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Laminar boundary layer self-induced separation in transonic flow

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To describe the self-induced separation of the laminar boundary, earlier mathematical models were created based on the use of asymptotic methods for the description of supersonic and subsonic regimes [1-2]. At the same time, the description of transonic modes remained unexplored, despite a number of papers in which individual regimes were considered.

Some progress could be made on the basis of results using the modified Khokhlov method, previously used to solve problems of nonlinear acoustics [3], including the description of nonlinear wave beams. The authors of [4] applied this method to solve non-linear non-stationary problems of gas dynamics at transonic velocities of flow around thin bodies.
The works noted above allow one to describe a self-induced separation of the laminar boundary layer at transonic flow rates. Earlier, the Karman – Guderley equation [5–6] was used to describe a perturbed transonic flow. The approach developed in [4] made it possible to obtain solutions of the Lin-Reisner-Tzyan equation, which expands the possibilities of describing viscous-inviscid interaction processes at transonic speeds over a certain range of variation of the Mach number. One of the important properties in the description of the self-induced separation is the separation of a three-layer structure of a perturbed flow, previously predicted in the work of V. Heisenberg [7], devoted to the analysis of the characteristics of linear hydrodynamic stability at high Reynolds numbers. This structure involves the selection of three characteristic regions containing streams of external inviscid flow, the main part of the flow in the boundary layer and the near-wall thin region in the boundary layer, in which the influence of viscosity and nonlinearity is significant. The following notation is used to denote the components of the velocity vector, pressure, density, dynamic viscosity coefficient, Cartesian coordinates, .- $u_\infty u, u_\infty v, \rho_\infty u_\infty^2 p, \rho_\infty \rho, \mu_\infty \mu, lx, ly$ respectively. The index refers to the functions in the unperturbed oncoming flow, l is the distance on the plate from the leading edge to the area of interaction and separation. It is assumed that the separation of the boundary layer may be caused by a fall of the shock on a laminar boundary layer, a break in the contour, or other reasons

After a series of transformations, the formula for determining the dimensionless pressure in the flow around a thin two-dimensional profile [4] can be represented as

$$p = 2(M_0 - 1)[1 - (1 + \frac{\dot{A}}{K^{3/2}})^{2/3}] \qquad (1)$$

where $K = 2(M_0 - 1)/(3\alpha\varepsilon_1)^{2/3}$ transonic similarity parameter, α -nondimensional body thickness, $\varepsilon_1 = (\gamma + 1)/2$

In this case, the boundary layer or, more precisely, the change in the thickness of the extrusion of the boundary layer acts as such a profile. Below it is assumed that the Mach number exceeds unity, so $K > 0$. It can be shown that for small values of perturbations, the formula transforms into the Akeret formula.

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Let us estimate the scales of perturbed functions in the near-wall region of the flow (region 3). Regions 1 and 2 refer to the external non-viscous flow and to the main part of the flow in the boundary layer. The magnitude of the longitudinal velocity is related to the scale of region 3 as follows.

$$u \sim y / \varepsilon$$

where $\varepsilon = \mathrm{Re}^{-1/2}$

Further, we denote $\beta = M_0 - 1$ where the M_0 Mach number of the unperturbed flow, then from (1) follows the estimate

$$p \sim \beta$$

From the longitudinal momentum equation, one can obtain an estimate for the magnitude and perturbation of the longitudinal velocity

$$u \sim \beta^{1/2}$$

The scale of the transverse size of region 3, as well as the scale of variation of the extrusion thickness, is determined as follows

$$y \sim A \sim \varepsilon\beta^{1/2}$$

From the formula for determining the pressure (1) it follows that

$$x \sim A \sim \varepsilon\beta^{1/2}$$

Another condition is related to the uniformity of orders of viscous and inertial terms in region 3

$$\frac{\partial p}{\partial x} \sim \varepsilon^2 \frac{\partial^2 u}{\partial y^2}, \quad \frac{p}{A} \sim \varepsilon^2 \frac{A}{\varepsilon\varepsilon^2\beta}, \quad \beta \sim \varepsilon$$

The last relation follows from the limit

$$K \sim 1$$

Where the ratio between the Mach number and the thickness of the boundary layer is valid

$$\beta \sim \varepsilon$$

Make the following transformations

$$u = \beta^{1/2}\rho_w^{-1/2}u_1, \;\; y = \varepsilon\beta^{1/2}a^{-1}\rho_w^{-1/2}, \;\; A = \varepsilon\beta^{1/2}a^{-1}\rho_w^{-1/2}A_1$$

$$p = \beta p_1, \;\; x = \varepsilon\beta^{1/2}a^{-1}\rho_w^{-1/2}K^{-3/2}x_1$$

Then the equations in the region 3 take the following form

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$$u_1 \frac{\partial u_1}{\partial x_1} + v_1 \frac{\partial u_1}{\partial y_1} + \frac{\partial p_1}{\partial x_1} = \Pi \frac{\partial^2 u_1}{\partial y_1^2}$$

