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Collection and stabilization of low-orbit small satellites in the formation

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Abstract
The article discusses the theoretical and practical aspects of the collection of small satellites in dense ordered group in the space. For this purpose each satellite coordinates are set in relation to the main satellite. The problem of control is the formation of the vector of control forces and moments, that provide its movement to a given point, taking into account restrictions. It is assumed that the satellites fly at a relatively close distance from several meters to one hundred meters from each other. The aim of this work is the development and study of optimal control system for collection of the group. At the last moment of control, each satellite must occupy a certain place in the formation relative to other objects in three-dimensional (3D) space.
Satellites for such a group are working together on the same problem. Group flights of small and relatively cheap satellites in many cases can reduce the cost of solving problems of Earth observation and study of various geophysical fields in comparison with the use of large satellites. This is facilitated by the possibility of placing sensors on small satellites at a much greater distance from each other than when using one large satellite. In addition, formation flights open up new possibilities for the development of unique scientific and technical programs using space assets.

1. Introduction

Usually, small satellites, from nano to micro satellites, are launched in groups, the initial distances between them are uncertain and can be from several metres to several kilometres. For this reason, the first task that needs to be addressed is the collection of satellites into a compact group. This problem was solved by the authors of the article as the optimal end-state control problem. However, the most effective mode of operation of electric rocket engines (ERE) is the maximum thrust mode. This mode of operation of the engine allows to organize a control system of satellites with maximum speed. This is especially important since electric rocket engines provide a very high specific impulse, but a small amount of thrust, which leads to slower control processes compared to liquid or solid-state jet engines. For these reasons, it is proposed to use a relay optimal controller synthesized using the Pontryagin maximum principle at the stage of collecting the group.
The signals of a spaceborne GPS are used to measure relatively large distances between satellites. For measuring small distances, the optical system of relative orientation and navigation is often used. Studies have shown that in all cases there is a strong uncertainty in the statistical characteristics of measurement errors when a group moves in orbit. For this reason, an original adaptive algorithm was proposed, whose work is based on spectral analysis of the residual. The reliability of the navigation system is ensured by the use of an inertial module, which allows interpolation of information in case of data receipt failures from satellite systems. In addition, the use of the inertial module allowed to reduce significantly the level of noise in navigation measurements. The simulation method demonstrated the performance and efficiency of the adaptive filter. It is proposed to use this filter in the control loop after the primary filter for positional measurements.
The issues of satellite control when changing the formation configuration by commands from the Earth are considered. This allows to expand the range of tasks hung by a group of satellites. The method of calculating the system of stabilization of a formation is given when a group of satellites are moving around the Earth. The dependences of the required thrust of electric propulsion depending on the parameters of the system and the orbit are investigated. Recommendations on the choice of engine modes are given. The accuracy of positioning of satellites in

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the formation was estimated depending on the errors of navigation sensors, the influence of the environment and the accuracy of reproduction of control forces.

The control algorithms considered in this article can be used in other areas of aviation and cosmonautics, where it is necessary to assemble a group of dynamic aircraft, including unmanned aerial vehicles. Most often, the purpose of unmanned aerial vehicles is to survey the surface of the Earth with high resolution.

To improve the quality of the transmitted video information, the flight should be carried out at a sufficiently low altitude. In this case, narrows the field of view of cameras mounted on vehicles. To monitor large areas it is necessary to use several such devices at the same time. In this case there is a problem of coordination of trajectories of devices and their relative mutual arrangement in the course of all flight. It is highly desirable that this coordination be automatic. In this case, the group will be managed by one person-operator. There are a number of problems with the implementation of such projects, including the dynamics of controlled vehicles, the restriction on their maneuverability and controllability, the problem of measuring the relative position and accurate maintenance of the spatial configuration. The main problem is to collect them in a compact formation, in which each unit is determined by the appropriate place.

Progress in space science and technology has led to the emergence of a different trend. Advances in microelectronics, improvements of onboard communications and control systems, and the appearance of micromechanical sensor elements stimulated the appearance of small SC. This approach can significantly (by 1-2 orders of magnitude) reduce expenses on development and operation of space objects, and shorten their design and manufacture time. The use of small satellites opens a variety of opportunities for development and improvement of forms and methods of space exploration: the delivery of small satellites into orbit as additional payload, together with a larger unit, the launch of a whole group of small satellites of the same or different class with a single carrier. Introducing group flights (Formation Flying) in addition to reducing the project cost leads to qualitatively new possibilities in terms of expanding research programs of surveying the earth's surface, the investigation of physical fields, mapping the magnetosphere, monitoring the ocean surface, tracking the traffic flows and migration of wild animal herds, short-term forecasting of earthquakes and so on.

It is now widely recognized that future near-earth space systems will use satellite formations as a distributed collaboration platform, and the role of existing systems based on the use of only one satellite will gradually decrease. This evolution of satellites requires a significant change in the approaches to design, implementation and operation for various missions [2], [4]. Two trends are typical of the modern development of unmanned space technologies and systems in satellite design. This is a significant decrease in the mass of each satellite, which makes the satellites smaller with the current increase in the number of satellites launched each year and the increasingly common practice of designing several small satellites as a distributed control system in which the satellites jointly solve common problems and set functional tasks. The number of the most common small satellites launched every year exceeds 100 and is constantly growing, but most of them are extremely simple and are not able to solve complex problems and any new significant tasks. The video camera and the simplest transmitter often form the basis of the "payload" of satellites for monitoring a surface from a low earth orbit.

