

Semi-analytical Method for Active Removal of Space Debris with Electrodynamics Tether System

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Abstract:

Growing the number of space debris has recently become a topic of eminent concern. Therefore, orbital debris removal has become a very grave issue for both of scientific and commercial space management in order to avoid loss any operational satellite. To prevent loss of spacecraft due to debris collision, it is very essential to address the aggregate risk which need an efficient way to remove or avoid collision with operating satellites.

This work develops a semi-analytical method for orbital decay using Electrodynamics tether system (EDT) as a technique of active debris removal to avoid collision with operational satellites. This efficient method can change the orbit of a satellite to be more elliptical in near polar orbits due to the higher-order perturbation of Earth's magnetic field and Lorentz force. Gaussian form of Lagrange Planetary equations is used to evaluate the orbital motion of EDT with environmental perturbations of electrodynamic force, aerodynamic drag and the effect of Earth's oblateness. Differential equations for the induced voltage-current across EDT are derived and solved with boundary conditions determined by mission objectives and duration of the deorbit. The change in electric current in the EDT due to its plasma environment and thermal conditions are considered. Analyses of different parameters of EDT dynamics with variations in the mass, type of the materials of the wire of tether, and the tether length were studied to classify the range of eccentricity of the elliptical orbit for possibility of deorbit the desired missions. Applied the current model of the EDT find that the orbit of a satellite deorbited will become elliptical in near polar orbits due to the higher-order perturbation Earth's magnetic field and Lorentz force. The effects of polarity on reverse of the induced voltage/current across EDT in near polar have been discussed. Comparison between air drag only EDT for orbital decaying time in case of equatorial and polar orbit are introduced.

Introduction:

Space debris has recently become a topic of elevated interest^[1] Currently, there are more than 17,000 space objects in the catalog of the United States Space Surveillance Network (SSN)^[2], with only about 1,200 being active satellites.^[3] A collision threat emanates from all the other, inactive a Space debris has recently become a topic of elevated interest^[1] currently, there are more than 17,000 space objects in the catalogue of the United States Space Surveillance Network (SSN)^[2], with only about 1,200 being active satellites.^[3] A collision threat emanates from all the other, inactive and uncontrollable objects: The space debris.^[1] The only natural self-cleaning mechanism in the near-Earth space is the residual atmospheric drag. However, the orbital lifetime of objects beyond 400 km is measured in decades or centuries. Therefore, it is likely that an object eventually collides with another object.^[1]

It is too late for simply stopping the littering, as it will not necessarily stop the debris population from growing. Moreover, launching heavily armored satellites is impractical for a variety of reasons: Shielding increases the systems' complexity, the launch masses, and the costs. Not all parts of a satellite can be shielded for technical reasons, and a lethal collision can never be ruled out. Besides, the additional shielding mass could end up as

additional debris when the satellite fails to perform PMD maneuvers.^[1] Finally, there are two options left: Actively clean up space, or be prepared to live with a high collision risk for spacecraft. There is a common understanding that natural resources should be used in a sustainable manner, which implicitly demands that near-Earth space should be preserved in a usable state at least. Conclusively, debris must be actively removed.^[1]

In this light, it does not surprise that the active removal of space debris has advanced to an essential part of the Security in Space within the last few years.^[1] The necessity and the effectiveness of active debris removal have been argued by a majority of researchers.^[4,5,6] However, the task of removing a piece of junk, regardless of its size, has unique characteristics: The target is uncooperative and, to a certain extent, unpredictable. There is a very high diversity in the object properties, such as size, shape, chemical composition, optical and electromagnetic properties, rotation velocity and orbit.^[1] A removal mission must not leave behind more debris than it removes. The legal requirements and implications are unclear, if not confusing, inconsistent or disputed.^[7,8,9] There is no political framework yet where such a mission could be performed. Finally, Removal concepts for larger junk usually require rendezvous operations with each targeted object. This is costly in terms of fuel and time, and therefore impractical when dealing with a high number of smaller objects.^[1]

The electrodynamic drag concept for deorbiting of LEO spacecraft is illustrated in Figure (1), conducting electrodynamic tether for use as a “Terminator Tether” for removing unwanted Low-Earth-Orbit (LEO) spacecraft from orbit at the end their useful lives.^[10,11] When a spacecraft fails, or has completed its mission and is no longer wanted, the Terminator Tether, weighing a small fraction of the mass of the host spacecraft, will be deployed. At both ends of the tether, a means of providing electrical contact with the ambient plasma will be provided to enable current to be transmitted to and from the ionospheric plasma.^[12] The electrodynamic interaction of the conducting tether moving at orbital speeds across the Earth’s magnetic field will induce current flow along the tether. The resulting energy loss from the heat generated by the current flowing through the ohmic resistance in the tether will remove energy from the spacecraft.^[12]

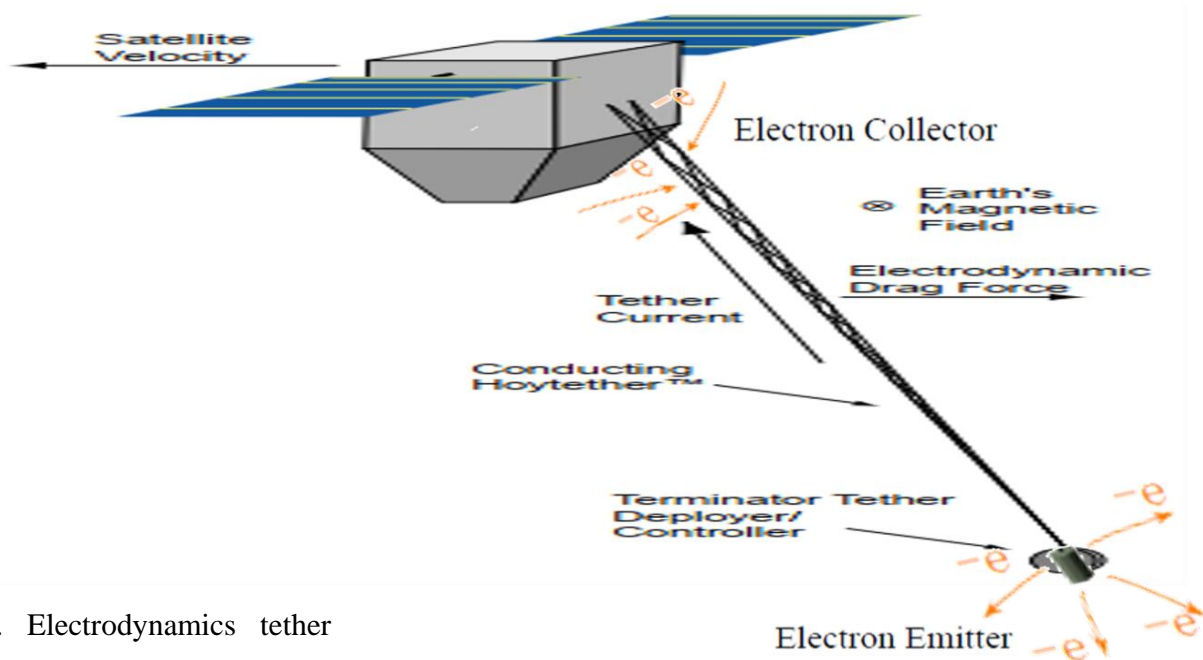


Fig 1. Electrodynamic tether system

Consequently, the orbital energy of the spacecraft will decay, causing it to deorbit far more rapidly than it due to atmospheric drag alone. Whereas a defunct spacecraft left in its orbit can take hundreds or thousands of years to deorbit due to atmospheric drag, a spacecraft with a Terminator Tether can be deorbited in weeks or months. The Terminator Tether thus is a low-mass means of reducing both the risk of spacecraft fratricide and the amount of orbital space debris that must be coped with in the future.^[12]

The basic effect of the electrodynamic force is changing the semi-major axis and eccentricity of the orbit secularly for elliptical orbit only, it depends upon two orbital elements, the semi-major axis (a) and the eccentricity (e), because it indicates the size and shape of the orbit. In the current work, we develop an analytical model for active debris removal using Electrostatic tether system for two different model. The main factor of the model depend on the length of the cable of the tether and the current produced. In addition the comparison between two models are considered concerning the size of debris, altitude, inclination a

Dynamical Model:

The first model:

Using (r, θ, φ) , where r the radius, θ the colatitudes angle and φ the azimuth are the spherical coordinates respectively.

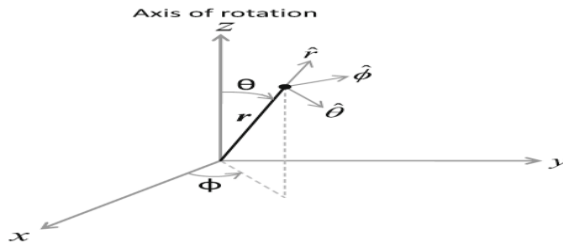


Figure (2): Inertial Cartesian and spherical coordinates

The Total Electrodynamic force per unit mass experienced by a charged particle with a charge q moving through the magnetic field of the Earth B vector, according to Streetman and Peck (2007a, 2007b), can be obtained due to the Magnetic field in inertial coordinates as^[14]:

$$\vec{F}_{EDT} = \frac{q}{m} l (\vec{v}_{rel} \times \vec{B}) \quad (1)$$

Where: v_{rel} The relative velocity of the satellite with respect to the magnetic field is given by^[15]

$$\vec{v}_{rel} = \vec{v} - \vec{\omega}_e \times \vec{r} = (\dot{r})\hat{r} + (r\dot{\theta})\hat{\theta} + (r\dot{\varphi} \sin \theta - r\omega_e \sin \theta)\hat{\varphi} \quad (2)$$

Where $\vec{\omega}_e$: is the Earth's angular velocity about its polar axis.^[15]

$$\vec{\omega}_e = \omega_e \hat{z} = \omega_e \cos \theta \hat{r} - \omega_e \sin \theta \hat{\theta} \quad (3)$$

Where: $\omega_e = 7.2921159 \times 10^{-5} \text{ rads}^{-1}$: average value of Earth's angular velocity.

$$\text{Since } \vec{v} = \dot{r}\hat{r} + r\dot{\theta}\hat{\theta} + r\dot{\varphi} \sin \theta \hat{\varphi} \quad (4)$$

$$\vec{r} = r\hat{r} \quad (5)$$

$$\hat{z} = \cos \theta \hat{r} - \sin \theta \hat{\theta} \quad (6)$$

The magnetic field B of a perfect dipole is (Griffiths 2004)^[15].

$$\vec{B} = \frac{B_0}{r^3} (2 \cos \theta \hat{r} + \sin \theta \hat{\theta}) \quad (7)$$

Where: $B_0 = 3.12 \times 10^{-5} T$: averaged value of the Earth's magnetic field at the Earth's surface on magnetic equator.

$$\vec{F}_{EDT} = \frac{q}{m} l (\vec{v}_{rel} \times \vec{B}) = \frac{q l B_0}{m r^2} [(\dot{\phi} - \omega_e)(-\sin^2 \theta \hat{r} + 2 \sin \theta \cos \theta \hat{\theta}) + \left(\frac{\dot{r}}{r} \sin \theta - 2\dot{\theta} \cos \theta\right) \hat{\phi}] \quad (8)$$

Now we going to calculate the Electrodynamics' Radial, Normal and Transverse component to use it in the Lagrange Planetary Equation.

