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Methodology to Identify a Mathematical Model for Predicting Cessna Citation X Cruise Performance in Cruise Regime using Flight Manual Data

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Abstract
This paper presents a methodology for identifying a performance model of an aircraft in cruise from a limited number of data. Starting from performance data published in the aircraft flight manual, a model for estimating the fuel flow of the aircraft in cruise was developed. The model was then combined with fundamental equations to predict the fuel consumption, specific range and economic speed of the aircraft. Validation of the proposed methodology was done by comparing data predicted by the model with data measured from a level-D flight simulator. Results have shown that the model was accurate with less than 5% of relative errors.

1. Introduction

In recent years, climate change and greenhouse gas emissions have become the major concerns of the aviation industry [1]. The impact of aviation on the environment is due to the fact that aircraft engines must consume a large amount of fuel in order to propel them. According to the U.S. Department of Transportation, more than 11 billion gallons of jet fuel was consumed by US carriers in 2016 (domestic operation only), which represented an increase of 3.97% compared to 2015. Since the number of passengers travelling around the world is expected to increase from 3.3 billion passengers in 2015 to 7.3 billion by 2034, aircraft fuel consumption is expected to continue to grow [2]. Faced with these statistics, the aerospace industry has recognized the need to meet the challenge of climate change, and has set a goal of reducing its carbon footprint by 50% by 2050 compared to the level recorded in 2005 [3,4].

In parallel to environmental motivations, there is also an economic motivation. According to several statistics, airlines spend more than 23% of their overall budget on fuel [5]. Given the unpredictable fluctuations in fuel prices, any strategy to reduce aircraft fuel consumption could therefore be a competitive advantage for an airline. In addition, by reducing the fuel consumption of their aircraft, airlines are helping to reduce the share of emissions from the aviation sector, creating a “win-win” scenario.

To address this dual ecological and economic challenge, universities and industry are conducting extensive research to provide solutions that can reduce aircraft fuel consumption and associated emissions. Some notable examples of promising solutions include the development of more efficient engine [6,7], the use of light material to reduce aircraft weight [8,9], and the design of new wing shapes to improve aerodynamic efficiency [10-14].

Flight trajectory optimization has been also identified as a very promising solution [15-18]. Indeed, Jensen *et al.* [19,20] analysed 217,000 flights within the U.S. domestic airspace and concluded that an average fuel savings of 1.93% could be obtained by optimizing the aircraft cruise speeds. In the same context, Félix Patrón *et al.* in [21] showed that selecting the optimal altitude/speed combination in cruise could improve the overall fuel efficiency of a flight and reduce flight costs (including fuel-related costs) by 2.57%. In another study, Turgut *et al.* [22] analyzed the fuel flow for different flights in Turkey and concluded that fuel consumption could be reduced by improving aircraft trajectories.

To calculate the optimum altitude and speed that the aircraft must fly in cruise, a typical flight planning system such as the Flight Management System (FMS) requires a mathematical model of the aircraft [23]. Such a model is essential to predict the trajectory of the aircraft, but also to estimate the amount of fuel required to perform the flight [16]. Today,

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the most widely accepted model for aircraft performance and trajectory analyses is the point-mass. This simplified mathematical model describes the forces acting on the aircraft, and it consequently requires a very good knowledge of the aero-propulsive characteristics of the aircraft/engine (i.e. thrust, lift, drag, etc.). Unfortunately, because of the high competitiveness of the aviation sector, these data are rarely available to researchers. Although it exists alternatives such as the Base of Aircraft Data (BADA) or the Global Aircraft Modeling Environment (GAME) [24], the use of these models is limited because of strict license agreements [25]. Moreover, as point out by several authors [16,24], the use of a mathematical model based on a point-mass representation is not practical for trajectory optimization applications because of the time required to evaluate the performance of the aircraft for given operating conditions. It is therefore of interest to develop a modelling technique to design a simple and reliable aircraft performance model with a minimum access to aircraft information, while retaining a good level of accuracy.

From this perspective, the objective of this study is to present a complete methodology to identify a performance model of an aircraft for the cruise phase using data published in aircraft flight manuals. The methodology was applied to the well-known Cessna Citation X business aircraft, for which the Flight Crew Operating Manual (FCOM) and a level-D Research Aircraft Flight Simulator (RAFS) available. According to the FAA (AC 120-40B), the level-D corresponds to the highest level for the flight dynamics and engine performance. The originality of this study lies in the fact that the model has been identified without any prior knowledge of engine and aerodynamic data.

Figure 1: Cessna Citation X Level-D Flight Simulator

2. Aircraft Mathematical Model and Background

The development of a model to predict the performance of an aircraft begins with the definition of a set of mathematical equations needed to describe its behavior. Within this context, the objective of this section is to present the theory and fundamental equations required to describe the flight characteristics of an aircraft in cruise. The section begins with the development of the equations of motion for the cruise phase. These equations are next supplemented with fundamental relationships required to quantify the aerodynamics of the aircraft. Finally, the main relationships used to describe the engine characteristics are presented.

2.1 Equations of Motion of Cruising Flight

Figure 2 schematically illustrates the external forces acting on an aircraft. As this figure shows, these forces can be grouped into three main components: aerodynamic, propulsive, and gravitational. By definition, the lift and drag, denoted by L and D respectively are the aerodynamic force components. The thrust denoted by F_n, is the net propulsive force produced by the two engines. Finally, the weight of the aircraft W is the gravitational component.