At the end of the last century and the beginning of this century, the latest achievements in electronics and other fields of science and technology led to the possibility of miniaturization of almost all spacecraft (SC) service systems without compromising their functional qualities. In this connection, it became possible to use spacecraft of small dimensions for solving rather complex scientific and applied problems. This is one of the factors that drives the increased interest of the industry and "non-profit" research institutions to replace the existing large space systems with small satellites and open up new opportunities for these small systems.

2. Relative navigation and instrumentation

There are two types of constellation groups: Formation Flying and Constellation. In group flights of the Formation Flying type, control is carried out taking into account the relative positions of the satellites moving at very close orbits, that is, the relative motion control is realized. When forming group flights of the Constellation type, individual control of each object is carried out.

Structure Formation Flying is a much more flexible system than a single satellite, because if one of them fails, the group mission can continue its functioning, in contrast to the single mission. The formation of satellites is more easily upgraded and is capable of solving problems that cannot be solved by a single spacecraft. When using the reconfiguration property, the constellation of satellites can be used in a single mission to solve several problems. In addition, this configuration is capable of providing a higher positioning accuracy of satellites in a group.

There are a number of problems for the implementation of such projects in terms of the dynamics of its constituent satellites. However, the ability to work together in the formation of satellites is determined by the ability to determine

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the relative position of each system object and the ability to control their relative position. The control of a group flight must be based on direct or indirect measurement of the relative position of the satellites in the group.

For the formation of group flights of small satellites for the relative navigation in the last decade used differential GPS, radio frequency and optical navigation techniques. However, new tasks in the use of groups of satellites put forward more stringent requirements for the accuracy of relative navigation and orientation for all six degrees of freedom. Requirements to reduce their weight, cost and limitations on available capacity determine the need for special studies to develop new hardware and software for relative navigation and motion control for the next generation of group flights of satellites. Recently, various structures for constructing groups consisting of identical and completely different satellites are considered, in this paper we discuss general questions of constructing such groups.

The paper considers a rather general scheme for the formation of a group of satellites. It initially consists of two satellites. One of them is the main (passive), and the other is active. The main satellite is controlled by commands from the Earth, occupies a certain place in orbit and is oriented in a certain way. The second satellite should take the prescribed position of the relative primary satellite in all six degrees of freedom. Since the number of active satellites is not limited, using this approach any configuration can be formed. . In addition, during the flight to the main satellite, a command can be sent from the Earth to change the configuration of the system (formation). In this case, new coordinates are transmitted from the main satellite for each active satellite and a new formation is formed.

The highest relative navigation accuracy is provided by the offline differential relative navigation GPS, which provides a position accuracy of at least 1 cm, and a real-time differential relative navigation system - no worse than 10 cm.

However, the cost and weight of such receivers do not allow use them for small and cheap satellites. For this reason, we will consider a combined navigation system, in which, for a long range, more than 30 m, a simple absolute navigation system is used, and with a shorter range, an optical system [3, 4, 5].

The bench for experimental verification of orientation and navigation algorithms based on the optical system is shown in figure 1.

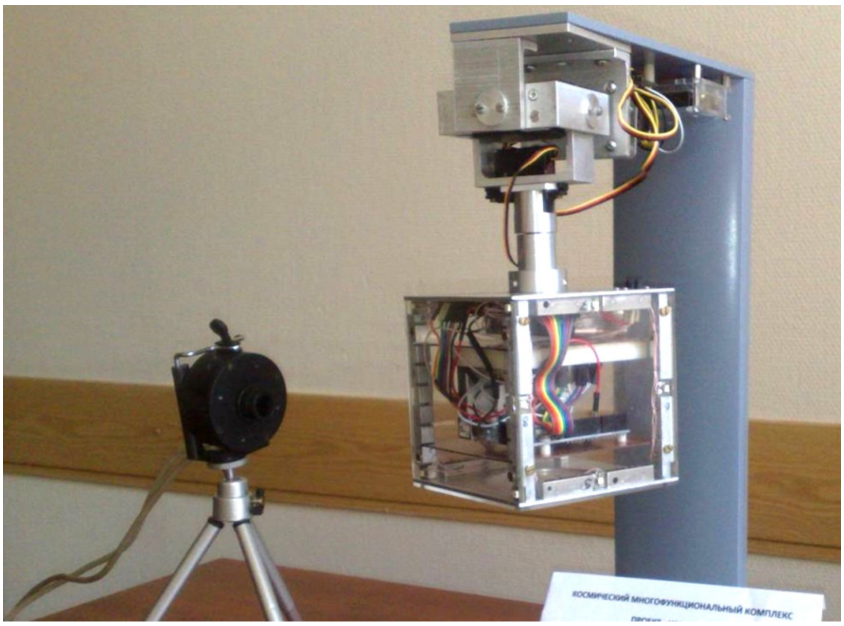

Figure 1: The model of the satellite on a three-axis controlled suspension and a movable video camera with variable focal length