$$\frac{\partial u_1}{\partial x_1} + \frac{\partial v_1}{\partial y_1} = 0, \quad \frac{\partial p_1}{\partial y_1} = 0$$

$$p_1 = 2[1 - (1 + \frac{\partial A_1}{\partial x_1})^{2/3}]$$

where

$$\Pi = \frac{2^{3/2} a^2 \rho_w^{1/2} \mu_w}{3K^3 \varepsilon_1^2}, \;\; K = \frac{2(M_0 - 1)}{(3\alpha\varepsilon_1)^{2/3}}, \;\; \varepsilon_1 = \frac{\gamma + 1}{2}, \;\; K = \frac{2\beta^{2/3} a^{2/3} \rho_w^{1/3}}{3^{2/3} \varepsilon_1^{2/3} \varepsilon^{2/3}}$$

With boundary conditions

$$x_1 \to -\infty \;\; u_1 = y_1, \;\; p_1 = 0$$

$$y_1 \to \infty \;\; u_1 = y_1 + A_1$$

For further analysis, we turn to the variable, assuming the monotonicity of the dependence of the induced pressure on the longitudinal coordinate. We introduce the flow function ψ

$$\psi = y_1^2 / 2 + f$$

Then the equation of motion takes the form

$$\frac{\partial p_1}{\partial x_1}[(f' + y)\frac{\partial f'}{\partial p_1} - (f'' + 1)\frac{\partial f}{\partial p_1}] + \frac{\partial p_1}{\partial x_1} = \Pi f'''$$

$$p_1 = 2[1 - (1 + \frac{\partial A_1}{\partial p_1}\frac{\partial p_1}{\partial x_1})^{2/3}], \quad \frac{\partial p_1}{\partial x_1} = [(1 - \frac{p_1}{2})^{3/2} - 1] / \frac{\partial A_1}{\partial p_1}$$

Consider the linear mode

$$\frac{\partial p_1}{\partial x_1}(y_1 \frac{\partial f'}{\partial p_1} - \frac{\partial f}{\partial p_1}) + \frac{\partial p_1}{\partial x_1} = \Pi f'''$$

$$p_1 = -\frac{4}{3}\frac{\partial A_1}{\partial p_1}\frac{\partial p_1}{\partial x_1}$$

Then the solution can be found in the form

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$$f = p_1 f_1$$

$$\frac{1}{p_1}\frac{\partial p_1}{\partial x_1}(y_1 f_1' - f_1) + \frac{1}{p_1}\frac{\partial p_1}{\partial x_1} = \Pi f_1'''$$

$$\frac{1}{p_1}\frac{\partial p_1}{\partial x_1} = -\frac{3}{4 f_1'(\infty)}$$

$$p_1 = c\exp(\alpha_1 x_1), \quad \alpha_1 = -\frac{3}{4 f_1'(\infty)}$$

$$\alpha_1(y_1 f_1' - f_1) + \alpha_1 = \Pi f_1'''$$

It is significant that the solution of the linear problem is defined up to a constant c. Depending on the sign of the constant, description or compression flow (with positive values of the constant) or expansiom flow (with negative values of the constant) is possible. The results of solving a linear problem are further used to numerically solve a nonlinear problem. A description of the numerical scheme and solution method is presented in the monograph [8].

Figure 1 shows the distribution of the induced pressure perturbation upstream from the separation point as a function of the longitudinal coordinate for the parameter value $\Pi = 1$. The solution was obtained in the region of positive values of surface friction up to the separation point.

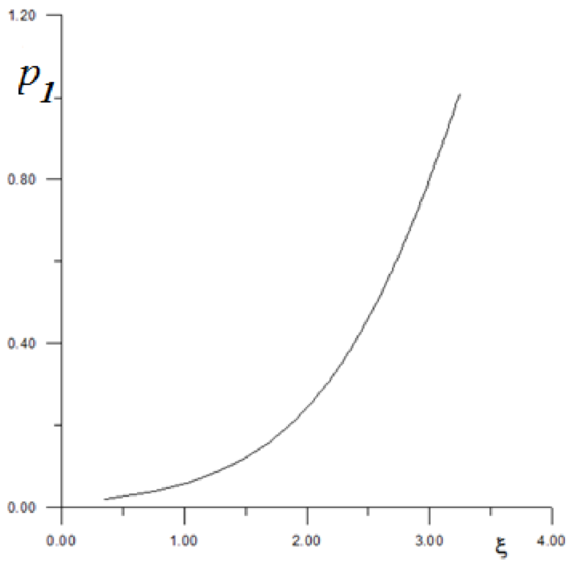

Figure 1 The dependence of the pressure perturbation from the longitudinal coordinate $\xi = x_1$

The figure 2 shows the distribution of surface friction upstream from the separation point.