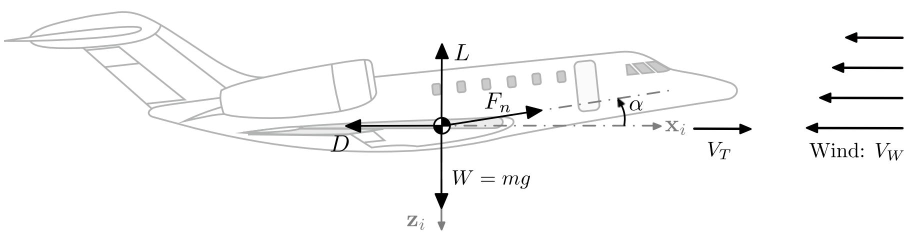

Figure 2: Forces Acting on the Aircraft in Cruise

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Since the cruise is a particular flight phase where the aircraft is supposed to fly at constant altitude and constant speed, there several assumptions that can be considered to simplify the aircraft mathematical model. In general, the angle of attack (denoted by α in Fig. 2) during cruise is very small in order to minimize drag, and prevent the aircraft from stalling. Therefore, it can be assumed that the thrust direction is aligned with the direction of flight. In addition, the aircraft is assumed to fly in an atmospheric wind field comprising only an horizontal component that is altitude-dependent. This fact implies that the aircraft is symmetric and that there is not drift angle. Finally, the atmosphere is assumed to be standard with temperature offsets (ΔISA), and its parameters (i.e., temperature, pressure, and density) are only functions of altitude.

Thus, by considering all these assumptions, and by obtaining the forces along and perpendicular to the flight path, the pertinent equations describing the motion of the aircraft in cruise can be stated as follows,

$$F_n - D = 0 \tag{1}$$

$$L - W = 0 \tag{2}$$

$$\dot{x} = V_T + V_W \cos\left(\psi_{AC/W}\right) \tag{3}$$

where $\dot{x}$ is the ground speed, V_T is the True Airspeed (TAS), V_W is the horizontal wind speed magnitude, and $\psi_{AC/W}$ is the wind direction relative to the aircraft (i.e., $\psi_{AC/W} = 0°$ corresponds to a tail wind, and $\psi_{AC/W} = 180°$ corresponds to a head wind).

2.2 Lift and Drag Fundamental Relationships

The lift and drag forces in Eqs. (1) and (2) constitute the two components of the aerodynamic force resultant acting on the aircraft. A more conventional and practical way of representing these two forces is to express their variations as a function of non-dimensional coefficients such as,

$$D = 1/2\,\rho V_T^2 S CD_s \tag{4}$$

$$L = 1/2\,\rho V_T^2 S CL_s \tag{5}$$

where ρ is the air density, S is the wind reference area and CL_s and CD_s are the lift and drag aerodynamic coefficients, respectively.

For most commercial aircraft, the lift and drag coefficients are related by a fundamental equation, called the *drag polar equation*. This equation reflects in a certain way the aerodynamics efficiency (or characteristics) of an aircraft, and is extensively used in aircraft performance analysis. Mathematically, the drag polar equation can be represented in a form of functional relationship as follows,

$$CD_s = \mathrm{f}(CL_s, M) \tag{6}$$

where M is the Mach number. It should be noted that the notation $\mathrm{f}(x, y)$ in the above equation is used in this context to simplify the general notation "a function of x and y".

2.3 Engine Parameter Fundamental Relationships

To complete the aircraft model, additional mathematical definitions for describing the characteristics of an engine are required. In general, for the study of aircraft performance, the desired engine characteristics are the thrust and the fuel flow. The former is used to predict the motion of the aircraft, while the latter is used to estimate the amount of fuel required to perform a specific maneuver (or mission).

According to various textbooks on engine performance [26-28], and as explained by the authors in previous studies [29,30], the thrust and fuel flow of a typical turbofan when expressed in corrected form, can be described by the following functional relationships,

$$F_n/\delta = \mathrm{f}\left(N/\sqrt{\theta}\,, M\right) \tag{7}$$

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$$W_f/(\delta\sqrt{\theta}) = \mathrm{f}(N/\sqrt{\theta}\,, M) \tag{8}$$

where δ is the pressure ratio, θ is the ambient temperature ratio, $N_c \equiv N/\sqrt{\theta}$ is the corrected engine rotational speed, and $F_{n,c} \equiv F_n/\delta$ and $W_{f,c} \equiv W_f/(\delta\sqrt{\theta})$ are the corrected thrust and corrected fuel flow, respectively.

3. Methodology: Identification of a Cruise Performance Model

Now that the main equations describing the aircraft flight characteristics have been introduced, the methodology developed in this study to determine a cruise performance model for the Cessna Citation X can be presented. For this purpose, this section begins with a brief description of the reference documentation used to gather information on the performance of the aircraft. The section then continues with several mathematical developments that were elaborated in order to determine a simplified model to estimate the performance of the aircraft in cruise.

3.1 Aircraft Performance Data Collection

The reference data used in this study to identify the fuel flow model for the Cessna Citation X was collected from the Flight Crew Operating Manual (FCOM). The FCOM is a comprehensive manual produced by the aircraft manufacturer to help pilots in evaluating the performance of the aircraft for various flight procedures. Among the data published in this document, the ones that remain by far the most complete and detailed are the performance data corresponding to the cruise phase. Table 1 shows an example of typical cruise performance data published in the FCOM of the Cessna Citation X. Note that the values presented in this table have been modified due to confidentiality concerns. Similarly, the table structure has been arranged to show only the essential information.

Table 1: Example of Cruise Performance Data published in the Cessna Citation X FCOM

Aircraft Weight	**Operating Conditions** 6 400 m / ISA+10°C					
12 000 kg	Mach	0.58$^{(1)}$	0.57	0.63	0.70	0.76$^{(2)}$
	W_f (kg/h)	901	956	990	1 025	1 079
12 500 kg	Mach	0.52$^{(1)}$	0.59	0.66	0.73	0.79$^{(2)}$
	W_f (kg/h)	771	946	1 109	1 327	1 606
⋮	⋮	⋮	⋮	⋮	⋮	⋮
16 000 kg	Mach	0.78$^{(1)}$	0.80	0.82	0.83	0.84$^{(2)}$
	W_f (kg/h)	638	663	691	726	733

The performance data shown in Table 1 specifies for a given altitude and temperature condition the fuel flow required to operate the aircraft in cruise at various combinations of weights and Mach numbers. The different Mach numbers presented in this table correspond to five thrust levels between the maximum cruise thrust (indexed by the number 1), and the maximum range thrust (indexed by the number 2). According to the description provided in the FCOM, the first one represents the thrust for which the aircraft flight speed is the highest, while the second one corresponds to the thrust for which the fuel flow is the lowest. Such a variation of the operating conditions allows therefore, to have a very good overview of the aircraft performance over its flight envelope in cruise.