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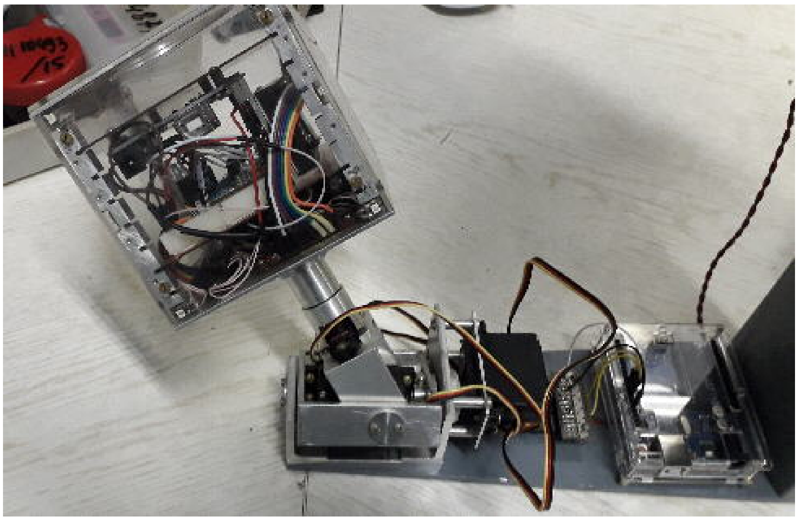

Figure 2: The layout of the stand control system (bottom view))

Infrared LEDs are installed on the surface of the satellite layout (CubeSat). The spectrum and light power of the LEDs is chosen from the condition of ensuring the stable operation of the optical system at distances up to 50 meters in low-orbital flight conditions, taking into account the influence of sun exposure.

The satellite model is rotated about three axes, and its orientation and distance projections from the video camera on the axis of the Cartesian coordinate system associated with the satellite are calculated from the video camera image using the algorithm described in [6,7]. In figure 2 shows a stand for visually observing the image from a video camera and studying the accuracy of relative orientation and navigation using an optical system.

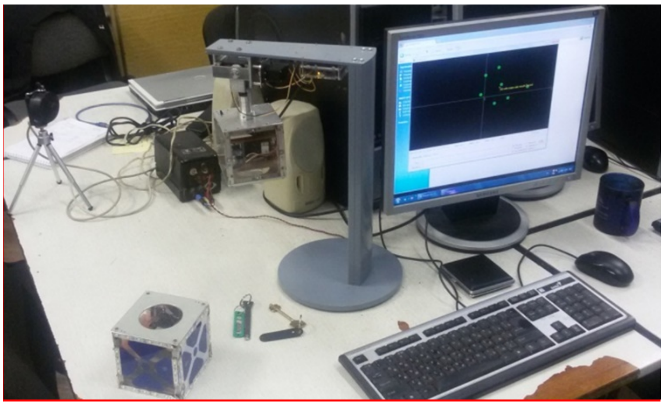

Figure 3: Stand for investigation of the accuracy of the relative orientation and navigation of the optical system.

Determination of the relative position of the satellites is one of the main problems of the group flight. To determine the relative state of apparatuses in a group, video image processing enabled by recording the neighboring satellite by a video camera installed on another satellite is used. There are several options for such an approach to the determination of the relative state. Relative navigation system based on image processing in the project PRISMA is implemented [1]. By processing frames, the program determines the relative distance and orientation of the devices.

This is analogous to the system used for navigation of transport spacecraft approaching the International Space Station (ISS). For distances of up to several hundred meters the GPS-data from the receivers installed on the ship and the ISS have been successfully used. Information on the status of the station is transmitted to the ship by radio channel. At close distances one uses automatic visual monitoring of the process of the approach and docking of spacecraft "Progress" with the ISS.

The base information is a video signal coming from the onboard cameras. The resulting sequence of images is processed in real time. In each frame there are special target details that are allocated, whose dimensions as well as

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the geometric parameters of the mutual arrangement are used as primary measurements. Based on these measurements the relative movement of the spacecraft and the space station is reconstructed.

The greatest accuracy is provided by the option in which differently colored LEDs are installed on one of the devices at certain positions. This option appears to be the least expensive for the navigation of small satellites and is the main scope of this article.

The objective of this work is to determine parameters of the relative orientation and navigation of satellites (6 parameters), as well as the synthesis of control laws for the center of mass motion for the controlled satellite when moved from an arbitrary position to a predetermined position relative to the uncontrolled target satellite. The main requirements to the control system being designed are its simplicity and low cost.

Consider the simplified system of optoelectronic relative orientation and navigation, consisting of two satellites. The first satellite is set eight infrared LEDs. LEDs are arranged at the vertices of a transparent cube and are numbered from 1 to 8. The second satellite, installed a video camera. Figure 3 shows the layout of satellites and readout system of coordinates.

On the sensitive matrix video camera images are infrared LEDs. The coordinates of these LEDs are measured and transmitted a special program to calculate the relative orientation angles and calculate the relative coordinates x, y, z. In solving the problems of the relative orientation and navigation is assumed that the camcorder can be "seen" all LEDs or arbitrary sets from 3 to 8 LEDs. Euler angles are measured sequential turning initial coordinate system successively at angles around the three axis.

The coordinates of the vertices of the cube have the images on the sensitive matrix of the video camera and a special program measures their coordinates. The measurements of each vertices of the cube except the first vertex allow creating two equations. The result is an overdetermined system of 14 equations with three unknowns. Often part of the infrared LEDs are hidden by satellite construction and not available for observation. In this case, the number of equations decreases.