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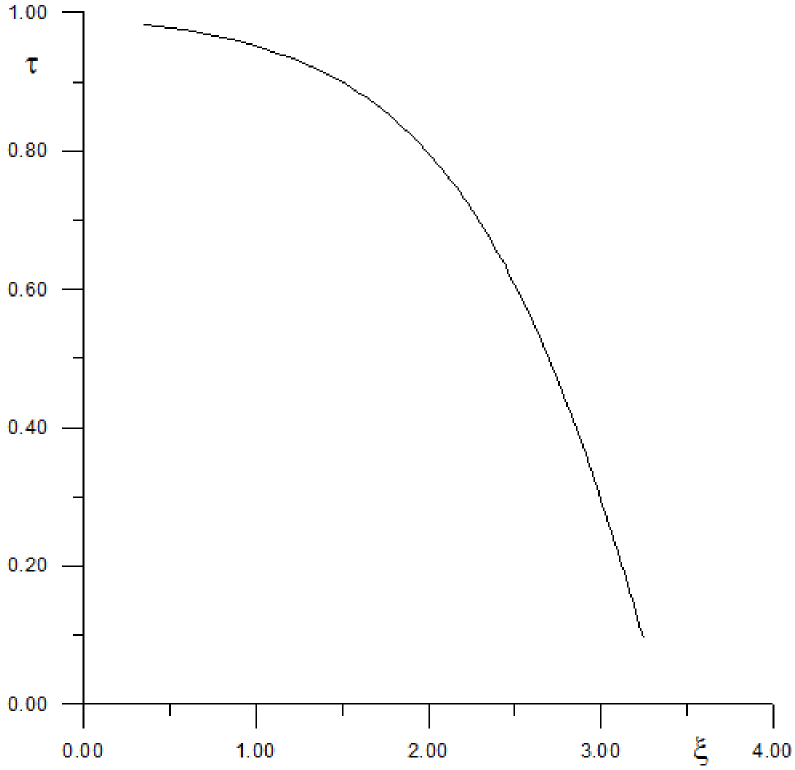

Figure 2 The dependence of surface skin friction $\tau = (\frac{\partial u_1}{\partial y_1})_w$ on the longitudinal coordinates $\xi = x_1$ for the parameter $\Pi = 1$

Finally, fig. 3 shows the dependence of the pressure perturbation ps at the separation point on the parameter value Π. It should be noted that when this parameter tends to zero, the pressure perturbation also tends to zero. Using the above equations, one can obtain the asymptotic behavior of pressure at the separation point for small and large values of the parameter Π. Small values of this parameter correspond to the transition to the description of separation on the basis of the theory of free interaction.

$$ps \sim \Pi^{1/2}$$

Larger parameter values Π correspond to the transonic limit when $K \to 0$

$$ps \sim \Pi^{1/5}$$

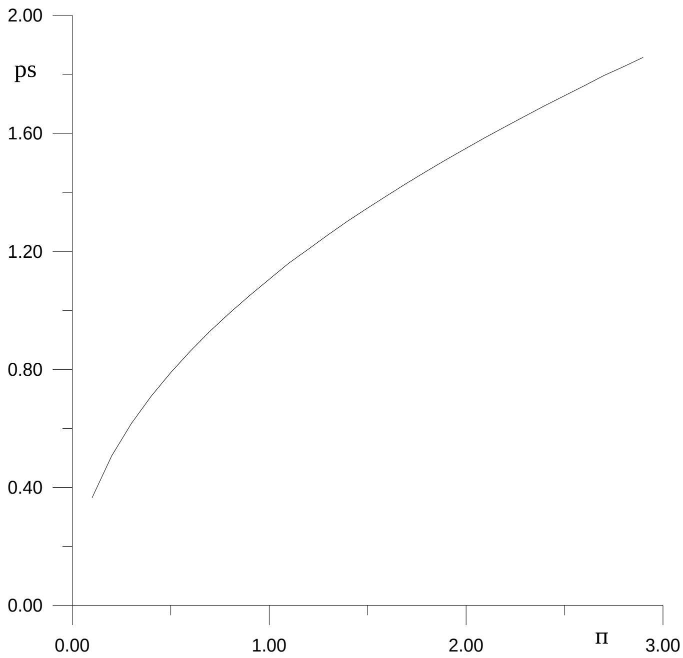

Fig 3. The dependence of pressure *ps* at separation point on the parameter Π .

Thus, the constructed model of the flow provided a description of the processes of viscous-non-viscous interaction and separation at transonic speeds in the conditions of the Mach number approaching unity. It is significant that for finite Mach numbers for supersonic flows, the model goes over to the previously constructed model of free interaction.

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