All the data contained in Table 1 were manually recopied into an Excel file. This process was repeated for all the 21 tables available in the FCOM (a table corresponds to different altitudes varying from 1 524 m [5 000 ft] to 15 544 m [51 000 ft]). Finally, the collected data was then imported into Matlab® for its reorganization, and to create an aircraft performance database describing the fuel flow of the Cessna Citation X over a wide range of operating conditions.

3.2 Simplified Performance Model in Cruise

The set of fundamental equations (1) to (8) are used to describe, and to represent the performance of an aircraft in cruise. However, identifying a performance model based on these equations would require access to thrust, drag and

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fuel flow data. Unfortunately, such information is not available in the FCOM. In addition, it worth noticing that using these equations to compute aircraft performance in a device with limited processing power, such as the Flight Management System (FMS), might be time consuming. It is therefore interesting to design a mathematical model that is simpler and easier to use, and above all that can be easily identified from the limited data available in the FCOM.

Starting from Eq. (1), and the definition of the drag force in Eq. (2), the following equation can be written,

$$F_n = D = 1/2\, \rho S V_T^2 CD_s \tag{9}$$

Then, by noting that the density of the air can be expressed as function of δ and θ such as,

$$\rho = (\delta/\theta)\rho_0 \tag{10}$$

where ρ_0 is the air density at mean sea level, Eq. (9) can be rewritten as follows,

$$F_n = D = 1/2\, (\delta/\theta)\rho_0 S V_T^2 CD_s \tag{11}$$

Recalling the relationship between the True Airspeed and Mach number,

$$V_T = a_0\sqrt{\theta}M \tag{12}$$

where a_0 is the speed of sound at mean sea level, Eq. (11) becomes,

$$F_n/\delta = 1/2\, \rho_0 a_0^2 S M^2 CD_s \tag{13}$$

By substituting the drag coefficient in the above equation with the drag polar in Eq. (6), Eq. (13) leads to,

$$F_n/\delta = 1/2\, \rho_0 a_0^2 S M^2 \mathrm{f}(CL_s, M) \tag{14}$$

By following a similar analysis, it can be demonstrated that Eq. (2) can be rearranged as follows,

$$W/\delta = 1/2\, \rho_0 a_0^2 S M^2 CL_s \tag{15}$$

Thus, by extracting the lift coefficient in Eq. (15), and by substituting it into Eq. (14), the following functional relationship can be obtained:

$$F_n/\delta = \mathrm{f}(W/\delta\, , M) \tag{16}$$

To finish the mathematical development, it remains to relate the result given in Eq. (16) to the “fuel flow”. This development can be done by combining Eq. (7) and Eq. (8), and by eliminating the parameter $N/\sqrt{\theta}$ to yield,

$$W_f/\left(\delta\sqrt{\theta}\right) = \mathrm{f}(F_n/\delta\, , M) \tag{17}$$

Finally, by substituting F_n/δ from Eq. (17) into Eq. (16), the following mathematical expression can be obtained,

$$W_f/\left(\delta\sqrt{\theta}\right) = \mathrm{f}(W/\delta\, , M) \tag{18}$$

The functional relationship in Eq. (18) between the corrected fuel flow rate, the corrected gross weight and the Mach number makes it possible to combine Eqs. (1) to (8) of the initial model into a single simpler three-variable equation. In addition, the only parameters explicitly included in this equation being the fuel flow, the gross weight and the Mach number, it is then possible to easily obtain a model from the data published in the FCOM.

3.3 Identification of a Mathematical Model for the Corrected Fuel Flow

The fundamental result in Eq. (18) was combined with the information collected in the FCOM in order to obtain a set of data describing the variation of the corrected fuel flow as a function of the corrected gross weight and the Mach number. This dataset was next partitioned into two subsets: identification and validation. The “identification” subset was used to identify a parametric model that best describe the trend of the data. The “validation” subset, meanwhile,

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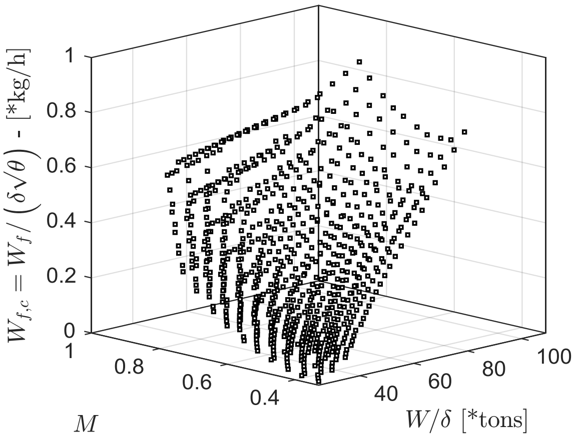
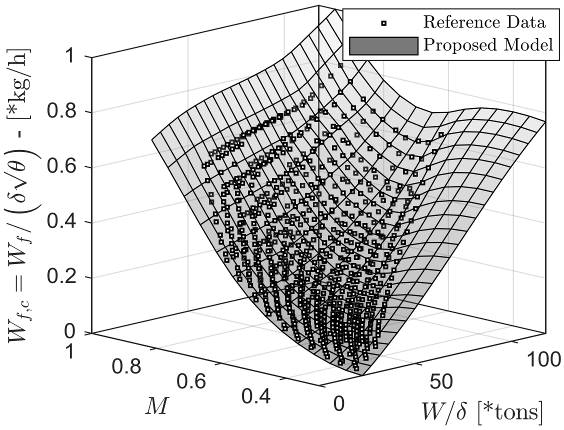
was used to evaluate the validity of the identified model. Separation between the identification and validation subsets was performed based on the Fisher-Yates shuffle algorithm in a proportion of 50% to 50%.