The Radial component is calculated from:

$$F_R = \vec{F}_{EDT} \cdot \hat{r} = -\frac{q l B_0}{m r^2} (\dot{\phi} - \omega_e) \sin^2 \theta \quad (9)$$

And the Normal component is calculated from:

$$F_N = \vec{F}_{EDT} \cdot \hat{n} = \frac{q l B_0}{m \sqrt{\mu p}} \left[-2\dot{\phi}(\dot{\phi} - \omega_e) \sin^2 \theta \cos \theta + \frac{\dot{r}\dot{\theta}}{r} \sin \theta - 2\dot{\theta}^2 \cos \theta \right] \quad (10)$$

$$\text{Where: } \hat{n} = \hat{h} = \frac{(r \times v)}{\sqrt{\mu p}} \quad (11)$$

Where: $\mu = 398600.4415 \times 10^6 m^3 s^{-2}$: The Earth Gravitational Parameter.

Then the Transverse component is calculated from:

$$F_T = \vec{F}_{EDT} \cdot \hat{t} = \frac{q l B_0}{m \sqrt{\mu p}} \left[2\dot{\theta} \omega_e \sin \theta \cos \theta + \frac{\dot{r}\dot{\phi}}{r} \sin^2 \theta \right] \quad (12)$$

$$\text{Where: } \hat{t} = \hat{n} \times \hat{r} = \frac{r^2}{\sqrt{\mu p}} [(\dot{\theta})\hat{\theta} + (\dot{\phi} \sin \theta)\hat{\phi}] \quad (13)$$

Relationship between spherical coordinates and orbital elements

$$r = \frac{a(1-e^2)}{1+e \cos v} \quad (14)$$

$$\dot{r} = \sqrt{\frac{\mu}{a(1-e^2)}} e \sin v \quad (15)$$

$$\cos \theta = \sin i \sin(\omega + v) \quad (16)$$

$$\sin \theta = \sqrt{1 - \sin^2 i \sin^2(\omega + v)} \quad (17)$$

$$\dot{\theta} = -\sqrt{\frac{\mu}{a^3(1-e^2)^3}} \frac{\sin i \cos(\omega+v)}{\sqrt{1-\sin^2 i \cos^2(\omega+v)}} (1 + e \cos v)^2 \quad (18)$$

$$\dot{\phi} = \sqrt{\frac{\mu}{a^3(1-e^2)^3}} \frac{\cos i}{1-\sin^2 i \sin^2(\omega+v)} (1 + e \cos v)^2 \quad (19)$$

Using Lagrange Planetary Equations in Gauss form to analyze the time rates of orbital elements resulting in acceleration from the total EDT force:

$$\frac{da}{dt} = \frac{2}{n\sqrt{1-e^2}} \left[e \sin(\nu) F_R + \frac{p}{r} F_T \right] \quad (20)$$

$$\frac{de}{dt} = \frac{\sqrt{1-e^2}}{na} \left[\sin(\nu) F_R + \left(\cos(\nu) + \frac{e \cos(\nu)}{1+e \cos(\nu)} \right) F_T \right] \quad (21)$$

$$\frac{di}{dt} = \frac{r \cos(\omega+\nu)}{na^2\sqrt{1-e^2}} F_N \quad (22)$$

$$\frac{d\Omega}{dt} = \frac{r \sin(\omega+\nu)}{na^2\sqrt{1-e^2} \sin(i)} F_N \quad (23)$$

$$\frac{d\omega}{dt} = \frac{\sqrt{1-e^2}}{nae} \left[-\cos(\nu) F_R + \left(1 + \frac{1}{a(1-e^2)} \right) \sin(\nu) F_T \right] - \cos(i) \frac{d\Omega}{dt} \quad (24)$$

$$\frac{dM}{dt} = \frac{1-e^2}{nae} \left[-\left(\cos(\nu) - 2e \frac{r}{a(1-e^2)} \right) F_R + \left(1 + \frac{r}{a(1-e^2)} \right) \sin(\nu) F_T \right] - \cos(i) \frac{d\Omega}{dt} \quad (25)$$

The second model:

The orbital coordinate system is denoted as x, y and z . The EDT is modeled as two masses connected by a massless tether. Its attitude is defined by α and β .

The Electrodynamic force acting on the tether due to the magnetic field is given as ^[16,17]

$$\vec{F}_{EDT} = I(\vec{L} \times \vec{B}) \quad (26)$$

The direction vector of the tether in the orbital coordinate is ^[16,17]

$$\vec{L} = l \left((\cos \beta \cos \alpha) \hat{i} + (\cos \beta \sin \alpha) \hat{j} + (\sin \beta) \hat{k} \right) \quad (27)$$

The geomagnetic field vector in the orbital coordinate is ^[16,17]

$$\vec{B} = \frac{\mu}{r^3} \left((-2 \sin i \sin \nu) \hat{i} + \sin i \cos \nu \hat{j} + (\cos i) \hat{k} \right) \quad (28)$$

$$\begin{aligned} \vec{F}_{EDT} = I(\vec{L} \times \vec{B}) = I & \left[(\cos \beta \sin \alpha \cos i - \sin \beta \sin i \cos \nu) \hat{i} - (\cos \beta \cos \alpha \sin i \cos \nu + \right. \\ & \left. 2 \sin \beta \sin i \sin \nu) \hat{j} + (\cos \beta \cos \alpha \sin i \cos \nu + \right. \\ & \left. 2 \cos \beta \sin \alpha \sin i \sin \nu) \hat{k} \right] \end{aligned} \quad (29)$$

Results and discussion:

form the first model, If we have a tether system with an eccentricity, $e = 0.02$, semi-major axis, $a = 6878000$ meters, terminator tether length, $L = 5000$ meters, spacecraft mass $m = 100$ kg and the Electric current induced in tether $I = 5$ Amps by using MATLAB setup we can get result:

The effect of the Electrodynamics force on the orbital elements in one day shown in figure (2) & (3)

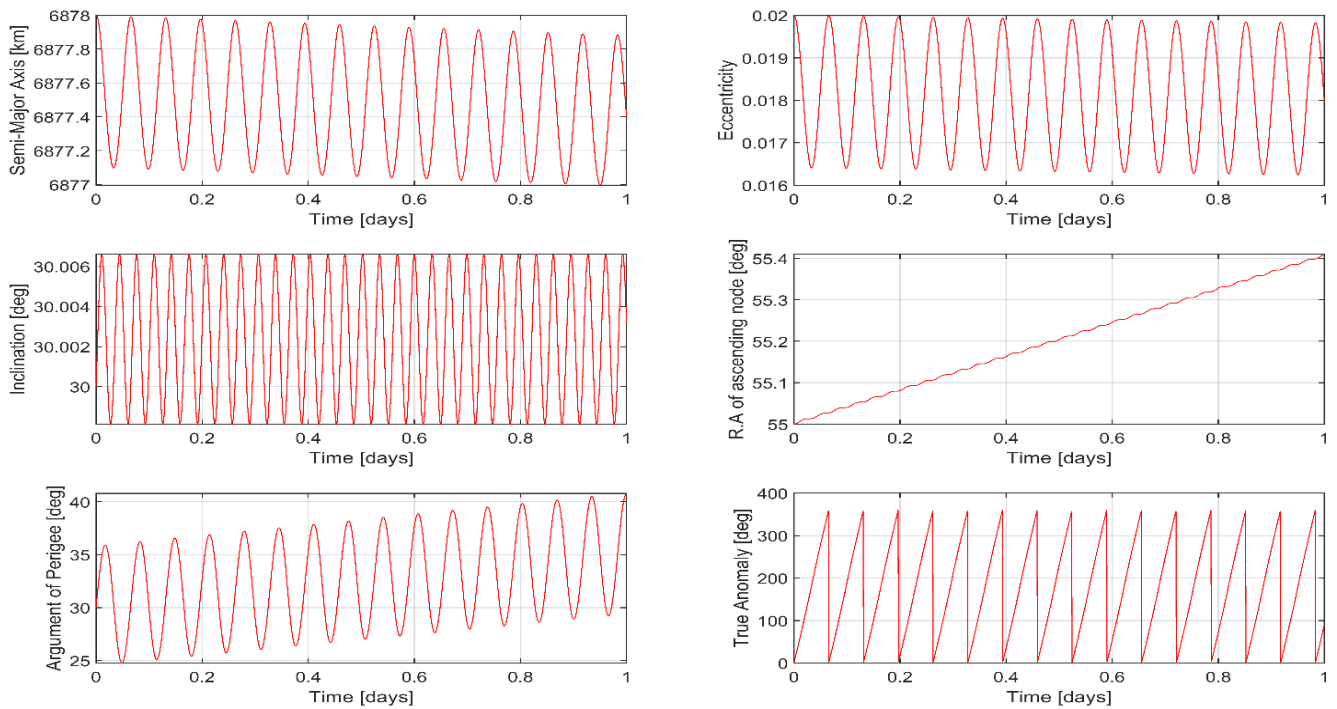


Figure (2): The Perturbations of the orbital elements (a , e , i , Ω , ω , M) due to the EDT force in one day

From Figure (2) we notice that semi major axis changed from 6878 km to 6877.4 km in one day, the eccentricity changed from 0.02 to 0.018 and the right ascension of the ascending node changed from 55 deg to 55.4 deg in one day, the other angles changed sinusoidally.

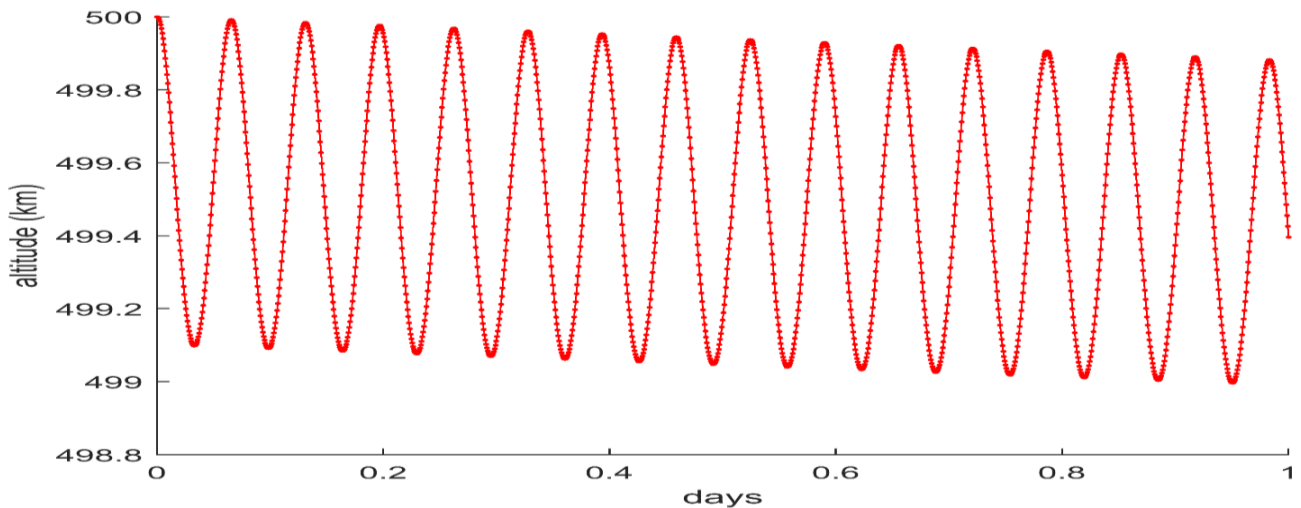


Figure (3): The Perturbations of the altitude due to the EDT force in one day

From the Figure (2) we notice that the altitude of the satellite or debris changed from 500 km to 499.4 km in one day.

