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Sensitivity of Spin Parameters to Uncertainties of the Aerodynamic Model of a Light Aircraft

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Abstract
Reliable prediction of aircraft post-stalled dynamics is entirely depend on the adequacy of the aerodynamic model. A mathematical model of the aerodynamic characteristics of a light aircraft is developed for a wide range of angles of attack, sideslip, and angular rates. It is augmented by a model of uncertainties based on a set of repeated measurements of static aerodynamic characteristics. The effect of aerodynamic uncertainties on steady-state and oscillatory spin modes is directly calculated using a continuation on a parameter technique. The influence of aerodynamic uncertainties on spin entry/recovery is also analyzed.

1. Introduction

Reliable prediction of aircraft spin, a clear non-linear and one of the most dangerous phenomena encountered in flight, is entirely related to the adequacy of the aerodynamic model used for analysis. The model must be valid in a wide range of angles of attack and sideslip with account of the intensive rotation effects. The procedure for developing a heuristic nonlinear aerodynamic model suitable for calculating the spin is generally known in the literature. It is usually based on several types of wind tunnel experiments. Since the experimental data contain different sources of errors, and the heuristic model contains a number of admissions, it is important to investigate the sensitivity of calculated bifurcation patterns of critical flight regimes and spin parameters to various uncertainties of the aerodynamic model. In other words, it is interesting to investigate whether the post-stalled aircraft dynamics remain close to the nominal one when the nominal aerodynamic model is perturbed, i.e. estimate the “robustness” of spin parameters to uncertainties of the aerodynamic model.

The robustness issues for normal flight regimes have been thoroughly studied during last three decades. A number of methods of robust analysis and synthesis were proposed: μ-analysis, Lyapunov-based robust methods, etc., and supplied by a user-friendly software [1]. They were intensively used for flight control design and robust stability/performance analysis in many engineering applications. The influence of aerodynamic model uncertainties on critical flight regimes was investigated substantially less, but this is an important issue of adequacy of the aerodynamic models for reliable prediction of post-stalled aircraft dynamics.

This work was carried out in the framework of the ONERA-TsAGI collaborative project “Improvement of representativeness of models” aimed to the accurate prediction of the aircraft post-stalled behaviour including spin. The investigations are based on the adequate mathematical modelling of the aircraft's aerodynamic characteristics. The mathematical model of the aerodynamic characteristics of a light aircraft is developed in ONERA for a wide range of angles of attack, sideslip angles, angular rates and various controls deflections, from ordinary flight to the post-stall region. The model is built based on the measurements of aerodynamic efforts in the vertical wind tunnel of ONERA-Lille during static, rotary balanced and oscillatory coning tests.

The generic aerodynamic model is augmented by a model of uncertainties based on a set of repeated measurements of static characteristics of the aircraft with various combinations of test parameters. Results for various values of angle of attack and sideslip, different methods of model suspension, various flow rates in the wind tunnel are taken into account. Significant uncertainty in the lateral aerodynamic characteristics is due to the uncertainty in the aerodynamic asymmetry caused by the flow separation.

Numerical prediction of spin parameters based on the developed aerodynamic model is performed directly using a technique of continuation on a parameter [2-9] and related methods of bifurcation and stability analysis. The equilibrium and oscillatory spin solutions are calculated for the full nonlinear equations of the aircraft motion

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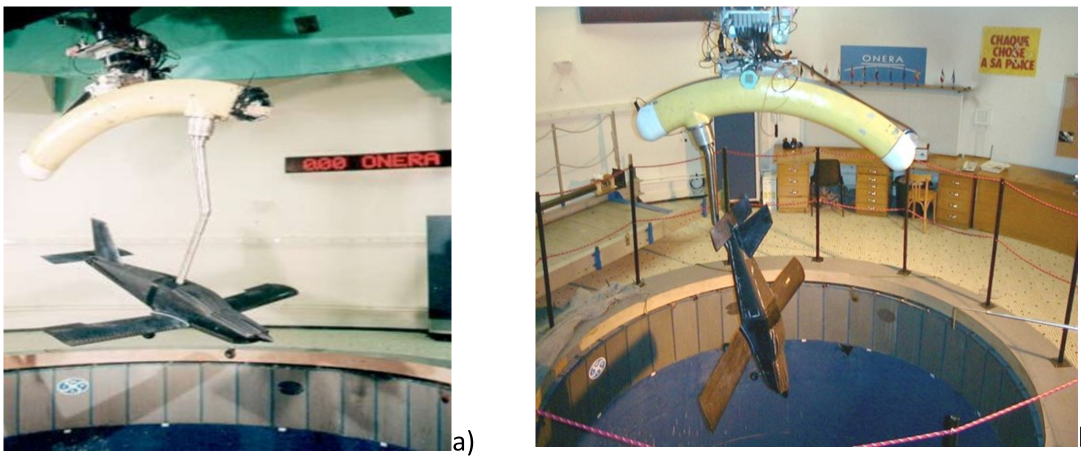
depending on the parameters that are deflection of the control surfaces. The calculated results are compared with the similar results for a reduced system of the 3-rd order (angle of attack, lateral slip, angular velocity around the velocity vector). For adequate stability prediction, the damping terms determined from the forced oscillation tests, are added to the aerodynamic model [10].
The effect of uncertainty of each aerodynamic coefficient and various combinations of uncertainties of coefficients is directly calculated and analysed for different stationary and oscillatory spin modes. The worst combinations of uncertainties, leading to the largest areas of stable spins, are calculated. The emphasis is made on the effects, essential for application to analysis of post-stalled aircraft behaviour, spin entry and spin recovery.
The paper is arranged as follows. The aerodynamic model structure, its development and augmentation by uncertainties is described in Sec. 2, spin parameters calculation in comparison with the approximate results is outlined in Sec. 3. Sensitivity of spin equilibrium and stable oscillatory solutions to uncertainties of the aerodynamic model, including spin entry/recovery is investigated in Sec. 4. Sec. 5 concludes the paper.