Figure 3(a) shows the corrected fuel flow as function of the corrected gross weight, and Mach number. Note that the data presented in this figure has been normalized between 0 and 1 for confidentiality reasons.



a) Corrected Fuel Flow Data



b) Illustration of the Proposed Model

Figure 3: Identification Results for the Corrected Fuel Flow

By analyzing the distribution of data in Fig. 3(a), it was found that the corrected fuel flow could be approximated by a polynomial structure of the following form,

$$W_{f,c} = p_{00} + p_{10}(W/\delta) + p_{01}M + \cdots + p_{nm}(W/\delta)^n M^m \tag{19}$$

where $\mathbf{p} = \{p_{00}, p_{01}, \dots, p_{nm}\}$ are coefficients that must be determined, and $\{n, m\}$ are the orders (i.e., degrees) of the polynomial with respect to the corrected gross weight and the Mach number.

The strategy adopted in this study to find the "optimal" polynomial structure that best fits the data while having a relatively low degree was based on a forward elimination procedure. This approach consisted of successively adjust the model in Eq. (19) by increasing the degree of the polynomial and evaluate the accuracy of the model at each step. For each combination of parameters $\{n, m\}$, the set of coefficients $\{p_{00}, p_{01}, \dots, p_{nm}\}$ was estimated using a least-squares fitting technique. The quality of the model was evaluated on the basis of the Sum of the Square Error (SSE) and the Root Mean Square Error (RMSE). Table 2 shows the results obtained for all polynomials tested.

Table 2: SSE and RMSE obtained in Identification and Validation

	Identification (50%)		**Validation (50%)**	
Polynomial Structure	SSE	RMSE	SSE	RMSE
$\#1 \to [n = 2, m = 2]$	1.3390×10^7	135.5	1.3643×10^7	132.2
$\#2 \to [n = 2, m = 3]$	7.4455×10^6	100.1	7.4455×10^6	102.6
$\#3 \to [n = 3, m = 2]$	7.2674×10^6	97.08	6.6462×10^6	92.30
$\#4 \to [n = 3, m = 3]$	7.3153×10^6	97.47	6.5124×10^6	91.37
$\#5 \to [n = 3, m = 4]$	3.3686×10^6	66.32	2.9845×10^6	61.86
$\#6 \to [n = 4, m = 3]$	3.3738×10^6	66.36	3.0299×10^6	70.26
$\#7 \to [n = 4, m = 4]$	3.2642×10^6	65.32	7.4455×10^6	62.32

As shown in Table 2, the most suitable model is the polynomial #5 with a degree 3 for the corrected gross weight, and a degree 4 for the Mach number. This model gives the lowest SSE and RMSE the in validation process. In addition, it is interesting to note that increasing the polynomial order does not necessarily lead to better results. Indeed, the value of the SSE and the RMSE in validation are higher for the polynomials #6 and #7 than those of the polynomial #5. This is the reason why the polynomial #5 has been selected.

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Finally, the polynomial #5 was transformed into a 2-D lookup table. A considerable advantage of expressing the model data in this form is that it allows calculating the corrected fuel flow without performing complex mathematical operations. For an avionics system such as the FMS, the savings in terms of processing time can be significant, as interpolating a value from a lookup table is often faster than using mathematical equations. An illustration of the lookup table model is illustrated in Fig. 3(b) for the convenience of the reader.

3.4 Aircraft Cruise Performance Prediction

Now that a prediction model of the fuel flow of the aircraft has been determined, it may be interesting to use it in order to deduce several parameters characterizing the performance of the aircraft in cruise.

3.4.1 Aircraft Fuel Consumption

One of the most important parameters required to analyze the performance of an aircraft in cruise is obviously the fuel consumption. Basically, the amount of fuel ΔF_B required to travel a given distance Δx can be expressed according to the following equation,

$$\Delta F_B = \frac{W_f \times \Delta x}{\dot{x}} = \frac{W_f \times \Delta x}{V_T + V_W \cos\left(\psi_{AC/W}\right)} \tag{21}$$

Recalling the expression of the Mach number $V_T = a_0\sqrt{\theta}M$, the above equation can be rewritten as follows,

$$\Delta F_B = \frac{W_f \times \Delta x}{a_0\sqrt{\theta}M + a_0\sqrt{\theta}M_W \cos\left(\psi_{AC/W}\right)} = \frac{W_f \times \Delta x}{a_0\sqrt{\theta}\left[M + M_W \cos\left(\psi_{AC/W}\right)\right]} \tag{22}$$

where M_W is the Mach number associated to the wind speed.

Then, the division of both sides of Eq. (22) by the pressure ratio δ leads to,

$$\frac{\Delta F_B}{\delta} = \frac{W_f \times \Delta x}{a_0\delta\sqrt{\theta}\left[M + M_W \cos\left(\psi_{AC/W}\right)\right]} \tag{23}$$

and by noting that $W_f/\left(\delta\sqrt{\theta}\right) = W_{f,c}$, Eq. (23) can be rearranged as follows,

$$\Delta F_B = \frac{\delta \times W_{f,c} \times \Delta x}{a_0\left[M + M_W \cos\left(\psi_{AC/W}\right)\right]} \tag{24}$$

It is important to mention that the result shown in Eq. (24) remains valid only for relatively small distances for which the variation of aircraft weight can be neglected. According to several studies, this approximation remains acceptable for distances of less than 46 km (25 nmi) [16]. Otherwise, it is necessary to use an iterative process as in the one shown in Algorithm 1 to improve the accuracy of calculations.

Algorithm 1. Aircraft Fuel Consumption Estimation

0. **Initialization –** Set the aircraft initial conditions: mass $m_{[1]}$ and weight $W_{[1]}$, and set the fuel burned $F_{B[1]}$ to zero.

1. **For the altitude h and ΔISA condition, compute:** δ, θ, M_w and $\psi_{AC/W}$.