So, The EDT force can remove or decay a satellite or debris 0.5 km of its semi major axis and 0.5 km of its altitude and 0.4 deg of the right ascension of the ascending node in one day, also it can change the shape of the orbit due to the change in the eccentricity.

Now change the current induced on the tether system from $I = 2 \text{ Amps}$, $I = 5 \text{ Amps}$ and $I = 7 \text{ Amps}$ in figure (4) & (5) shown the effect of the Electrodynamics force on the elements in one day with change in the current induced.

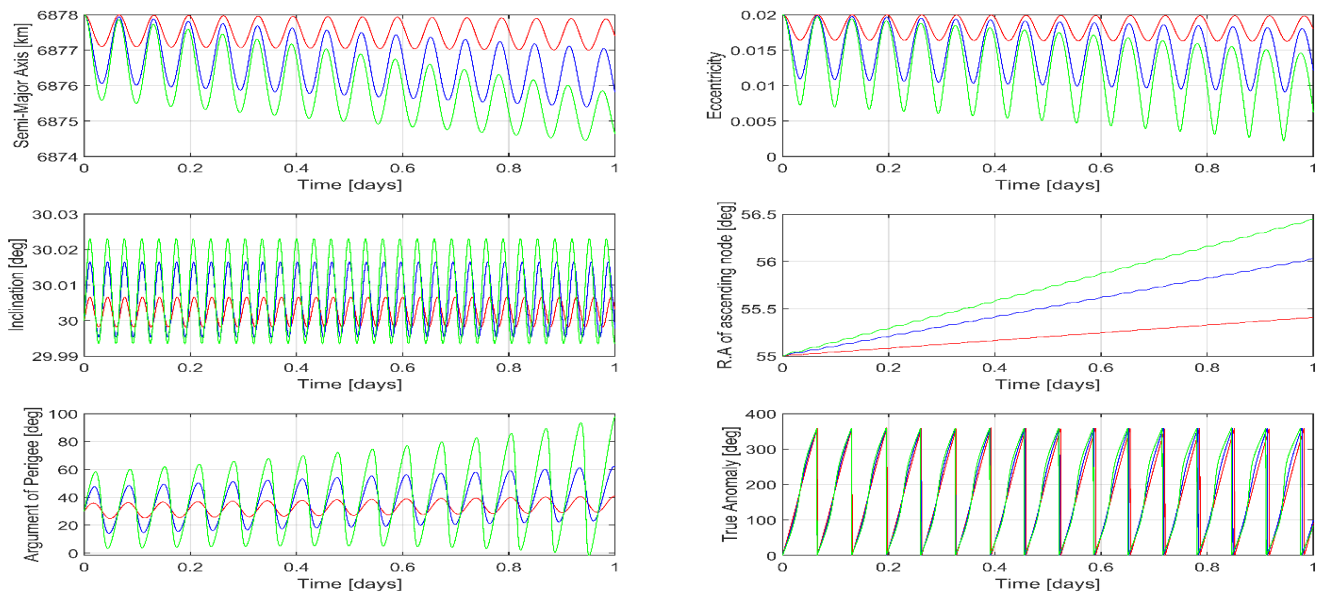


Figure (4): The Perturbations of the orbital elements (a, e, i, Ω , ω , M) due to the EDT force with change the current induced in one day.

From Figure (4) we notice that semi major axis changed from 6878 km to 6875 km, the eccentricity changed from 0.02 to 0.005 and the right ascension of the ascending node changed from 55 deg to 56.5 deg in one day, with the increasing of the current induced on the tether wire from $I = 2 \text{ Amps}$, $I = 5 \text{ Amps}$ and $I = 7 \text{ Amps}$ in one day.

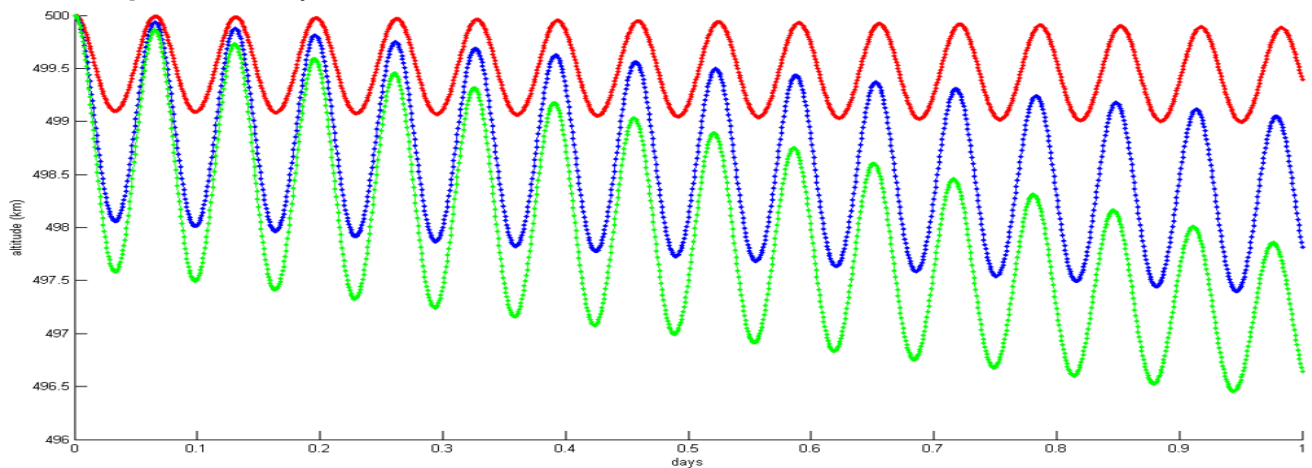
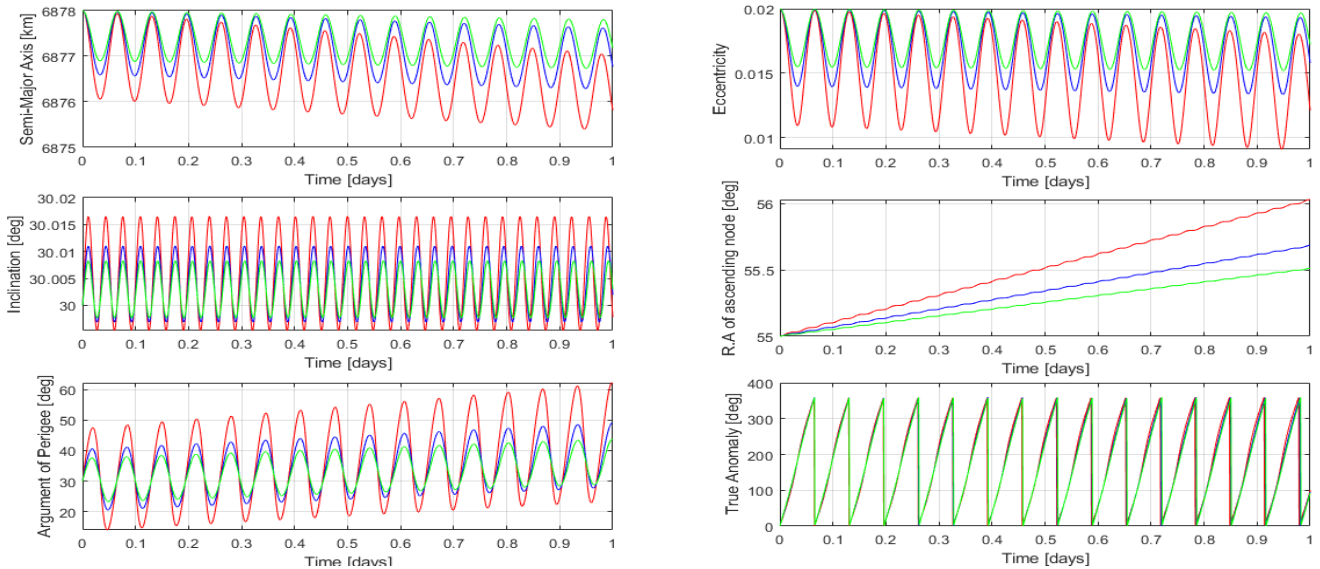


Figure (5): The Perturbations of the altitude due to the EDT force with change the current induced in one day.

From the Figure (4) we notice that the altitude of the satellite or debris changed from 500 km to 496.5 km with the increasing of the current induced on the tether wire from $I = 2 \text{ Amps}$, $I = 5 \text{ Amps}$ and $I = 7 \text{ Amps}$ in one day.

So, the change of the current induced on the tether wire increase the EDT force effect on removing or decaying a satellite or debris by 3 km of its semi major axis, 3.5 km of its altitude and 1.5 deg of the right ascension of the ascending node in one day, also it can change the shape of the orbit due to the change in the eccentricity.

Now change the mass of the spacecraft from $m = 100 \text{ Kg}$, $m = 150 \text{ Kg}$ and $m = 200 \text{ Kg}$, the results



shown in Figure (6) & (7).

Figure (6): The Perturbations of the orbital elements ($a, e, i, \Omega, \omega, M$) due to the EDT force with change spacecraft's mass in one day.

From Figure (6) we notice that semi major axis changed from 6878 km to 6876 km, the eccentricity changed from 0.02 to 0.015 and the right ascension of the ascending node changed from 55 deg to 56 deg in one day, with the increasing of the mass of the spacecraft from $m = 100 \text{ Kg}$, $m = 150 \text{ Kg}$ and $m = 200 \text{ Kg}$.