2. Aerodynamic model

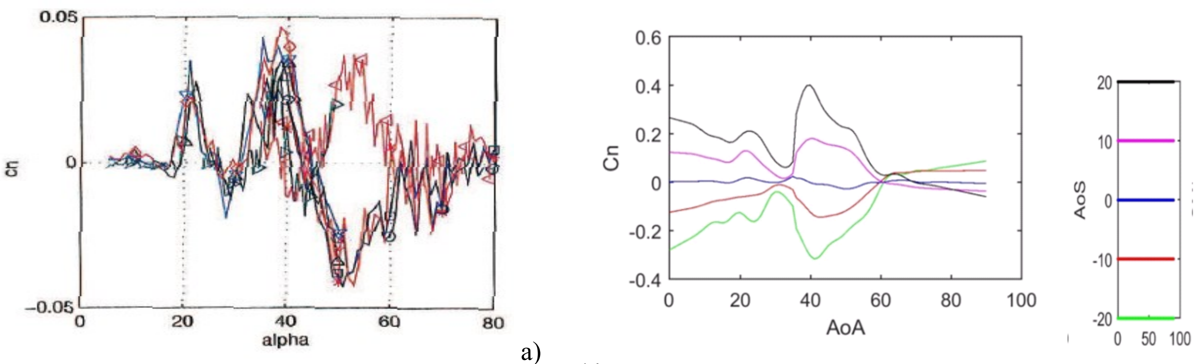
2.2 The aerodynamic model structure

The model of aerodynamic characteristics was obtained on the base of a large number of experiments in the ONERA-Lille vertical wind tunnel: static tests in a wide range of angles of attack, sideslip, and conical rotation of the aircraft model near the velocity vector. The wind tunnel diameter is 4m, and an investigated aircraft has a span 2m. To investigate the total range of angles of attack two mountings were used: a dorsal mounting for the range of angles of attack $\alpha \in [45, 90^o]$, and a rear sting for the range $[-45, 45^o]$. They are shown in Figure 1. Measurements on rotary balance were performed with rotation rate about velocity axis from -600 to +600°/s. An example of static yaw moment coefficient dependence on angle of attack (for zero sideslip angle β) is presented in Figure 2 demonstrating a high level of uncertainty, and lateral asymmetry due to flow separation. The aerodynamic data are combined from the two data bases obtained with the two mountings.



a) b)

Figure 1: Vertical wind tunnel and two mountings used for tests: a) dorsal mounting b) rear mounting



a)
b)

Figure 2: Static yaw moment coefficient a) experiment, b) modelling

DOI: 10.13009/EUCASS2019-537

The ONERA aerodynamic model is essentially nonlinear, and the dimensionless coefficients of the aerodynamic forces and moments have the following structure:

$$\begin{aligned}
C_X &= C_{X0}(\alpha)\\
C_Y &= C_{Y0}(\alpha,\beta) + \Delta C_Y(\alpha,\beta,\Omega^*) + \Delta C_Y(\alpha,\delta_a) + \Delta C_Y(\alpha,\delta_r)\\
C_Z &= C_{Z0}(\alpha,\beta) + \Delta C_Z(\alpha,\beta,\Omega^*) + \Delta C_Z(\alpha,\delta_e)\\
C_l &= C_{l0}(\alpha,\beta) + \Delta C_l(\alpha,\beta,\Omega^*) + \Delta C_l(\alpha,\delta_a) + \Delta C_l(\alpha,\delta_r)\\
C_m &= C_{m0}(\alpha,\beta) + \Delta C_m(\alpha,\beta,\Omega^*) + \Delta C_m(\alpha,\delta_e)\\
C_n &= C_{n0}(\alpha,\beta) + \Delta C_n(\alpha,\beta,\Omega^*) + \Delta C_n(\alpha,\delta_a) + \Delta C_n(\alpha,\delta_r)
\end{aligned} \tag{1}$$