2. **Divide the total distance x_r into N sub-segments Δx = 43 600 m (25 nmi).**

3. **For** i from 1 to N
    a) Compute the corrected fuel flow: $W_{f,c} = f\left(W_{[i]}/\delta\,, M\right)$,
    b) Compute the amount of fuel burned: $F_{B[i+1]} = F_{B[i]} + \Delta F_B$,
    c) Update the aircraft mass: $m_{[i+1]} = m_{[i]} - \Delta F_B$.

   End For

3. **Return the total fuel burned F_B**

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3.4.2 Aircraft Specific Range

Another parameter that is also of interest when analysing the cruise performance of an aircraft is the *Specific Range* (SR). Also known as *fuel mileage*, the SR is an instantaneous measure of the aircraft fuel efficiency in cruise. This parameter quantifies the distance (or range) that the aircraft can travel for a given fuel quantity. It is typically expressed in nautical mile per kilogram of fuel (i.e., nmi/kg). Mathematically, the SR for a given gross weight, altitude, and speed can be determined according to the following equation,

$$\mathrm{SR} = \frac{\dot{x}}{W_f} = \frac{V_T + V_W \cos\left(\psi_{AC/W}\right)}{W_f} \tag{25}$$

Similarly to the fuel consumption estimation, it is interesting to express the specific range directly as function of the corrected fuel flow. For this purpose, Eq. (25) is rewritten as follows,

$$\mathrm{SR} = \frac{a_0\sqrt{\theta}M + a_0\sqrt{\theta}M_W \cos\left(\psi_{AC/W}\right)}{W_f} = \frac{a_0\sqrt{\theta} \times \left[M + M_W \cos\left(\psi_{AC/W}\right)\right]}{W_f} \tag{26}$$

Then, multiplying both side of Eq. (26) by the pressure ratio δ leads to,

$$\mathrm{SR} \times \delta = \frac{a_0\delta\sqrt{\theta} \times \left[M + M_W \cos\left(\psi_{AC/W}\right)\right]}{W_f} \tag{27}$$

Finally, rearranging Eq. (27), it can be demonstrated that,

$$\mathrm{SR_c} \equiv \frac{\mathrm{SR} \times \delta}{a_0} = \frac{\left[M + M_W \cos\left(\psi_{AC/W}\right)\right]}{W_{f,c}} \tag{28}$$

where $\mathrm{SR_c}$ is the corrected specific range. It should be noted that since the pressure ratio δ is constant for a given altitude, the SR and $\mathrm{SR_c}$ have similar characteristics. This means that the locus of the maximum values of the $\mathrm{SR_c}$ is coincident with the locus of the maximum values of the SR. Therefore, the $\mathrm{SR_c}$ can be used to find the speed in cruise that results in the greatest SR, also known as the Maximum-Range Cruise (MRC) speed.

3.4.3 Flight Cost and Economic Speed

In practice, aircraft do not necessary fly at the MRC speed. Indeed, the selection of the speed in cruise depends mainly on the economic strategy of the airlines [26]. For example, flying at a relatively low Mach number (i.e., low speed) reduces fuel consumption, but increases flight time. Conversely, flying at a relatively high Mach number (i.e., high speed) reduces flight time, but increases fuel consumption. To solve this dilemma, it is necessary to find a compromise between these two extreme cases, and to select the optimal flight speed that minimizes the overall flight cost.

The Mach number that results in the lowest flight cost is called the *economic speed*. This speed, usually abbreviated as the ECON speed, can be determined by minimizing the following Cost Function (CF),

$$\mathrm{CF} = \frac{W_f + 60 \times \mathrm{CI}}{V_T + V_W \cos\left(\psi_{AC/W}\right)} \tag{29}$$

where CI is the Cost Index. This parameter is a constant value that quantifies the compromise between minimizing the fuel and time costs. A high CI gives priority to the flight time without considering the fuel-related costs, while a small CI gives priority to the fuel-related costs without considering the time-related costs. Typical CI values can range from 0 to 99 or 0 to 999 depending on the FMS manufacturer [31].

In the same way as the previous analyses, it may be interesting to express the CF as function of the corrected fuel flow. For this purpose, Eq. (29) is rewritten as follows,

$$\mathrm{CF} = \frac{W_f + 60 \times \mathrm{CI}}{\delta a_0\sqrt{\theta}M + a_0\sqrt{\theta}\cos\left(\psi_{AC/W}\right)} = \frac{W_f + 60 \times \mathrm{CI}}{a_0\sqrt{\theta}\left[M + M_W \cos\left(\psi_{AC/W}\right)\right]} \tag{30}$$

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Then, by dividing both sides of Eq. (30) by the pressure ratio δ leads to,

$$\frac{a_0 \times \mathrm{CF}}{\delta} = \frac{W_f + 60 \times \mathrm{CI}}{\delta\sqrt{\theta}\left[M + M_W \cos\left(\psi_{AC/W}\right)\right]} \tag{31}$$

Finally, by introducing the corrected cost index $\mathrm{CI_c} \equiv \mathrm{CI}/\left(\delta\sqrt{\theta}\right)$ definition, Eq. (31) can be written in its corrected form as follows,

$$\mathrm{CF_c} \equiv \frac{a_0 \times \mathrm{CF}}{\delta} = \frac{W_{f,c} + 60 \times \mathrm{CI_c}}{M + M_W \cos\left(\psi_{AC/W}\right)} \tag{32}$$

where $\mathrm{CF_c}$ is the corrected cost function.

Once again, it is interesting to note that at a given altitude (i.e., δ = constant), CF and $\mathrm{CF_c}$ are proportional each other. As a result, the ECON speed can be determined by minimizing $\mathrm{CF_c}$. In this paper, the minimization of the corrected cost function was performed using the Golden-Search technique shown in Algorithm 2. This algorithm has been chosen among many others for its ease of use, but also because it is a free derivative algorithm. In addition, the implementation of the Golden-Search algorithm to optimize the flight path of an aircraft in previous studies conducted by LARCASE researchers has yielded excellent results.