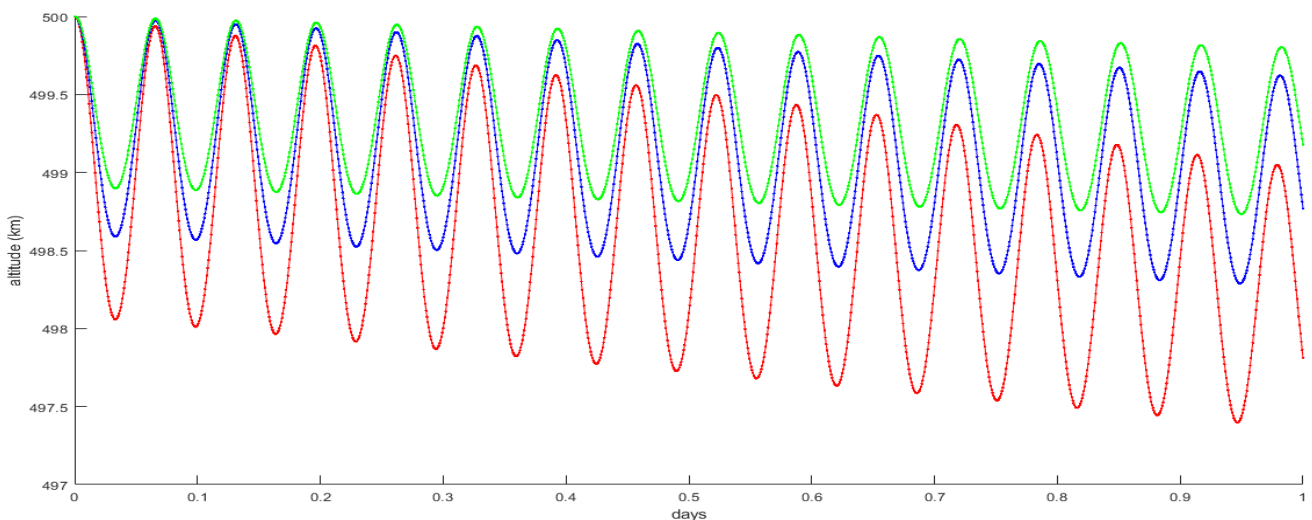


Figure (7): The Perturbations of the altitude due to the EDT force with change spacecraft's mass in one day.

From the Figure (7) we notice that the altitude of the satellite or debris changed from 500 km to 497.8 km in one day due to increasing of the mass of the spacecraft from $m = 100 \text{ Kg}$, $m = 150 \text{ Kg}$ and $m = 200 \text{ Kg}$.

So, the change of the mass of the spacecraft increase the EDT force effect on removing or decaying a satellite or debris by 2 km of its semi major axis, 2.2 km of its altitude and 1 deg of the right ascension of the ascending node in one day, also it can change the shape of the orbit due to the change in the eccentricity.

Now change the length of the terminator tether from $L = 5000 \text{ meters}$, $L = 7000 \text{ meters}$ and $L = 10000 \text{ meters}$ the results shown Figure (8) & (9).

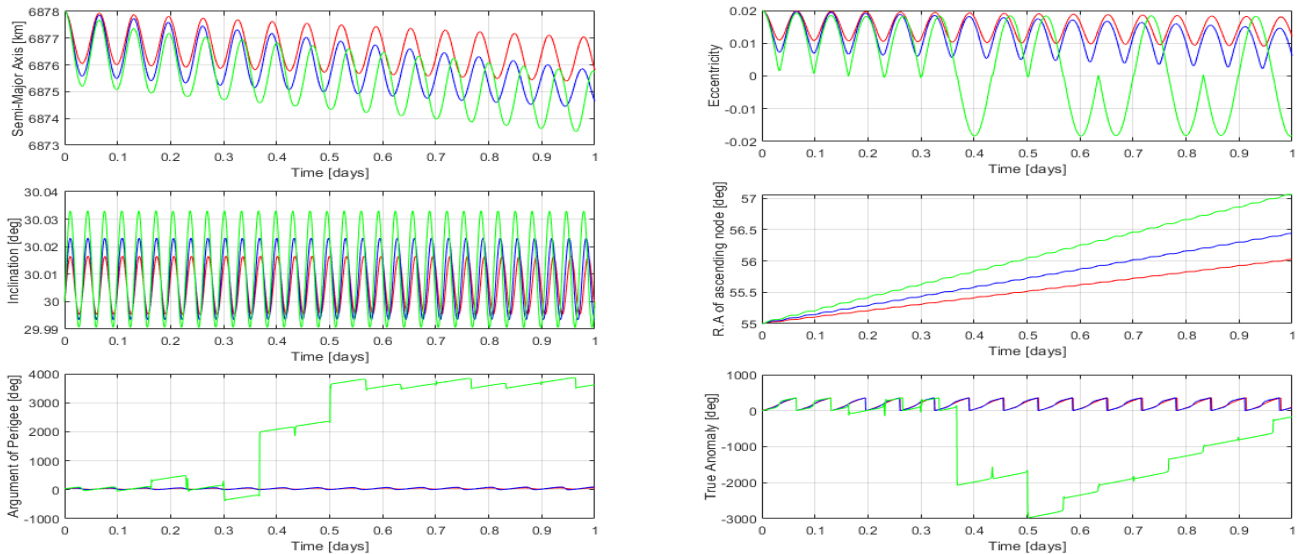


Figure (8): The Perturbations of the orbital elements ($a, e, i, \Omega, \omega, M$) due to the EDT force with change terminator's length in one day.

From Figure (8) we notice that semi major axis changed from 6878 km to 6875 km, the eccentricity changed from 0.02 to 0.00 and the right ascension of the ascending node changed from 55 deg to 57 deg in one day due to increasing of the length of the terminator tether system from $L = 5000 \text{ meters}$, $L = 7000 \text{ meters}$ and $L = 10000 \text{ meters}$.

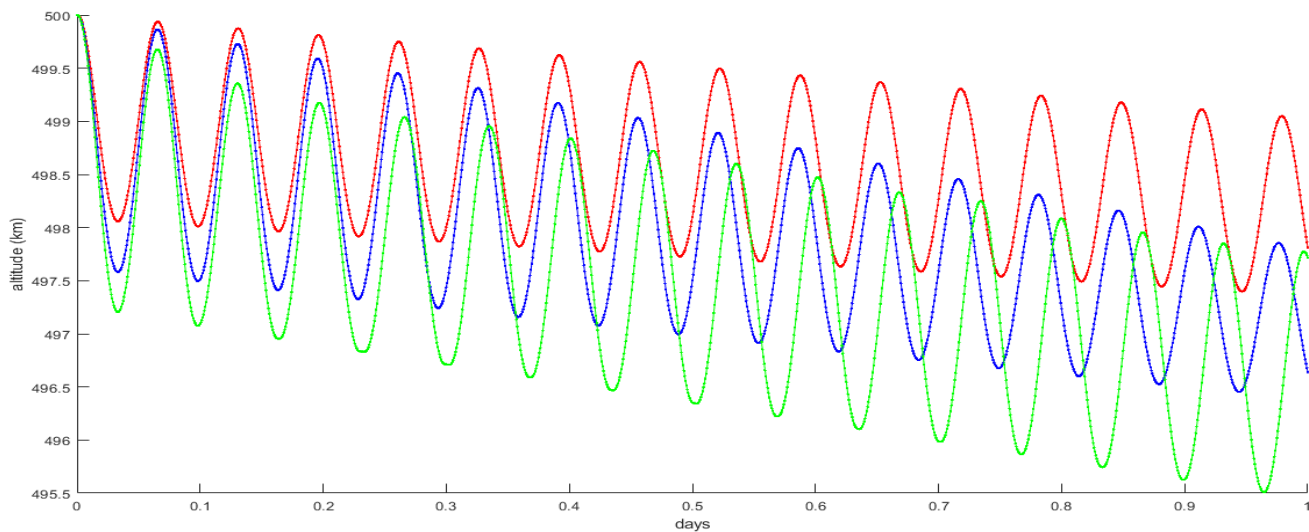


Figure (9): The Perturbations of the altitude due to the EDT force with change terminator's length in one day.

From the Figure (9) we notice that the altitude of the satellite or debris changed from 500 km to 496.5 km in one day with the increasing of the length of the terminator tether system from $L = 5000 \text{ meters}$, $L = 7000 \text{ meters}$ and $L = 10000 \text{ meters}$.

So, the change of the length of the terminator tether system increase the EDT force effect on removing or decaying a satellite or debris by 3 Km of its semi major axis, 3.5 Km of its altitude and 2 deg of the right ascension of the ascending node in one day, also it can change the shape of the orbit due to the change in the eccentricity.

Now change the period from one day to one week, firstly without any change in current, mass or length and secondly with change in current then mass then length of the terminator tether system and notice the change of the EDT force and its effect on the orbital element perturbation.

The effect of the Electrodynamics force on the elements in one week shown in figure (10) & (11)

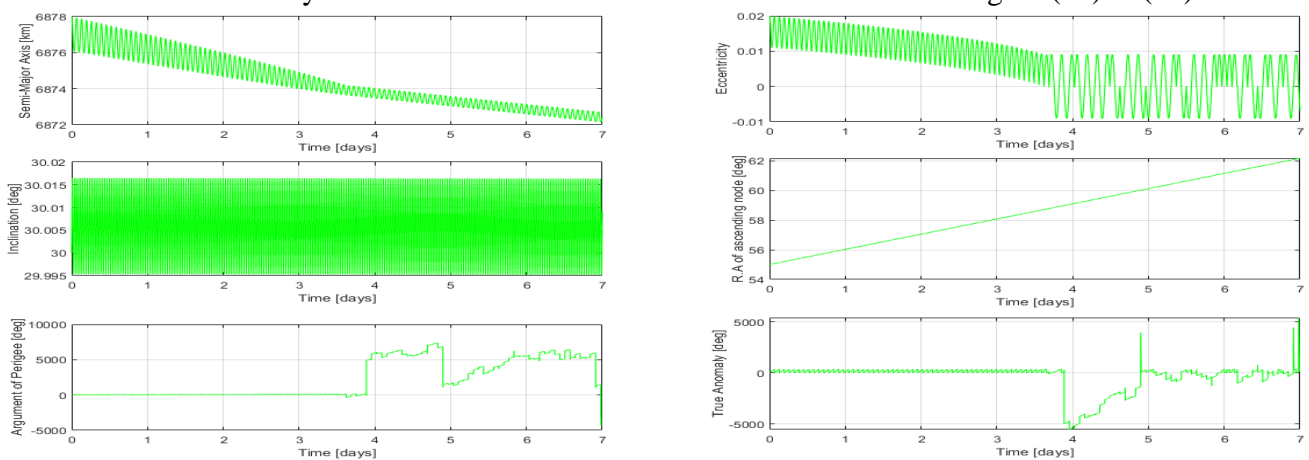


Figure (10): The Perturbations of the orbital elements (a , e , i , Ω , ω , M) due to the EDT force in one week.

From Figure (10) we notice that semi major axis changed from 6878 km to 6872 km, the eccentricity changed from 0.02 to -0.01 and the right ascension of the ascending node changed from 55 deg to 62 deg in one week, the other angles changed sinusoidally.

