and acceptable states as correct operating states (with an expected level of demand (w) of at least 50%), the trajectory is correct in 57.35% of cases. On the other hand, the trajectory is degraded or in an incorrect state in 42.65% of cases. In addition, the time it takes the system to achieve the stationary distribution is 698 seconds (with an error of the estimation of 0.0255%). It should be noted that simulations have been developed without considering any correcting measures for the variables throughout the flight, so the parameters tend to degrade with flight time.
We use three indicators to describe the system performance: mean instantaneous performance, mean instantaneous deficiency and mean instantaneous reliability. that are described below.

Mean instantaneous performance (Et)

In order to obtain indicators that characterize the average MSS output performance, we can use the performance

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expectation. The mean value of MSS instantaneous output performance at time t is determined as [75]:

$$E_t = \sum_{k=1}^{t} g_k p_k(t) \tag{17}$$

Being N the total number of states, g_k is the performance rate associated with state k and $p_k(t)$ is the probability that the system is at state k at time t. The steady-state expected performance can also be obtained substituting in Equation (4) the stationary distribution values. Indeed, the value of the steady-state mean performance is the asymptotic value of the mean instantaneous performance (Figure 12 left). This graph shows the mean instantaneous performance evolution. The system is initially functioning perfectly (the initial state is selected as a reference with a 100% performance rate), and then evolves towards the mean instantaneous performance value for stationary distribution (43.35 %).

Mean instantaneous deficiency (D_t)

The mean instantaneous deficiency or deviation is defined as a weighted average between the system probability to be found in each state and the service levels associated to these states. A weighted average of the value of a random variable where the probability function provides weights can be understood as the expected value [75]. In case the difference is negative the average is weighted with a zero. That is because in those cases the system is meeting the expected demand and the aim of the index is to assess the cases when the system is not fulfilling the demand.

$$\mathrm{D_t} = \sum_{\mathrm{i}=1}^{\mathrm{N}} \mathrm{p_i(t)} \max\left(\mathrm{w} - \mathrm{g_i}; 0\right) \tag{18}$$

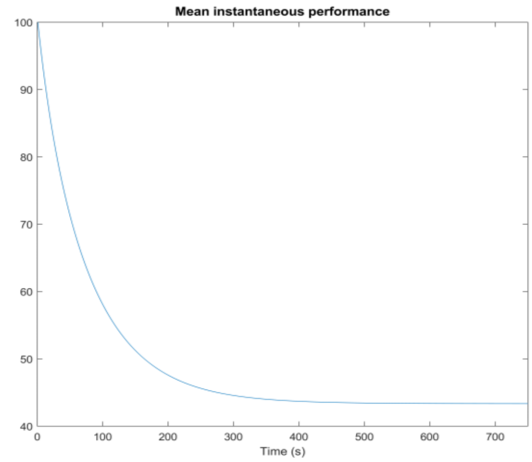
Where $p_i\ (t)$ is the probability that the system is in state i at t-th time, w is the expected demand and g_i is the level of performance associated to state i.
The value of mean deficiency for stationary distribution is 21.33% (Figure 12 (right). This indicator shows the evolution of degradation of the trajectory. Starting from a state defined as optimal or ideal that satisfies the expected demand, it evolves to a degraded and unacceptable state. This confirms the outputs obtained from the mean instantaneous performance indicator.

Mean instantaneous reliability (R)

The reliability of a system (R) is defined as the system's ability to remain in acceptable states during the operation period. Therefore, the reliability function can be defined as the probability that the system is not in its unacceptable states [75].

$$\mathrm{D_t} = \sum_{\mathrm{i}=1}^{\mathrm{N}} \mathrm{p_i(t)} \max\left(\mathrm{w} - \mathrm{g_i}; 0\right) \tag{19}$$

Where i is the number of unacceptable states.
Figure 13 shows the evolution of the probability that the 4D trajectory is in the correct states (optimal and acceptable). Reliability first decreases rapidly with time and later reaches a stable value, stationary distribution, 57.35%. The result coincides with the results obtained in the calculation of the stationary distributions.



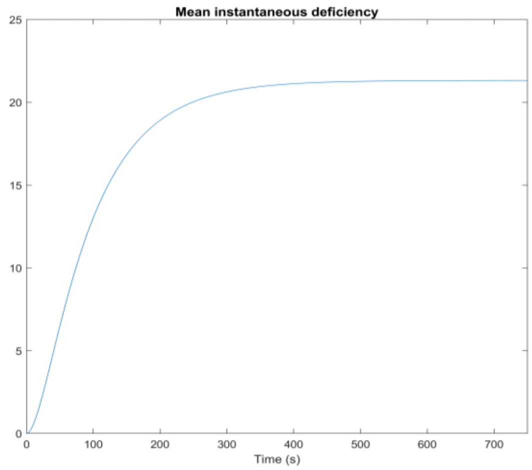


Figure 12: Mean instantaneous performance (left) and Mean instantaneous deficiency (right)