Algorithm 2. Golden-Search Method for the Minimization of the Cost Function

0. **Initialization –** Set altitude h, aircraft weight W, wind conditions V_w and $\psi_{AC/W}$, temperature ΔISA, and cost index CI. Set the initial interval $[M^{\min}, M^{\max}]$, and the golden ratio $\Phi = \left(\sqrt{5} - 1\right)/2$.

1. **For the current altitude *h*, compute:** δ, θ, M_w, W_c and $\mathrm{CI_c}$.

1. **Compute initial parameters:**
   a) $M_1 = M^{\max} - (M^{\max} - M^{\min}) \times \Phi$ | c) $\mathrm{CF}_{c,1} = \mathrm{CF_c}(M_1)$
   b) $M_2 = M^{\min} + (M^{\max} - M^{\min}) \times \Phi$ | d) $\mathrm{CF}_{c,2} = \mathrm{CF_c}(M_2)$

2. **While** $M^{\max} - M^{\min} \geq 0.001$ **do**
   If $CF_{c,1} \leq CF_{c,2}$
   a) $M^{\max} = M_2$ and $M_2 = M_1$ | c) $\mathrm{CF}_{c,2} = \mathrm{CF}_{c1}$
   b) $M_1 = M^{\max} - (M^{\max} - M^{\min}) \times \Phi$ | d) $\mathrm{CF}_{c,1} = \mathrm{CF_c}(M_1)$
   Else
   a) $M^{\min} = M_1$ and $M_1 = M_2$ | c) $\mathrm{CF}_{c,1} = \mathrm{CF}_{c2}$
   b) $M_2 = M^{\min} + (M^{\max} - M^{\min}) \times \Phi$ | d) $\mathrm{CF}_{c,2} = \mathrm{CF_c}(M_2)$
   End If
   End While

3. **Return the ECON Mach:** $M^{\mathrm{ECON}} = (M^{\max} + M^{\min})/2$

4. Results and Validation of the Methodology

The last section of this paper presents the methodology validation results. For this purpose, three different analyses were conducted. The first analysis consisted in evaluating the accuracy of the fuel flow model identified in **Section 3.3**. The second analysis was conducted to verify the accuracy of the model in predicting the SR and MRC over a wide range of flight conditions. Finally, the third analysis aimed to validate the ECON speed estimation method proposed in **Section 3.4.3**.

4.1 Fuel Flow Model Validation

The first analysis consisted in evaluating the accuracy of the corrected fuel flow model. To this end, a series of flight tests was conducted with the Cessna Citation X RAFS available at the LARCASE. These flight tests aimed to measure the aircraft fuel flow in cruise for various flight conditions and aircraft configurations. As a basis for comparison and validation of the model, 110 flight conditions were selected within the flight envelope of the aircraft. These conditions

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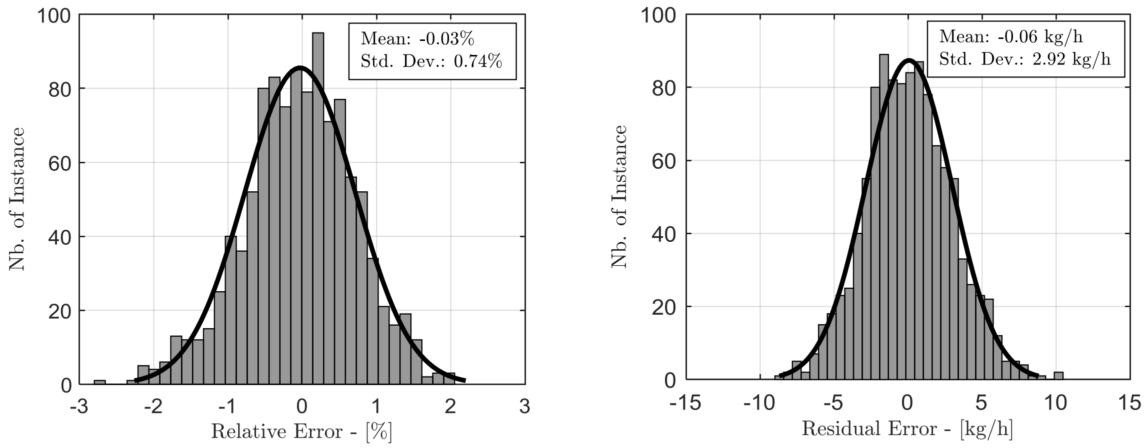
were determined by varying the altitude from 7 620 m (25 000 ft) to 13 716 m (45 000 ft) with an increment of 609 m (2 000 ft), and by selecting 20 different Mach numbers for each altitude. In addition, the flight tests were reproduced for five aircraft weight configurations. The combination of all these parameters led to a total of 1 100 flight tests.

For each flight, the fuel flow of both engines was sampled every 30 seconds over a period of five minutes. The instantaneous fuel flow was then estimated by calculating the average value over the ten sampled data. In parallel, the corrected fuel flow model was used to compute the fuel flow for the same flight conditions. The criterion established to validate the model was that the predicted and measured fuel flow agree within 5%. This criterion was determined based on information provided in the manual of criteria for the qualification of flight simulators (Section Performance – 4.1). The results comparison for all flight tests are presented in Fig. 4.



a) Fuel Flow Relative Errors b) Fuel Flow Residual Errors

Figure 4: Fuel Flow Errors Distribution over 1 100 Flight Tests

From a general point of view, the results show that there is a very good agreement between the fuel flow measured with the RAFS and that computed by the model. Indeed, it can be seen from Fig. 4(a) that the fuel flow is well predicted with less than 3% of error. Regarding the residual errors, Fig. 4(b) shows that the maximum error obtained over the 1 100 flight tests was less than 10 kg/h. This means that the potential error in predicting fuel consumption for a four-hour cruise would be approximately 40 kg. Clearly, compared to the total fuel burned during a typical cruise, an error of 40 kg is very small, and can be considered negligible.