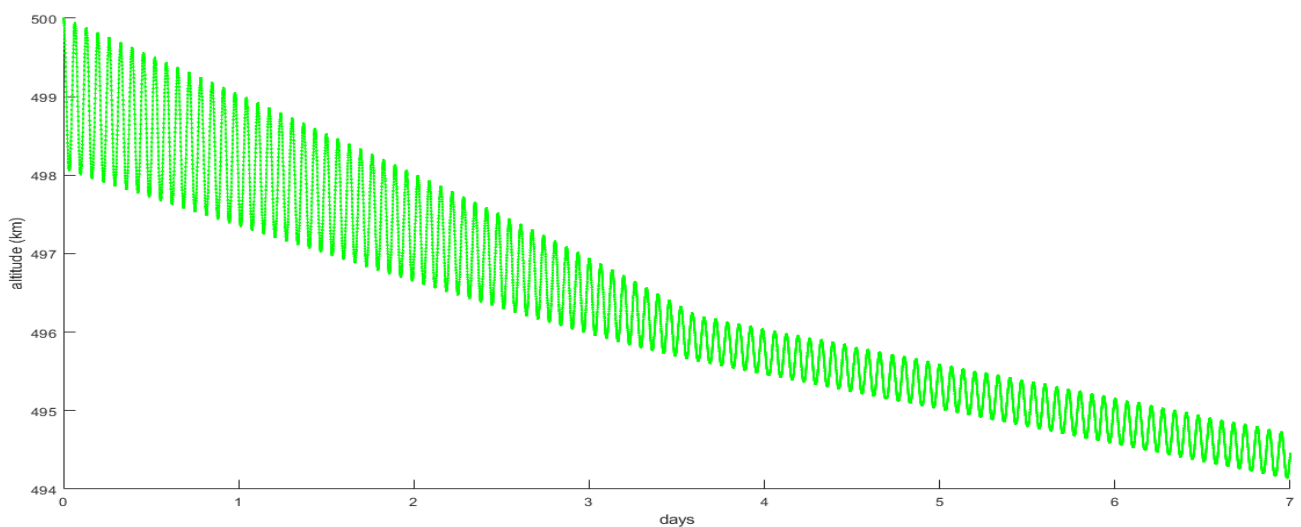


Figure (11): The Perturbations of the altitude due to the EDT force

From the Figure (11) we notice that the altitude of the satellite or debris changed from 500 km to 494 km in one week.

So, The EDT force can remove or decay a satellite or debris 6 km of its semi major axis and 6 km of its altitude and 7 deg of the right ascension of the ascending node on one week, also it can change the shape of the orbit due to the change in the eccentricity.

Now change the current induced on the tether system from $I = 2 \text{ Amps}$, $I = 5 \text{ Amps}$ and $I = 7 \text{ Amps}$ Figure (12) & (13) shown the effect of the Electrodynamics force on the elements in one week with change in the current induced.

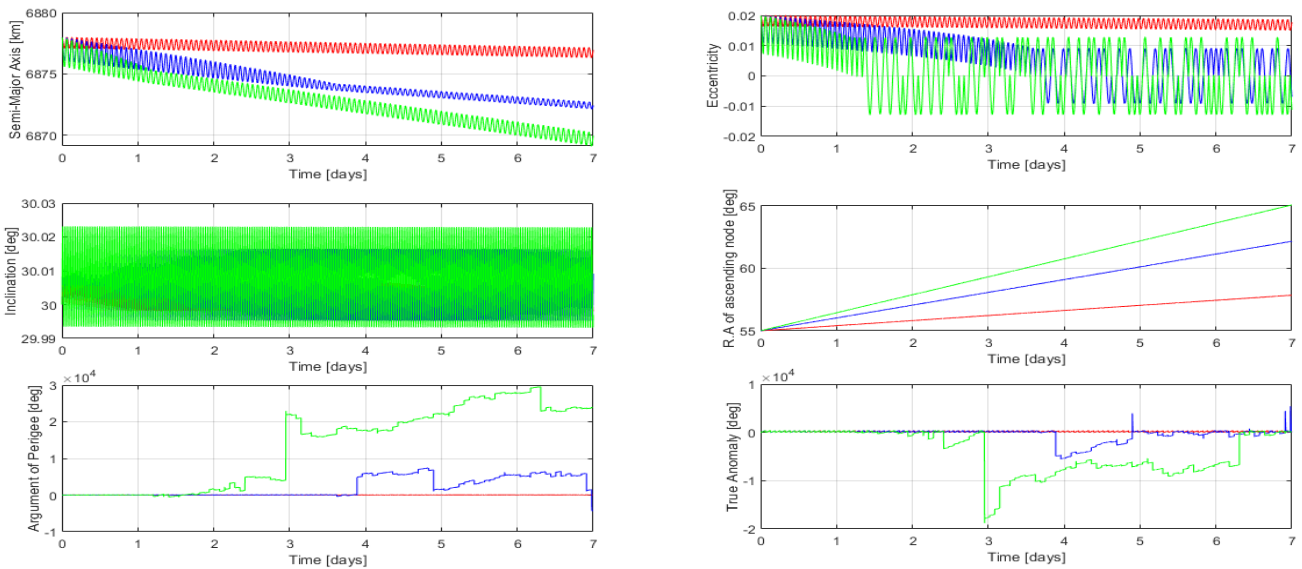
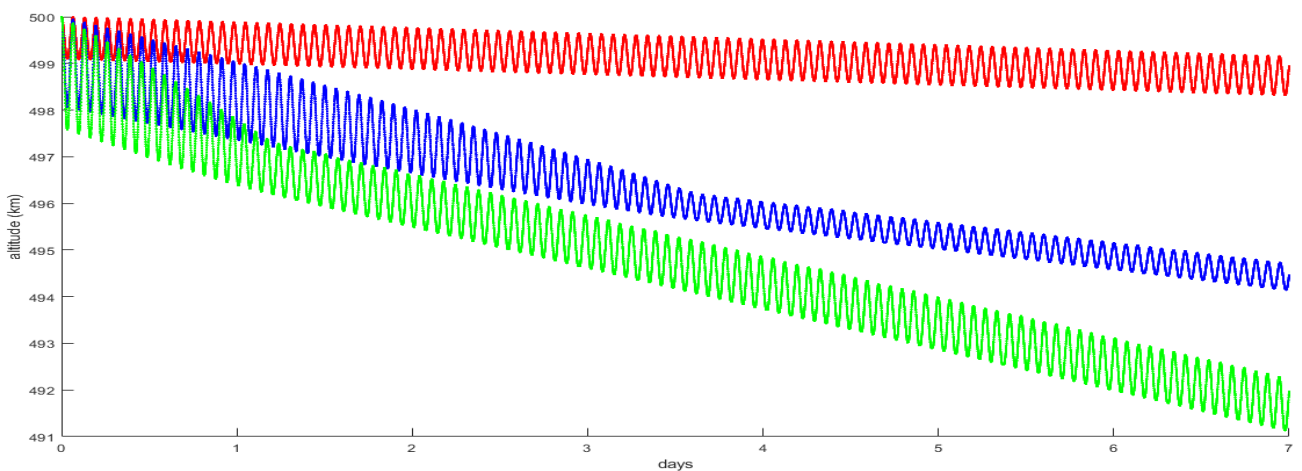


Figure (12): The Perturbations of the orbital elements (a , e , i , Ω , ω , M) due to the EDT with change the current induced in one week.

From Figure (12) we notice that semi major axis changed from 6878 km to 6870 km, the eccentricity changed from 0.02 to -0.012 and the right ascension of the ascending node changed from 55 deg to 65 deg in one week, with the increasing of the current induced on the tether wire from $I = 2 \text{ Amps}$, $I = 5 \text{ Amps}$ and



$I = 7 \text{ Amps}$.

Figure (13): The Perturbations of the altitude due to the EDT with change the current induced in one week.

From the Figure (13) we notice that the altitude of the satellite or debris changed from 500 km to 491 km in one week due to increasing of the current induced on the tether wire from $I = 2 \text{ Amps}$, $I = 5 \text{ Amps}$ and $I = 7 \text{ Amps}$.

So, the change of the current induced on the tether wire increase the EDT force effect on removing or decaying a satellite or debris by 8 km of its semi major axis, 9 km of its altitude and 10 deg of the right ascension of the ascending node in one week, also it can change the shape of the orbit due to the change in the eccentricity.

Now change the mass of the spacecraft from $m = 100 \text{ Kg}$, $m = 150 \text{ Kg}$ and $m = 200 \text{ Kg}$ Figure (14) & (15) shown the effect of the Electrodynamics force on the elements in one week with change in the mass of the spacecraft.

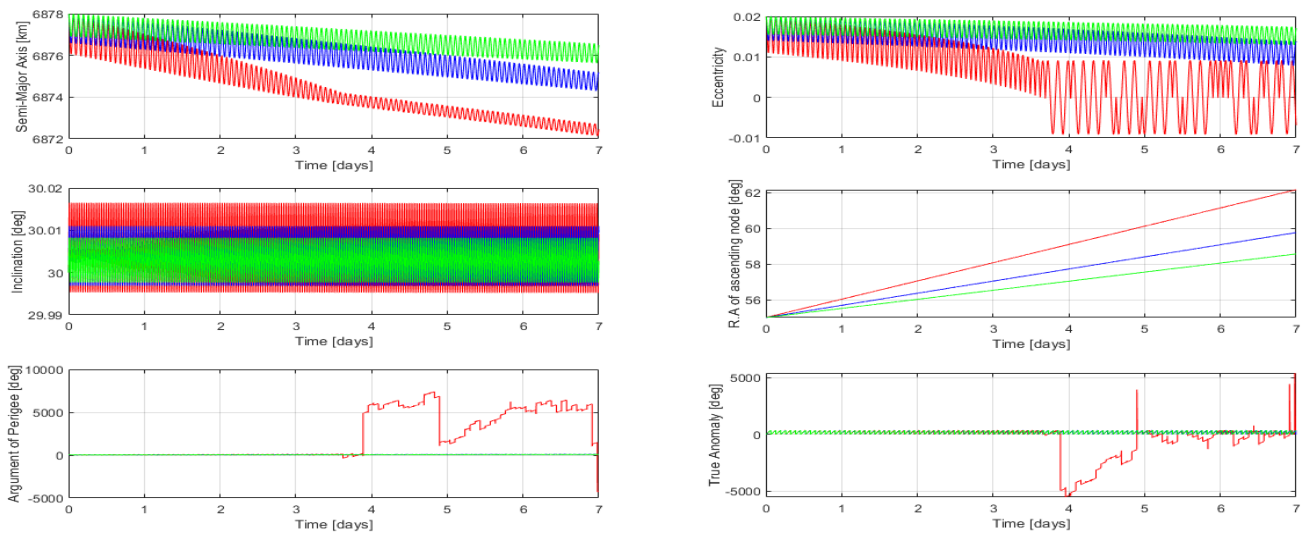


Figure (14): The Perturbations of the orbital elements ($a, e, i, \Omega, \omega, M$) due to the EDT with change the spacecraft's mass in one week.