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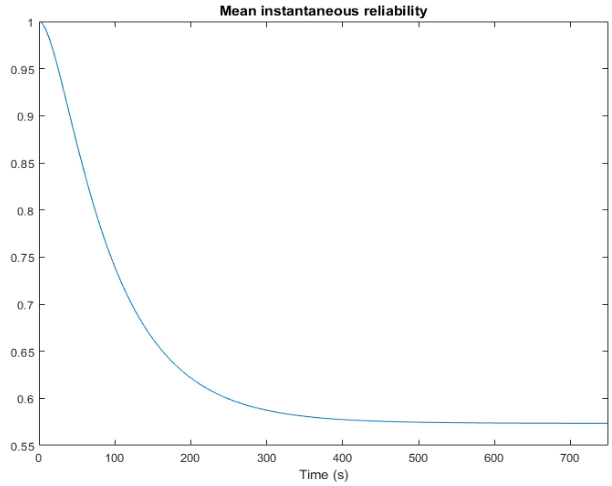

Figure 13: Mean instantaneous reliability

5. Conclusions

4D trajectories and their associated Trajectory Based Operations (TBO) concept require high accuracy and reliability in trajectory monitoring and forecasting. In the first stage of the study, we defined a 4D trajectory prediction model and studied the influences of different parameters in the estimation of checkpoints. This model and a Monte Carlo technique allow us to perform 1500 simulations and evaluate, through the information obtained from the simulations, the evolution of trajectories through a stochastic approach. So, the primary conclusion of this work is that we have modelled and characterised TWs for 4D trajectories and we realise that the function of the evolution of TWs is not lineal and it depends largely on the flight phase. The results show that, with 95% probability, the aircraft reaches 75NM (middle of climb phase) within a range of ±20 seconds, whereas it reaches 303NM (middle of cruise phase) within a range of ±10 seconds and 530NM (middle of descent phase) within a range of ±6 seconds. The climb phase entails more degradation than descent phase and this, at the same time, more than the cruise stabilised phase. This is because when aircraft is climbing or descending, the uncertainties of the parameters are greater. The application of these TW is promising because it could serve for different strategical analysis as conflict detection or pre-flight sequencing.

With the Bayesian Networks we have determined and quantified the impact of the different influence factors, being the most important the speed, the thrust, the temperature and the aircraft mass. BNs have proven to be an excellent method for analysing causality in trajectory prediction because they are "white boxes in the sense that the components of the model (variables, links, probability and utility parameters) are open to interpretation. This means that it is possible to perform a wide range of analyses of the network, e.g., causal interactions, conflict analysis, (in)dependence analyses, sensitivity analysis, and value of information analysis. Furthermore, they have the following advantages: (a) They allow data to be used in conjunction with expert knowledge/judgment; (b) They enable inference to be performed efficiently in models with a large number of variables; (c) They employ a probabilistic approach in decision making and in managing uncertainty which is consistent with the treatment of stochastic processes; and (d) They make cross-inference (several control variables) possible so that conclusions may be derived when multiple sources of information and complex interaction patterns are involved.

Then, we developed a reliability analysis using multi-state systems theory and Markov Chain model. For this, first of all, we defined the trajectory as a MSS, composed of the most influential parameters that were identified in the 4D-trajectory model. These parameters (mass, speed, thrust, range and temperature) were identified with the help of the causal analysis performed in the first part of the study. In addition, the causal analysis allowed us to define different blocks or elements of study. The performance rates were defined as: optimal, acceptable and degraded. Subsequently, a Markov Chain approach was used to define the transition between the different states (instant times) of the system. Transition matrices showed that the probability of remaining in the current state is about 97%, but with a clear tendency to the degraded state as time grows. This result coincides with the outcomes obtained in the simulations: a degradation arises as the distance flown by the aircraft grows. For the global model, in order to define the global transition matrix, we introduced two blocks of parameters: fundamental and non-fundamental. The fundamental block is the most restrictive: if it is degraded, the trajectory state is considered degraded. In case the fundamental block is not degraded, the other parameters are taken into account for the calculation of the global transition matrix. We defined as the correct state the one that is imposed by the optimal and acceptable state, and the degraded state being the incorrect one. Next, other results are also obtained from the global transition matrix. The reliability analysis showed the system evolution in time. With this analysis, we found a huge degradation towards an incorrect state. The probability of being in a correct state is 57.35%. Another output of the reliability model is the mean performance ratio in the stationary distribution,

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which is 43.35%, below acceptable status. Finally, time to achieve this stationary state is 698 seconds. In conclusion, during the flight the aircraft suffers disturbances that cause a great degradation of the trajectory. Hence, it is necessary to take appropriate actions to meet the agreed targets within the TBO concept.

The main contribution of this paper is the development of a tool for assessing 4D-trajectories reliability, which is associated with trajectory degradation. This model can be useful for the flight operators or the ANSPs to predict and manage 4D-trajectories, as the model makes it possible to assess, through different flight parameters, the evolution of trajectory degradation. With this information, it is possible to make trajectory updates at the appropriate time, to help deviation management and predictability of 4D-trajectory.

6. Future Works

A possible deviation of the trajectory means greater future deviations, which implies a non-fulfilment of the trajectory requirements. Furthermore, the impact of sources of uncertainty is cumulative along the trajectory. To avoid this circumstance, it is necessary to propose corrective measures. So, in future works we will analyse trajectory degradation to define and introduce the necessary corrective measures. Moreover, we will study the conditions and requisites to implement 4D-trajectories and we will propose a tolerance windows that mark when will it be necessary to apply the corrective measures for the trajectory. Future work will also include a feedback controller to emulate the actions that ATCOs and pilots may take to satisfy the constraints imposed on TWs.

Research lines for the future are focused on determining when the Air Traffic Control (ATC) should intervene tactically on the 4D-trajectory to ensure adherence to subsequent TWs and not to degrade neither trajectories nor the ATM system.

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