In any case, the overdetermination of the system can allows to increase the accuracy of its solutions in the presence of measurement errors of coordinates of points on the matrix of camcorder. Usually the solution of transcendental equations use the method successive approximations, which requires the initial conditions. In practice, as these conditions it will be used evaluating orientation parameters in the previous step of tracking.

3. Adaptive filtering of the measurements

The use of light and cheap meters inevitably leads to a high level of noise component of navigation measurement errors. Such measurements cannot be directly used in the control system without additional filtering. The complexity of synthesizing such filters is determined by the fact that the real statistical properties of measurement errors depend on the relative position of the navigation satellites and the continuously changing arrangement of the controlled group of satellites.

From a practical point of view it appears that the statistical properties of the measurement errors are uncertain and change with time. Moreover, the measurement error in the process can vary by tens of times. This fact significantly reduces the effectiveness of simple filtration methods.

When using robust filtering methods have to immediately give up the goal of achieving the maximum achievable accuracy at every stage of the flight. Known effective adaptive algorithms are quite complex and require a lot of time to process incoming information and to identify real conditions for the flight of satellites. Their effectiveness in conditions of sufficiently rapid changes in flight conditions does not always give the desired result. For this reason, the use of such filters requires special studies.

Here considers an adaptive algorithm for filtering relative-range measurements. Statistical characteristics of these measurements are not specified and may vary within wide limits. The differential equations of Hill as a mathematical model of the relative motion of satellites are used. It is taken into account that external perturbations act on closely spaced satellites, which depend on the shape of each satellite and its orientation in space. The differences in these forces that cause relative displacements of satellites in the group are small, but their real values are unknown.

In addition to the requirements for accuracy, such an algorithm requires simplicity of computation and a sufficiently high speed. The algorithm is based on the analysis of spectral properties of the difference between measurement and measurement estimation (residuals).

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Let, in the general case, the equation of autonomous navigation means be given in the form

$$\dot{x}(t) = Fx(t) + Gw(t), \tag{1}$$

where $x(t)$ is the system state vector, F and G are the system state matrices and input actions, respectively, $w(t)$ is the vector of Gaussian white noises, with characteristics

$$M[w(t)] = 0,\ M[w(t), w(\tau)] = Q\delta(t-\tau).$$

The equation of the correction system we write in the form

$$z(t) = h(x,t) + \xi(t), \tag{2}$$

where $\xi(t)$ is the vector of Gaussian measurement noise with zero expectation and the correlation function $M\left[\xi(t),\xi(\tau)\right] = R\delta(t-\tau)$, $h\ (x,\ t)$ is a vector function reflecting the transformation of the estimated vector $x(t)$ into the basis of the correction system.

The measuring system described by equations (1) and (2) corresponds to the generalized Kalman filter equation

$$\begin{aligned} &\dot{x}(t) = Fx(t) + K\left[z(t) - h(x,t)\right], \\ &K = P\left(\frac{\partial h}{\partial x}\right)^T R^{-1}, \\ &\dot{P} = FP + PF^T + GQG^T - P\left(\frac{\partial h}{\partial x}\right)^T \cdot R^{-1}\left(\frac{\partial h}{\partial x}\right) \cdot P. \end{aligned} \tag{3}$$

In problems of synthesis of adaptive algorithms, only the steady state filtering mode is usually considered. Then for a priori given values of $R = R_a$ and $Q = Q_a$ we will have

$$K_a = P_a\left(\frac{\partial h}{\partial x}\right)^T R_a^{-1},$$

where P_a is the only positive definite solution of the algebraic Riccati equation

$$P_a + P_a F^T + GQ_a G^T - P_a\left(\frac{\partial h}{\partial x}\right)^T R_a^{-1}\left(\frac{\partial h}{\partial x}\right)P_a = 0. \tag{4}$$

It is known that under the estimated conditions the updated process (the residual signal) $v(t) = z - h(x,t)$ is white noise, while if the actual noise characteristics of the gauges deviate relative to the calculated ones, a correlation is found between its individual points. This leads to deformation of the spectral characteristics of the updated process. To determine the nature of the change in the discrepancy spectrum, a formula for its correlation functions $S(\omega)$ was obtained, which has the form [10]:

$$\begin{aligned} C(t-\tau) = C(\eta) = &\frac{\partial h}{\partial x}\left[1(\eta)\Phi(\eta)P_\infty + 1(-\eta)P_\infty\Phi^T(-\eta)\right]\left(\frac{\partial h}{\partial x}\right)^T - \\ -1(\eta)&\frac{\partial h}{\partial x}\Phi(\eta)K_a R^{-1}(-\eta)RK_a^T\Phi^T(-\eta)\left(\frac{\partial h}{\partial x}\right)^T + R\delta(\eta). \end{aligned} \tag{5}$$

From the correlation functions, it is easy to go to the energy spectra of the updated processes.