From Figure (14) we notice that semi major axis changed from 6878 km to 6872 km, the eccentricity changed from 0.02 to -0.01 and the right ascension of the ascending node changed from 55 deg to 62 deg in one week due to increasing of the mass of the spacecraft from $m = 100 \text{ Kg}$, $m = 150 \text{ Kg}$ and $m = 200 \text{ Kg}$.

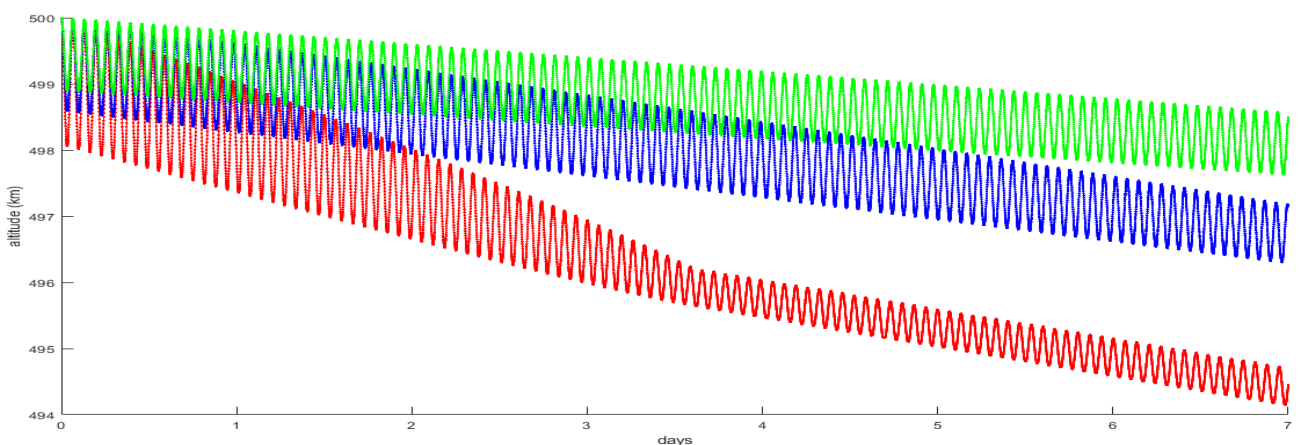


Figure (15): The Perturbations of the altitude due to the EDT with change the spacecraft's mass.

From the Figure (15) we notice that the altitude of the satellite or debris changed from 500 km to 494.2 km in one week due to increasing of the mass of the spacecraft from $m = 100 \text{ Kg}$, $m = 150 \text{ Kg}$ and $m = 200 \text{ Kg}$.

So, the change of the mass of the spacecraft increase the EDT force effect on removing or decaying a satellite or debris by 6 km of its semi major axis, 5.8 km of its altitude and 7 deg of the right ascension of the ascending node in one week, also it can change the shape of the orbit due to the change in the eccentricity.

Now change the length of the terminator tether from $L = 5000 \text{ meters}$, $L = 7000 \text{ meters}$ and $L = 10000 \text{ meters}$ Figure (16) & (17) shown the effect of the Electrodynamics force on the elements in one day with change in the length of the terminator tether system.

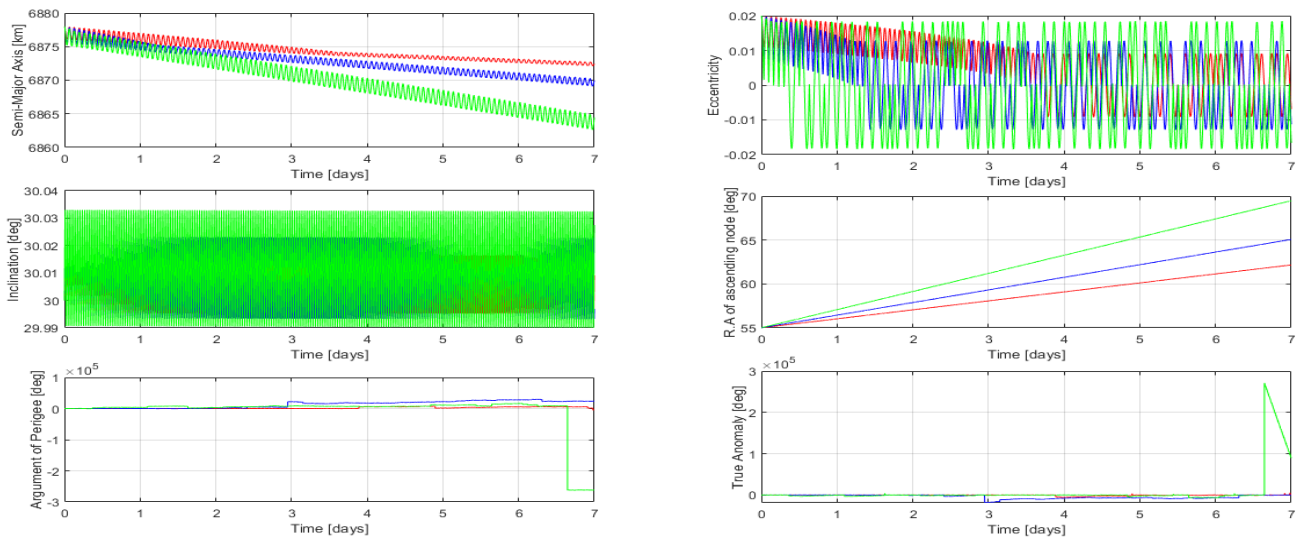
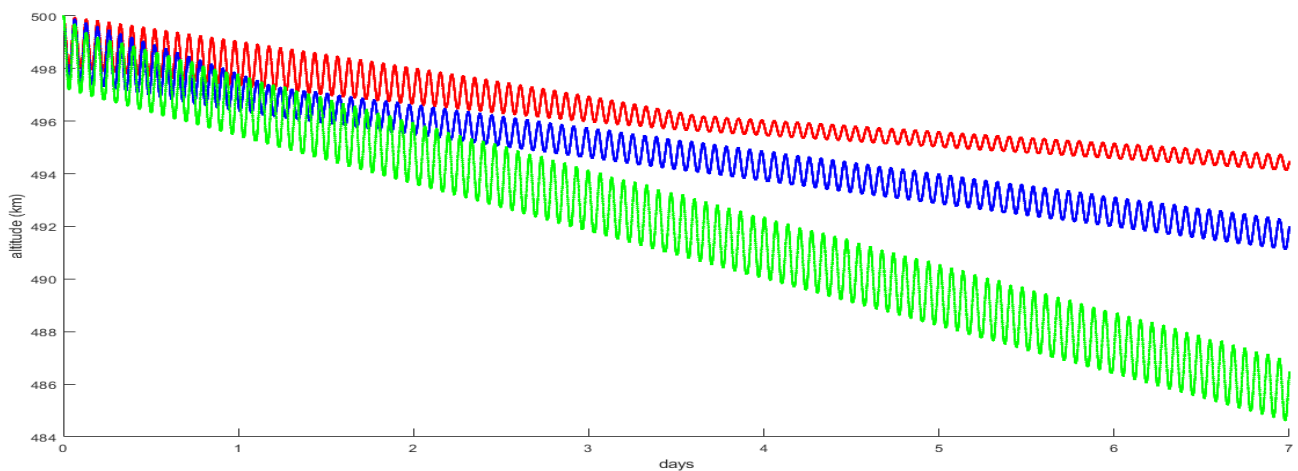


Figure (16): The Perturbations of the orbital elements ($a, e, i, \Omega, \omega, M$) due to the EDT with change the terminator's length.

From Figure (16) we notice that semi major axis changed from 6878 km to 6863 km, the eccentricity changed from 0.02 to -0.02 and the right ascension of the ascending node changed from 55 deg to 70 deg in one week due to increasing of the length of the terminator tether system from $L = 5000 \text{ meters}$, $L = 7000 \text{ meters}$



and $L = 10000 \text{ meters}$.

Figure (17): The Perturbations of the altitude due to the EDT with change the terminator's length in one week.

From the Figure (17) we notice that the altitude of the satellite or debris changed from 500 km to 484.5 km in one week due to increasing of the length of the terminator tether system from $L = 5000 \text{ meters}$, $L = 7000 \text{ meters}$ and $L = 10000 \text{ meters}$.

So, the change of the length of the terminator tether system increase the EDT force effect on removing or decaying a satellite or debris by 15 km of its semi major axis, 15.5 km of its altitude and 15 deg of the right ascension of the ascending node in one week, also it can change the shape of the orbit due to the change in the eccentricity.

Now change the period from one week to one month, firstly without any change in current, mass or length and secondly with change in current then mass then length of the terminator tether system and notice the change of the EDT force and its effect on the orbital element perturbation.

The effect of the Electrodynamics force on the elements in one month shown in figure (18) & (19)

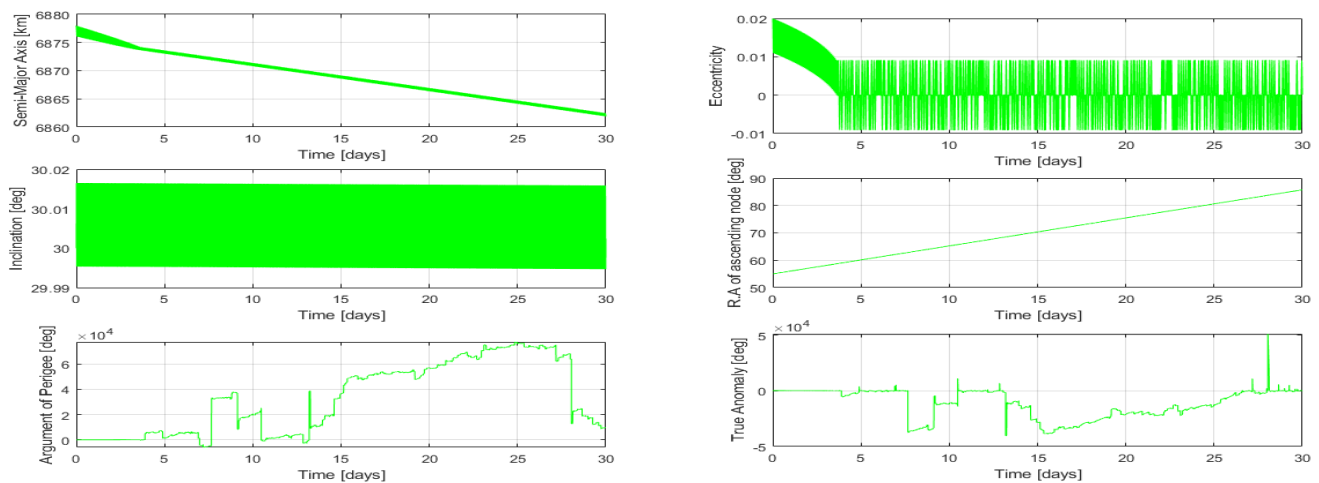


Figure (18): The Perturbations of the orbital elements ($a, e, i, \Omega, \omega, M$) due to the EDT force in one month.