In the light of these very good results, it can be concluded that the model identified in this study reflects very well the aircraft fuel flow in cruise. These results also demonstrate that the data published in the FCOM is reliable enough to develop a mathematical model to predict the fuel flow of an aircraft in cruise.

4.2 Specific Range Analysis

The next analysis conducted in this study consisted in verifying if the model identified was reliable enough to predict the SR and MRC speed over a wide range of operating conditions. For this purpose, the fuel flow measured during the flight tests was used to compute the SR according to Eq. (25). The data was then approximated with a smoothing-spline to estimate the optimal SR, and to find the corresponding Mach number. In parallel, the SR parameter was calculated based on the result in Eq. (28), and by using the Golden-Search algorithm developed in **Section 3.4.3** with CI = 0.

Figure 5 shows an example of comparison results obtained for four altitudes and five weight configurations. On each graph shown in Fig. 5, the SR values computed with the RAFS data are represented by square markers, while the solid lines represent the SR values predicted by the model. Figure 5 also shows the Maximum-Range Cruise (MRC) and Long-Range Cruise (LRC) speeds for each altitude/weight combination. The LRC speed was calculated by finding the highest Mach number corresponding to 99% of the maximum SR.

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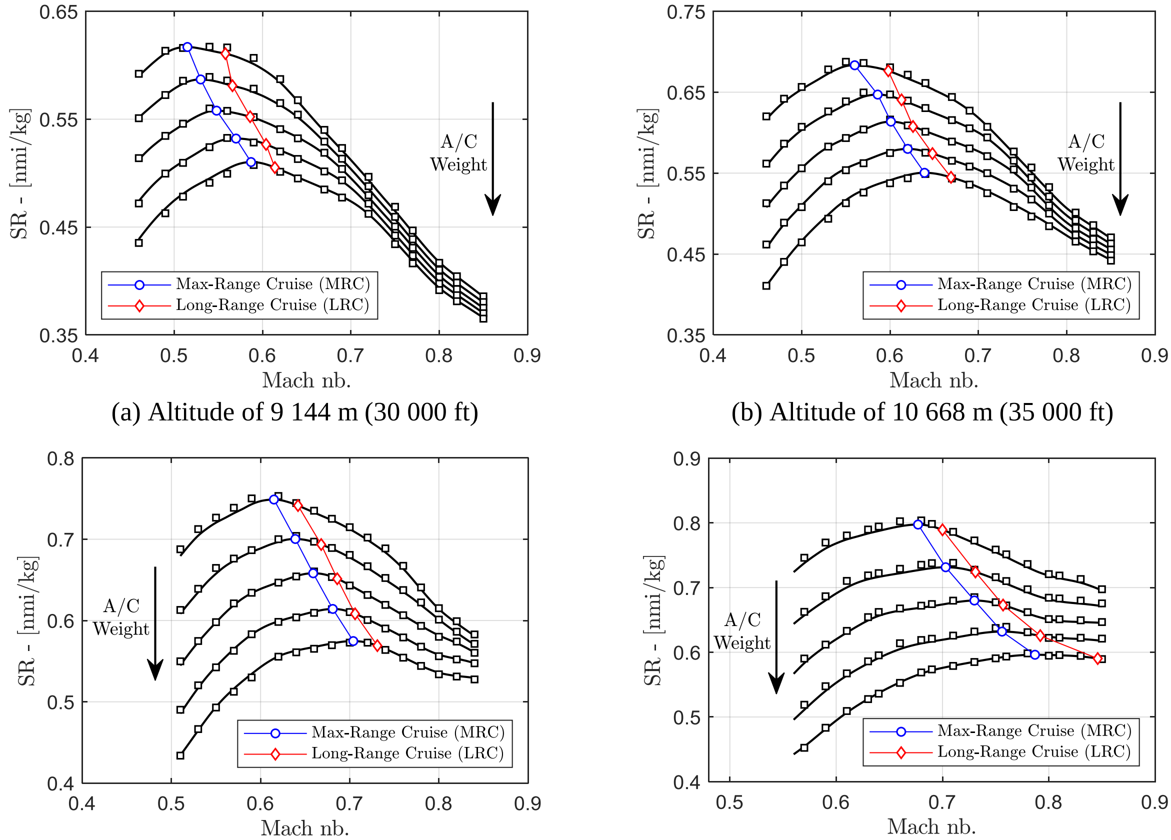


(a) Altitude of 9 144 m (30 000 ft)

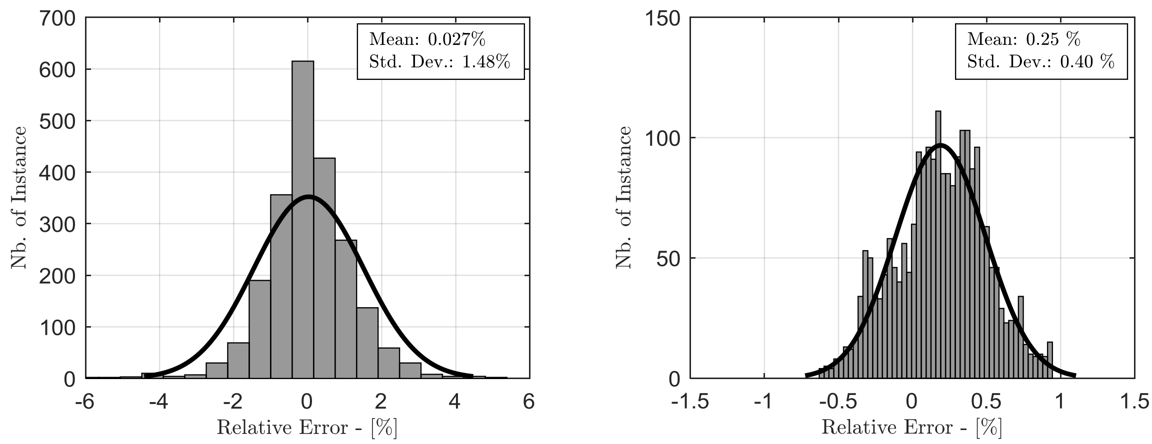
(b) Altitude of 10 668 m (35 000 ft)