If the Kalman filter is used for estimation and the model exactly corresponds to the actual process, then the spectral density of the residual is white noise. In real flight conditions, the spectral density of the residual can be calculated by the following formula:

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$$S(\omega)=\frac{\partial h}{\partial \hat{x}}W(j\omega)P_{\infty}\left(\frac{\partial h}{\partial \hat{x}}\right)^{T}+\frac{\partial h}{\partial \hat{x}}P_{\infty}W^{T}(-j\omega)\left(\frac{\partial h}{\partial \hat{x}}\right)^{T}-\frac{\partial h}{\partial \hat{x}}W(j\omega)K_{a}R-RK_{a}^{T}W^{T}(-j\omega)\left(\frac{\partial h}{\partial \hat{x}}\right)^{T}+R, \qquad (6)$$

where $W(j\omega)$ is the frequency response of the system:

$$W(j\omega)=\left[j\omega I-\left(F-K_{a}\frac{\partial h}{\partial x}\right)\right]^{-1},$$

and P_{∞} is a solution of the algebraic Riccati equation

$$P_{\infty}\left(F-K_{a}\frac{\partial h}{\partial x}\right)^{T}+\left(F-K_{a}\frac{\partial h}{\partial x}\right)P_{\infty}+K_{a}RK_{a}^{T}+GQG^{T}=0.$$

If we take into account that

$$\lim W(j\omega)_{\omega\to 0}=\left(-F+K_{a}\frac{\partial h}{\partial x}\right)^{-1} \quad \text{and} \quad \lim \mathrm{W}(j\omega)_{\omega\to\infty}=0,$$

then for $\omega \to 0$ will get:

$$S(\omega)=\frac{\partial h}{\partial x}\left(-F+K_{a}\frac{\partial h}{\partial x}\right)^{-1}\cdot\left(P_{\infty}\left(\frac{\partial h}{\partial x}\right)^{T}-K_{a}R\right)+\left(\frac{\partial h}{\partial x}P_{\infty}-RK_{a}^{T}\right)\cdot\left(-F^{T}+K_{a}^{T}\left(\frac{\partial h}{\partial x}\right)^{T}\right)^{-1}\left(\frac{\partial h}{\partial x}\right)^{T}+R \qquad (7)$$

and for $\omega \to \infty$ we will obtain $S(\omega)=R$.

The spectrum can be divided into two components with the help of low-frequency (LPF) and high-frequency filters (HPF), the input of which is supplied with a residual signal. The cut-off frequency for the filters is established by analyzing the graphs of the deformation of the energy spectrum when the actual noise characteristics of the meters differ from the a priori specified.

The functional diagram of the adaptive filter based on the spectral analysis of the residual signal in the Kalman filter is shown in figure 4. In the HPF and LPF1 scheme, high and low frequency filters dividing the residual signal into frequency components, and the LPF2 and LPF3 filters are used as averaging when receiving ratings $\hat{R}$ and $\hat{Q}$.

When selecting and shaping the filters of the LPF2 and LPF3, can use the classical algorithms of statistical processing, use the averaging method in the sliding sample window, or use the simplest dynamic filters. The criterion here is the time of obtaining reliable estimates of the determined dispersions and the level of the residual fluctuation component. The resulting variance estimates are then used to reconfigure the Kalman filter coefficients.

To evaluate the efficiency of the adaptive filter, a simulation was performed to compare the effectiveness of the Kalman filter and the proposed adaptive filter for large changes in the statistical characteristics of errors in measuring the relative range of satellites. These changes are shown in figure 5. During the first 100 seconds of flight, the a priori noise intensity of the measurement noise is equal to real intensity =1. In the time range from 100 sec to 400 sec = 100, and then jump to 40.

The results of the simulation showed that the proposed algorithm effectively estimates the intensity of errors of measuring the range and the time of estimating the actual value of the error intensity of the meter does not exceed 100 sec.

Figure 6 illustrates the effectiveness of the filter adaptation to changes in the distance of random measurement errors. Curve "not adaptive" illustrates processes with the Kalman filter with constant coefficients corresponding to the first 100 seconds of flight. Curve "adaptive" corresponds to the operation of the adaptive filter.

Another approach to the filtering of the measurements of the relative distances between the satellites was described in [6].

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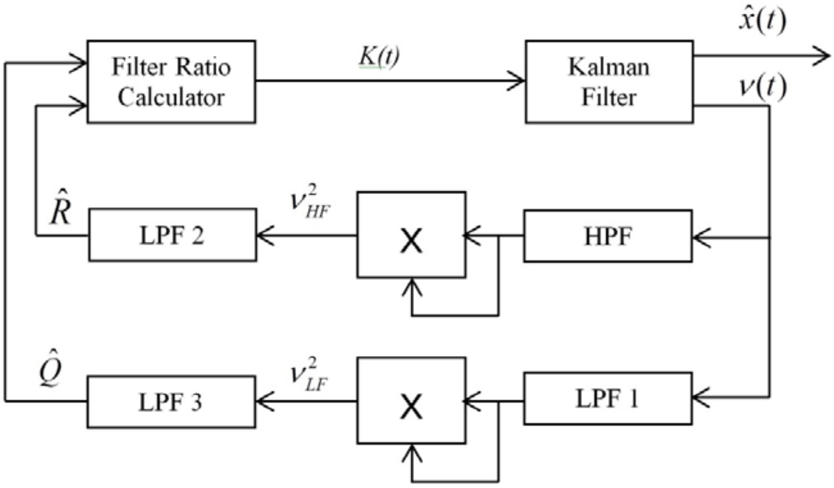