From Figure (18) we notice that semi major axis changed from 6878 km to 6863 km in one month, the eccentricity changed from 0.02 to -0.01 and the right ascension of the ascending node changed from 55 deg to 87 deg in one month, the other angles changed sinusoidally.

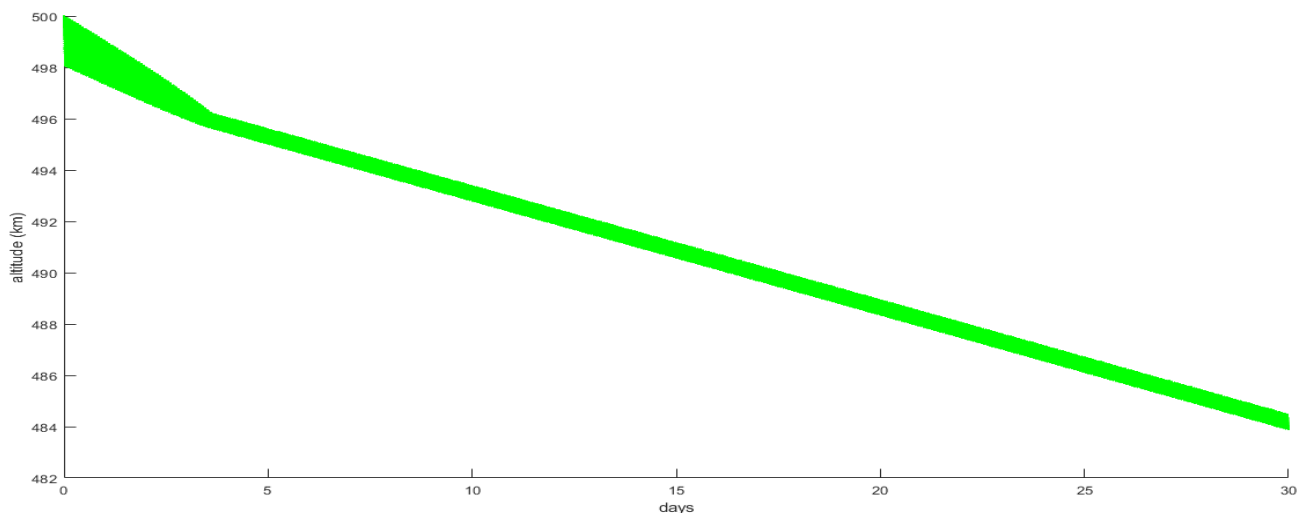


Figure (19): The Perturbations of the altitude due to the EDT force in one month.

From the Figure (19) we notice that the altitude of the satellite or debris changed from 500 km to 484 km in one month.

So, the EDT force can remove or decay a satellite or debris 15 km of its semi major axis, 16 km of its altitude and 32 deg of the right ascension of the ascending node in one month, also it can change the shape of the orbit due to the change in the eccentricity.

Now change the current induced on the tether system from $I = 2 \text{ Amps}$, $I = 5 \text{ Amps}$ and $I = 7 \text{ Amps}$ Figure (20) & (21) shown the effect of the Electrodynamics force on the elements in one month with change in the current induced.

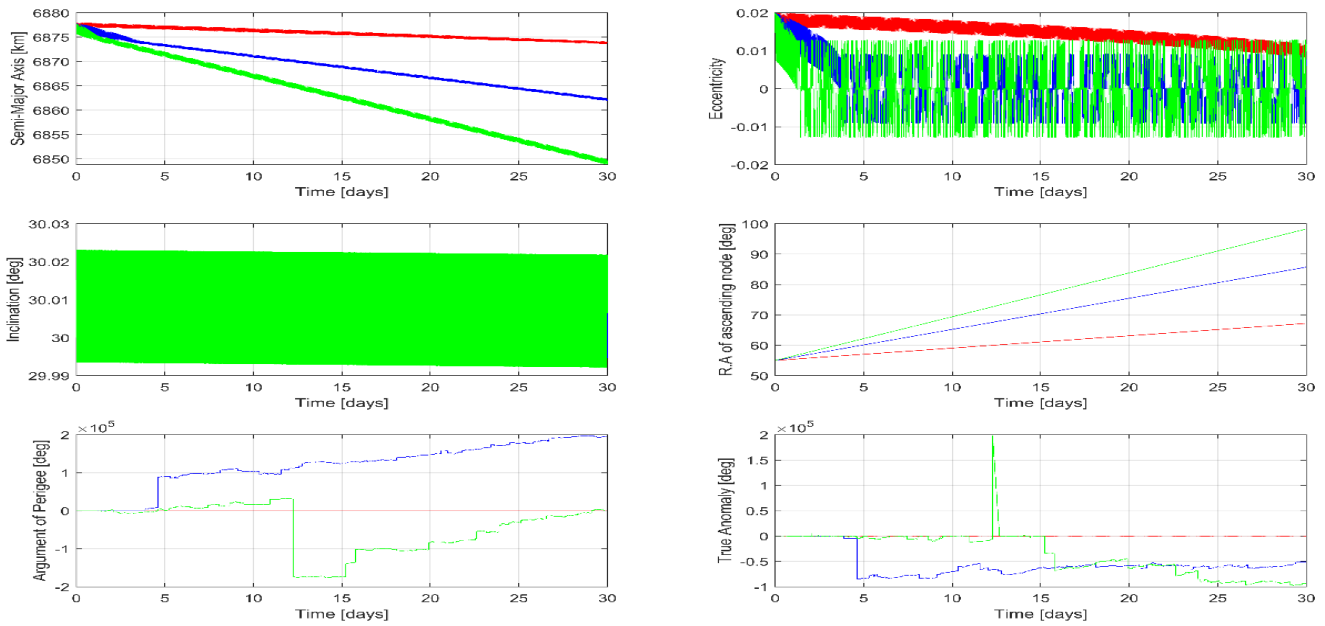
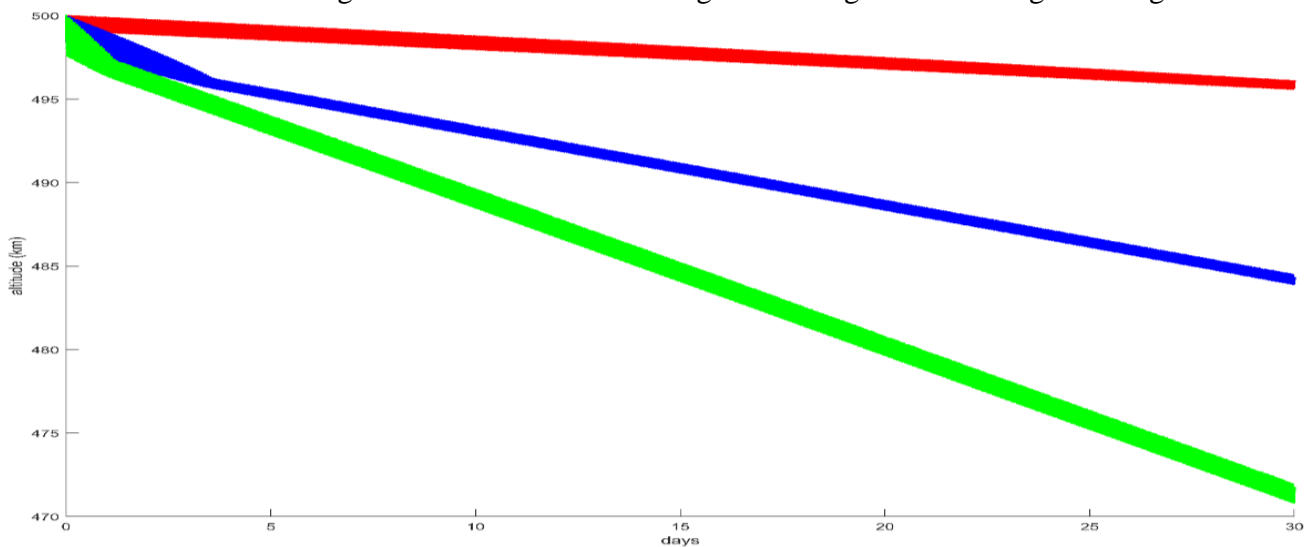


Figure (20): The Perturbations of the orbital elements (a , e , i , Ω , ω , M) due to the EDT with change the current induced in one month.

From Figure (20) we notice that semi major axis changed from 6878 km to 6850 km, the eccentricity changed from 0.02 to -0.015 and the right ascension of the ascending node changed from 55 deg to 99 deg in one month



due to increasing of the current induced on the tether wire from $I = 2 \text{ Amps}$, $I = 5 \text{ Amps}$ and $I = 7 \text{ Amps}$.

Figure (21): The Perturbations of the altitude due to the EDT with change the current induced in one month.

From the Figure (21) we notice that the altitude of the satellite or debris changed from 500 km to 471 km in one month due to increasing of the current induced on the tether wire from $I = 2 \text{ Amps}$, $I = 5 \text{ Amps}$ and $I = 7 \text{ Amps}$.

So, the change of the current induced of the terminator tether system increase the EDT force effect on removing or decaying a satellite or debris by 28 km of its semi major axis, 29 km of its altitude and 44 deg of the right ascension of the ascending node in one month, also it can change the shape of the orbit due to the change in the eccentricity.

Now change the mass of the spacecrafts from $m = 100 \text{ Kg}$, $m = 150 \text{ Kg}$ and $m = 200 \text{ Kg}$ Figure (22) & (23) shown the effect of the Electrodynamics force on the elements in one month with change in the terminator's mass.

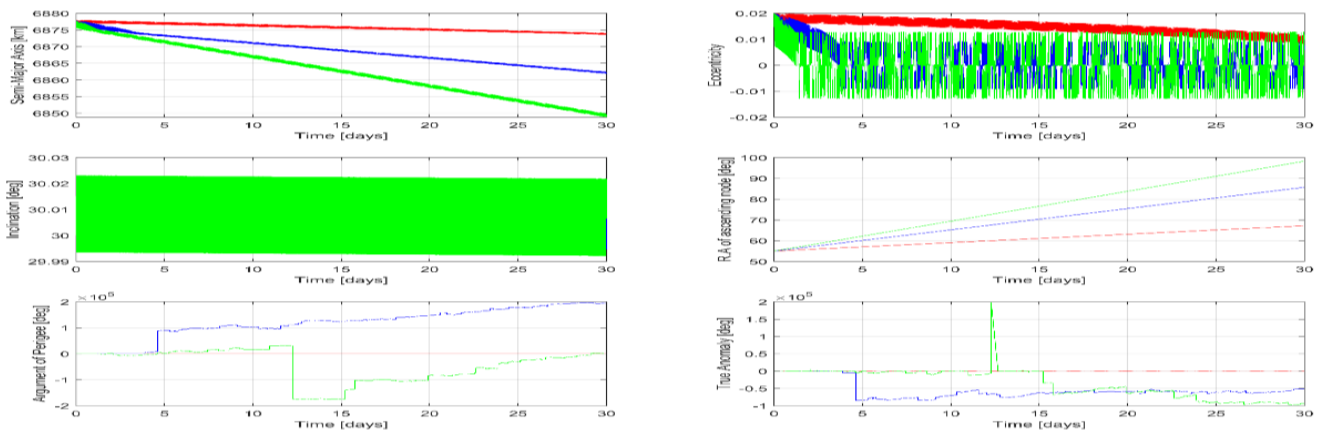


Figure (22): The Perturbations of the orbital elements ($a, e, i, \Omega, \omega, M$) due to the EDT with change the spacecraft's mass in one month.