(c) Altitude of 12 192 m (40 000 ft)

(d) Altitude of 13 716 m (45 000 ft)

Figure 5: Example of SR and MRC Comparison for two Altitudes

The comparisons illustrated in Fig. 5 were repeated for all altitude/weight combinations used for the validation of the fuel flow model in **Section 4.1**. In addition, different wind conditions were also considered by varying the wind speed from 0 to 100 m/s, and the wind direction from 0° to 180°. It worth noticing that since the wind does not influence the fuel flow in cruise, it was not necessary to realize additional flight tests with the RAFS. For the sake of simplicity, the calculation of the SR in presence of wind was done based on the fuel flow measured from the flight simulator in still air. Figure 6 shows the results comparison obtained for the estimation of the MRC speed.



a) MRC Speed Relative Errors

b) Maximum SR Relative Errors

Figure 6: Error Analysis for the MRC Speed Estimation

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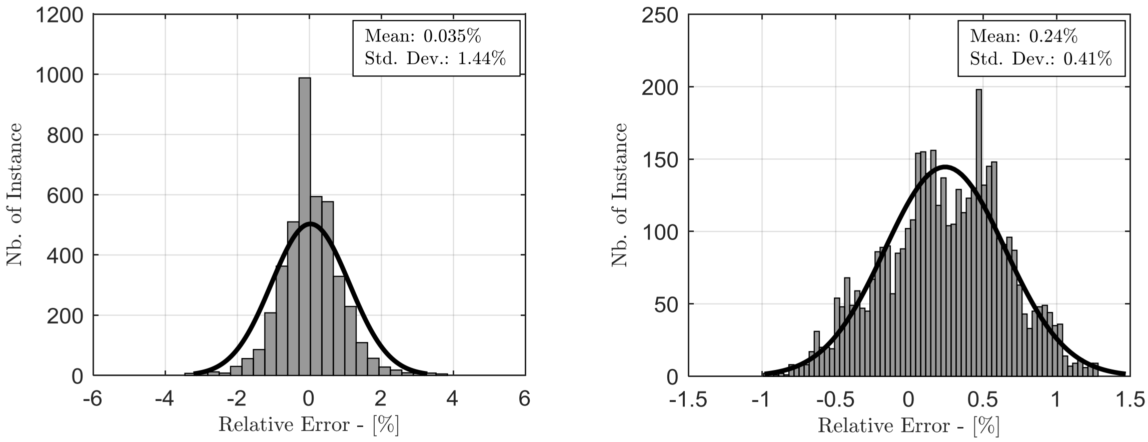
As it can be seen in Fig. 6(a), the MRC speed is well predicted with less than 4% error and with an average error of 0.027%. In terms of residual error, it was found that the maximum deviation was around Mach 0.04. Although such a difference may seem large, it can be explained by the shape of the SR curve. Indeed, as seen on Fig. 5(b), at high altitude and heavy weight, the SR curve exhibits a “flat shape” around its maximum. This characteristics makes it difficult to find the optimal value because several Mach numbers may have SR values that are almost identical. This observation was further corroborated by the results in Fig. 6(b), which show the distribution of the relative errors between the maximum SR measured with the flight simulator and the maximum SR predicted by the model. As seen, the relative error is less than 1%, which is clearly negligible.

4.3 Economic Speed Analysis

The final analysis aimed to evaluate the reliability of the model to predict the ECON speed over a wide range of operating conditions. For this purpose, the fuel flow measured during the flight tests was used to compute the cost function according to Eq. (29) considering different wind conditions and cost index values (from 0 to 99). Using the same technique as for the MRC speed, the ECON speed was then calculated by approximating the CF values using a smoothing spline and looking for the Mach number for which CF was the lowest. In parallel, the ECON speed for the same operating conditions was calculated using the Algorithm 2. The results comparison between ECON speeds measured with the RAFS data, and ECON speeds predicted by the model are shown in Fig. 8



a) ECON Speed Relative Errors   b) ECON Cost Relative Errors

Figure 7: Error Analysis for the ECON Speed Estimation

As can be seen in Fig. 8(a), the results obtained for the ECON speed are generally the same as those obtained previously for the MRC speed. Indeed, the maximum error for ECON speed estimation is smaller than 4%, with an average error of 0.035% and a standard deviation of around 1.44%. Similarly, in terms of residual error, it was observed that the maximum error was of the order of Mach 0.05. Again, this difference can be explained by the shape of the CF curve which has flat regions, especially at high altitudes. However, the maximum error on the ECON cost function is less than 1%, which remains acceptable.

5. Conclusion

In this paper, a complete modelling technique for determining the performance of an aircraft in cruise was presented. The methodology was applied to well-known business jet aircraft Cessna Citation X, for which a level-D Research Aircraft Flight Simulator (RAFS) was available.

Starting from available data published in the Flight Crew Operating Manual (FCOM), a model describing the fuel flow of the aircraft was firstly identified. This model was subsequently combined with several fundamental equations in order to propose different techniques to predict three performance parameters: fuel consumption, specific range, and flight costs. A golden search algorithm was also developed in order to estimate the economical speed of the aircraft in cruise. Validation of the study was done by comparing the predictions obtained from the model with performance data measured with the RAFS. More than 1 000 flight tests were performed in order to cover as much as possible the entire cruise envelope of the aircraft. From a general point of view, it was shown that the predicted performance matched the

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measured performance within 5%. The use of the modeling technique could therefore help aerospace researchers to develop their own cruise performance model from a limited amount of data.

The model presented in this study was limited to cruise phase at constant altitude. However, during the cruise, step climbs or descents (change of altitude) can be performed in order to reduce the aircraft fuel consumption. As a result, it would be interesting to complete the methodology and proposed a technique to determine the fuel required to perform step climbs or descents, and therefore allow a complete analysis of the cruise phase.

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