Figure 4: Functional diagram of the adaptive filter based on the spectral analysis of the updated sequence

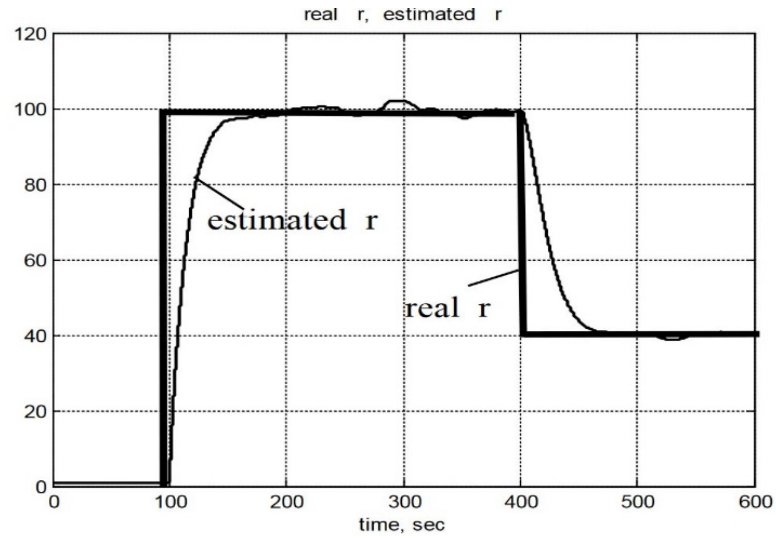

Figure 5: Process of the noise intensity estimation

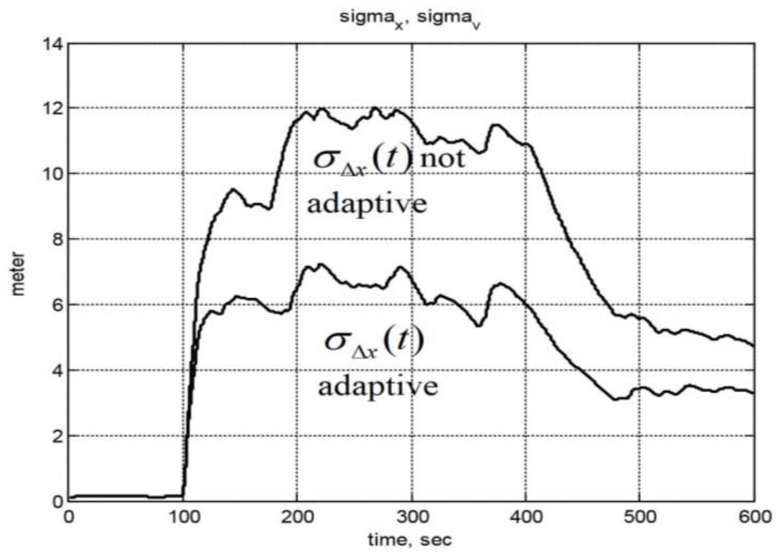

Figure 6: Changes of the projections of the relative velocity of the satellites

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4. Control system simulation

To describe the motion of an active satellite, we use an orbital coordinate system whose axis *Oz* is directed along the radius vector connecting the centers of mass of the Earth and the satellite; the axis *Ox* is perpendicular to the *Oz* axis and lies in the plane passing through the radius vector and the velocity vector of the satellite's center of mass, the *Oy* axis complements the coordinate system to the right orthogonal system.

Within the huge amount of literature on the dynamics of close formations in small eccentricity and circular/quasi circular orbits, papers and research approaches are considered here, which can be most useful for the formation designer. The simplest model to describe deputy motion with respect to the chief in terms of relative position and velocity coordinates is represented by Hill's equations, also called Clohessy-Wiltshire equations (HCW) [1–4]. HCW equations are based on the assumptions of central gravitational field as the only external force, Keplerian circular orbit for the chief satellite, small distance between chief and deputy. They can be derived by writing the dynamic equation of the deputy with respect to the chief and then linearizing the gravitational acceleration around the chief position. In typical cases, HCW modeling error tends to increase with time because of the underlying assumptions. Hill's equations can be derived in differential form as follows:

$$\begin{aligned} &\ddot{x} + 2\cdot\omega\cdot\dot{z} = 0, \\ &\ddot{y} + \omega^2\cdot y = 0, \\ &\ddot{z} - 3\cdot\omega^2\cdot z - 2\cdot\omega\cdot\dot{x} = 0. \end{aligned} \tag{8}$$

From the point of view of control theory, the satellite output to a given point is a typical task of terminal control. This means that at a fixed or arbitrary instant of time, the state vector of the control object must be close to the specified value. At the same time, acceptable system behavior should be ensured throughout the entire control process. Usually, this problem is formulated as an optimization problem and solved by methods of variation calculus. In the general case, the solution of such problems leads to a two-point boundary value problem. The boundary conditions for the equations of the object are specified at the initial moment of time, and the boundary conditions for the adjoint equations are specified at the final moment of time. Solving such a problem requires a large amount of computation, the resulting algorithm is more likely to have theoretical value and is not suitable for implementation in real-time systems.

In this section, a satellite control algorithm is considered that ensures its removal to a given point in space, focused on practical implementation in the on-board control system. For this purpose, the entire process of approach management is divided into three stages.