From Figure (22) we notice that semi major axis changed from 6878 km to 6850 km, the eccentricity changed from 0.02 to -0.013 and the right ascension of the ascending node changed from 55 deg to 99 deg in one month due to increasing of the mass of the spacecraft from $m = 100 \text{ Kg}$, $m = 150 \text{ Kg}$ and $m = 200 \text{ Kg}$.

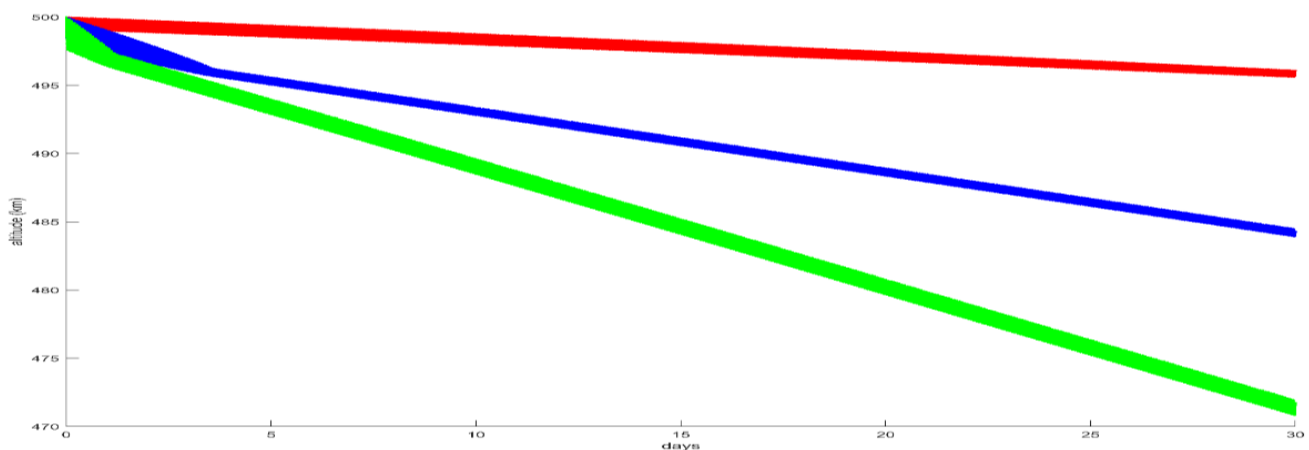
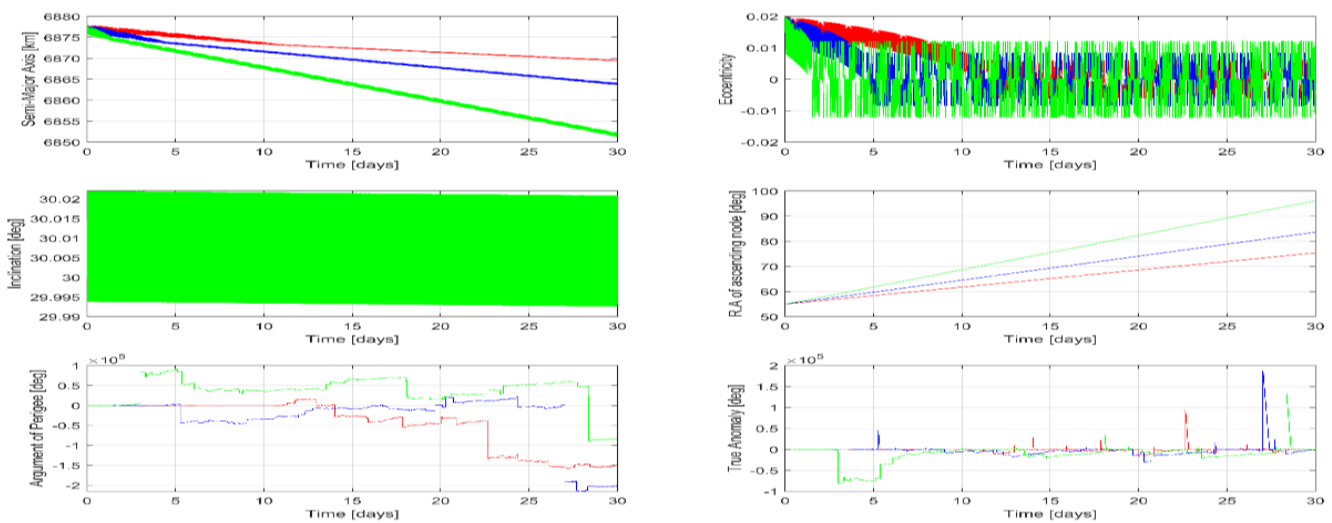


Figure (23): The Perturbations of the altitude due to the EDT with change the spacecraft's mass in one month.

From the Figure (23) we notice that the altitude of the satellite or debris changed from 500 km to 471 km in one month due to increasing of the mass of the spacecraft from $m = 100\text{ Kg}$, $m = 150\text{ Kg}$ and $m = 200\text{ Kg}$.

So, the change of the mass of the spacecraft increase the EDT force effect on removing or decaying a satellite or debris by 28 km of its semi major axis, 29 km of its altitude and 44 deg of the right ascension of the ascending node in one month, also it can change the shape of the orbit due to the change in the eccentricity.

Now change the length of the terminator tether from $L = 5000\text{ meters}$, $L = 7000\text{ meters}$ and $L = 10000\text{ meters}$ Figure (24) & (25) shown the effect of the Electrodynamics force on the elements in



one month with change in the length of the terminator tether system.

Figure (24): The Perturbations of the orbital elements ($a, e, i, \Omega, \omega, M$) due to the EDT with change the terminator's length in one month.

From Figure (24) we notice that semi major axis changed from 6878 km to 6853 km, the eccentricity changed from 0.02 to -0.01 and the right ascension of the ascending node changed from 55 deg to 97 deg in one month due to increasing of the length of the terminator tether system from $L = 5000$ meters, $L = 7000$ meters and $L = 10000$ meters.

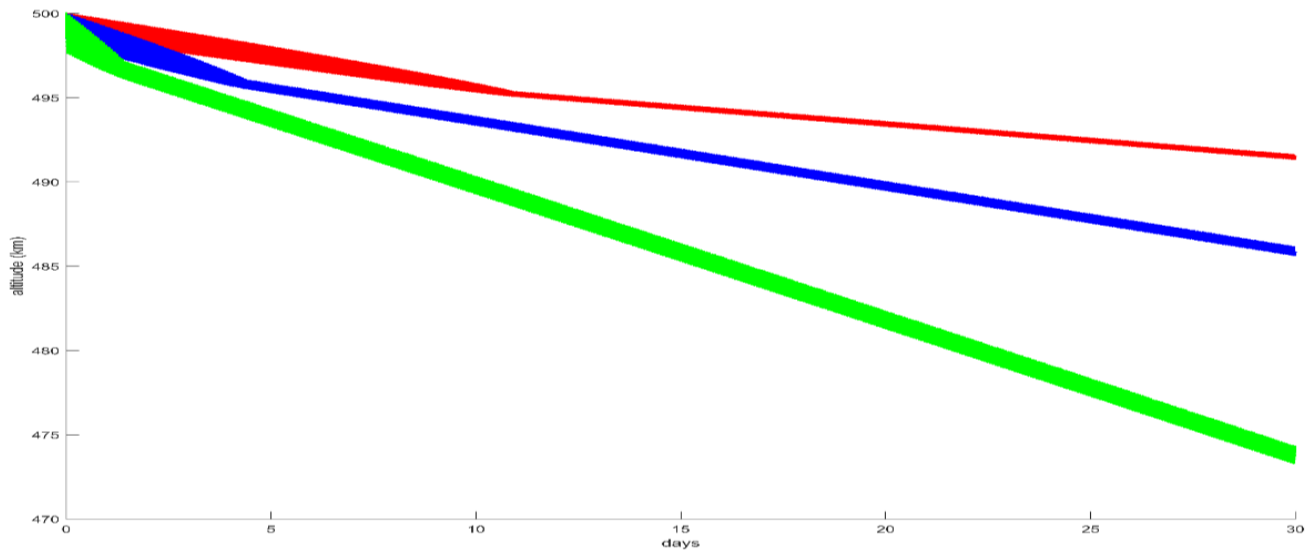


Figure (25): The Perturbations of the altitude due to the EDT with change the terminator's length in one month.

From the Figure (25) we notice that the altitude of the satellite or debris changed from 500 km to 473 km in the month due to increasing of the length of the terminator tether system from $L = 5000$ meters, $L = 7000$ meters and $L = 10000$ meters. So, the change of the length of the terminator tether system increase the EDT force effect on removing or decaying a satellite or debris by 25 km of its semi major axis, 27 km of its altitude and 42 deg of the right ascension of the ascending node in one month, also it can change the shape of the orbit due to the change in the eccentricity. We concluded the preliminary results of our model in table (1).

Table (1) showing the variation of the orbital element due to EDT

EDT force effect		$I = 5$ Amps $m = 100$ kg $L = 5000$ meters	$I = 7$ Amps $m = 100$ kg $L = 5000$ meters	$I = 5$ Amps $m = 150$ kg $L = 5000$ meters	$I = 5$ Amps $m = 100$ kg $L = 7000$ meters
One Day	a	0.5 km	3 km	2 km	3 km
	e	0.002	0.015	0.005	0.02
	Ω	0.4 deg	1.5 deg	1 deg	2 deg
	h	0.5 km	3.5 km	2.2 km	3.5 km
One Week	a	6 km	8 km	6 km	15 km
	e	0.03	0.032	0.03	0.04
	Ω	7 deg	10 deg	7 deg	15 deg

	h	6 km	9 km	5.8 km	15.5 km
One Month	a	15 km	28 km	28 km	25 km
	e	0.03	0.035	0.033	0.03
	Ω	32 deg	44 deg	44 deg	42 deg
	h	16 km	29 km	29 km	27 km

Conclusion:

An analytical model for active debris removal using Electrostatic tether system has been developed for two different model. A preliminary results for the first model confirm the capability of the system which can be used to decay the dangers debris. The main factor of the results depend the length of the cable of the tether and the current produced. We are still working on the model and the second model, and will compare both of them depend the size of debris, altitude, inclination and length of tether.

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