At the first stage, the satellite is brought to the finish line. At the second stage, the rotation around the center of mass is performed and the satellite is installed in a given orientation relative to the target. At the same time, its center of mass stabilizes at the finish line. At the third stage, the satellites approach each other when moving along the finish line.

At all three stages, control is carried out according to similar algorithms, which facilitates the implementation of a multimode control system. The most complex control algorithm corresponds to the third stage of satellite approach control. For this reason, the study of this stage of traffic control will be focus.

At this stage of the flight, the system of equations (8) can be simplified and presented in the form of three simple independent second-order equations for each of the axes

$$\begin{aligned} &\dot{x}_1(t) = 0, \\ &\dot{x}_2(t) = a(t). \end{aligned}$$

Quadratic criterion for terminal problem is

$$J = \frac{1}{2}\left([x_1, x_2]\begin{bmatrix} s_{1f} & 0 \\ 0 & s_{2f} \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) + \frac{1}{2}\int_{t_0}^{t_f} (a^2(t))dt.$$

The solution of this problem is

$$a(t) = -\frac{1/s_{2f}\,(t_f - t) + 1/2\,(t_f - t)^2}{D(t_f - t)} x_1(t) - \frac{1/s_{1f} + 1/s_{2f}\,(t_f - t)^2 + 1/3\,(t_f - t)^3}{D(t_f - t)} x_2(t), \tag{9}$$

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where

$$D(t_f - t) = \left[\frac{1}{s_{1f}} + \tfrac{1}{3}(t_f - t)^3\right]\left[\frac{1}{s_{2f}} + t_f - t\right] - \tfrac{1}{4}(t_f - t)^4 .$$

An alternative way to control the approach of satellites is to use the relay optimal control laws obtained using the Pontryagin maximum principle **[9].** In this case, the control law for each of the three projections of the projections of the distance between the satellites can be written in the form of the following form:

$$a(x_1, x_2) = -a_{\max} sign(x_1 + \frac{x_2 |x_2|}{2a_{\max}}) . \qquad (10)$$

The use of this law to control large space objects is discussed in [ ]. The advantage of this control law is that it assumes the use of maximum thrust during the entire control time interval. Such a mode is preferable when using electro-jet engines.

Figure 7 shows a program for modeling the processes of controlling the collection of a group of satellites using the acceleration control law (9).

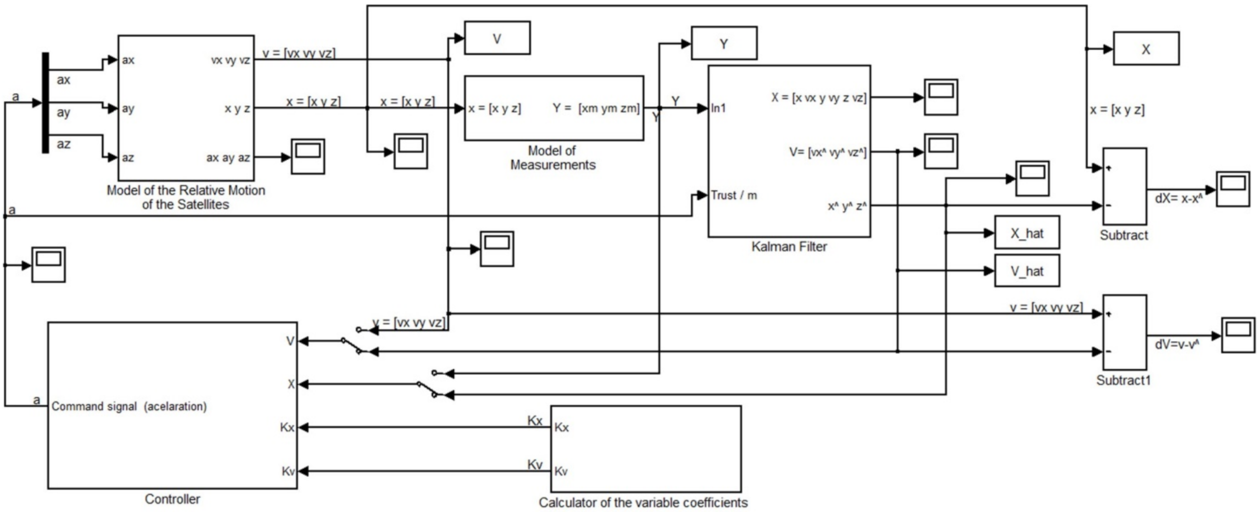

Figure 7: Structure of the program for simulating the processes of controlling the approach of satellites with an adaptive Kalman filter and without a filter

Figures 8 - 10 show the simulation results of bringing one of a group of satellites to a given point in space using and without using the adaptive Kalman filter.

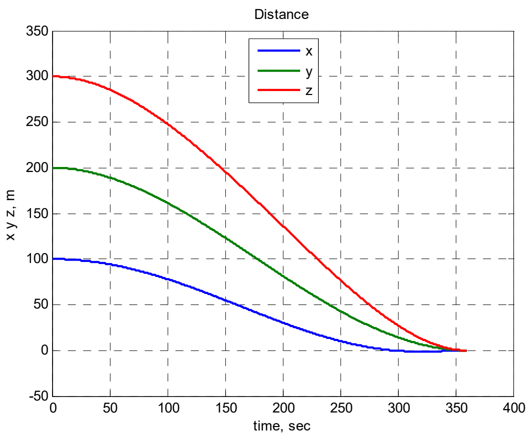

Figure 8: The projections of the relative distance between satellites for standard deviation $\sigma_x = \sigma_y = \sigma_z = 3$ m and $\sigma_{vx} = \sigma_{vy} = \sigma_{vz} = 3$ m/sec without filtration of measurements

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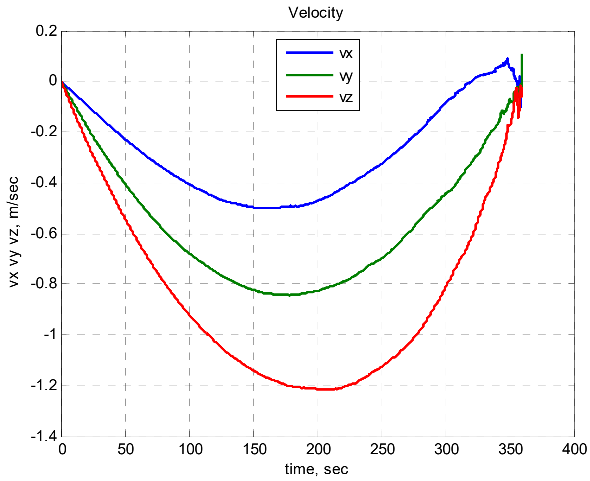


Figure 9: The projections of the relative velocities between satellites for standard deviation $\sigma_x = \sigma_y = \sigma_z = 3$ m and $\sigma_{vx} = \sigma_{vy} = \sigma_{vz} = 3$ m/sec without filtration of measurements

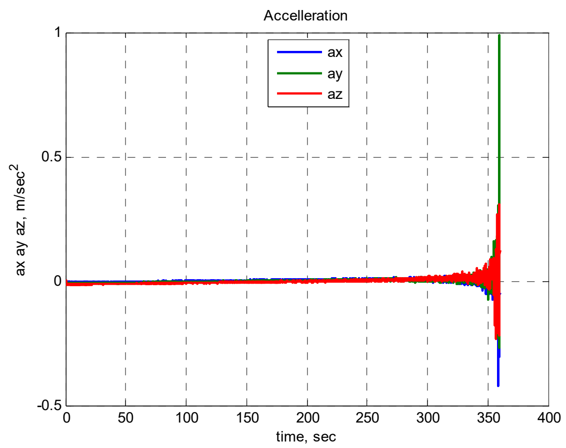
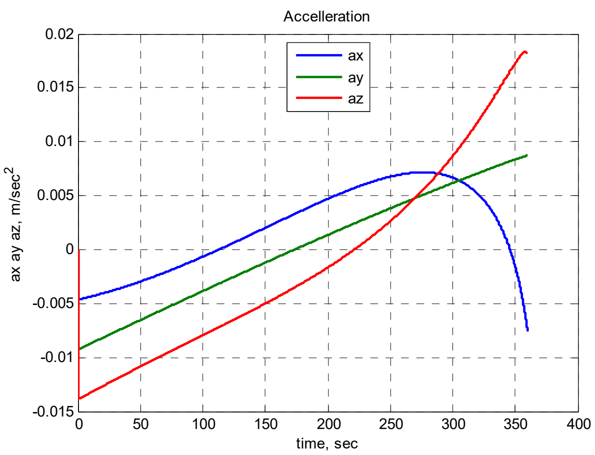
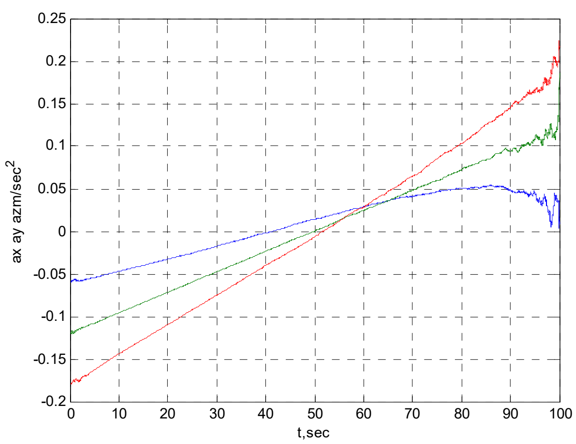


Figure 10: The projections of the relative velocities between satellites for standard deviation $\sigma_x = \sigma_y = \sigma_z = 3$ m and $\sigma_{vx} = \sigma_{vy} = \sigma_{vz} = 3$ m/sec without filtration of measurements





a) b)

Figure 11: The projections of the relative accelerations between satellites for two cases :
a) without measurement errors
b) standard deviation of measurement errors $\sigma_x = \sigma_y = \sigma_z = 3$ m and $\sigma_{vx} = \sigma_{vy} = \sigma_{vz} = 3$ m/sec with adaptive filter

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6. Conclusions

The simulation results confirmed the effectiveness of the proposed control algorithms for the construction of a dense formation. The implementation of these algorithms imposes minimal requirements for on-board computing devices. This allows them to be used as part of small satellites navigation and control systems.

Use in the navigation part of the proposed adaptive filter algorithm using an inertial module can significantly improve the accuracy of estimating the relative navigation parameters in the group.

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