The model of aerodynamic characteristics (1) depends on angles α, β, elevator, aileron and rudder deflections δ_{re}, δ_a, δ_r, full non-dimensional angular rate $\Omega^* = \Omega\,\bar{c}\,/\,V$, but not depend on each of the components *p, q, r* of angular rate separately. As was shown in [10], such a model makes it possible to predict well enough the parameters of spin modes, but not their stability: all the calculated spin modes turned out to be unstable due to the lack of sufficient damping in the aerodynamic model. The unreliable picture of stability of the spin modes does not allow to adequately predict the behavior of the aircraft in the case of loss of control of normal flight. For a more adequate prediction of the stability characteristics of the spin, in [10] it was proposed to supplement the ONERA aerodynamic model with additional damping terms taken from the results of experiments with forced oscillations in pitch, roll and yaw, available at TsAGI for an airplane similar to the ONERA light aircraft.
The mathematical model of aerodynamic characteristics (1) is based on the results of static tests and steady conical rotation around a velocity vector. However, in the case of stalling the vector of angular rate, as a rule, is not collinear to the velocity vector of the aircraft. Such movements cannot be modeled in a wind tunnel. For their analysis, a hybrid approach is used [8,9], in which the angular motion is decomposed into the conical rotation around the velocity vector, and an additional one, with smaller angular rates. To build such a more complete aerodynamic model, body-axis components of aircraft angular rate *p, q, r* are recalculated into wind axes:

$$\begin{aligned}
p_a &= p\cos\alpha\cos\beta + r\sin\alpha\cos\beta + q\sin\beta\\
r_a &= p\sin\alpha - r\cos\alpha\\
q_a &= -p\cos\alpha\sin\beta - r\sin\alpha\sin\beta + q\cos\beta.
\end{aligned}$$

Here $p_a = \Omega$ coincides with the rate of rotation in rotary balance tests, dependence of the aerodynamic coefficients on this component is substantially nonlinear and described by terms $\Delta C_i(\alpha,\beta,\Omega^*),\ i = Y,Z,l,m,n$ in model (1). r_a, q_a - components represent misalignment between velocity and angular velocity vectors. The misalignment components are normally small with respect to rotary balance rate. So, a linear dependence on these components is assumed. In terms of dimensionless angular velocities, we have:

$$C_i^d(\alpha,\beta,\bar{p},\bar{q},\bar{r},\delta) = C_i(\alpha,\beta,\bar{p}_a,\delta) + C_{i\bar{r}_a}\bar{r}_a + C_{i\bar{q}_a}\bar{q}_a,\ i = l,m,n \tag{2}$$

Here $C_i(\alpha,\beta,\bar{p}_a,\delta) \equiv C_i(\alpha,\beta,\Omega^*,\delta)$ are the total dimensionless aerodynamic coefficients (1), and $C_i^d(\alpha,\beta,\bar{p},\bar{q},\bar{r},\delta)$ are the aerodynamic coefficients of the modified aerodynamic model with the added damping. Rotational aerodynamic derivatives in wind-axes system $C_{i\bar{q}_a}, C_{i\bar{r}_a},\ i = l,m,n$ for equations (2), are determined by recalculating from aerodynamic derivatives in a body-axis system obtained in small-amplitude forced oscillation tests. The dependence of these damping derivatives on angle of attack is shown in Figure 3. This dependence is non-linear at moderate and high angles of attack; there are ranges of anti-damping. Therefore, the replacement of these aerodynamic data with the data for a similar aircraft can give only approximate results, which, however, are important for understanding the qualitative picture and assessing the sensitivity of spin calculation results to the damping in the aerodynamic model.

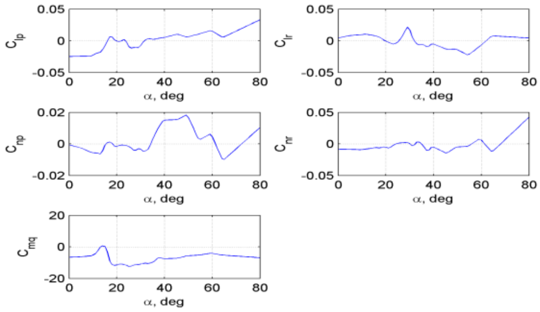


Figure 3: Damping derivatives versus angle of attack

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When analyzing the post-stalled dynamics, two options for loading the aircraft are considered:

1. Case 1: the nominal characteristics of the aircraft, mass m = 1000kg, the position of the center of gravity is r_x = 20% of the chord, moments of inertia I_x=1883kg m^2, I_y=2205 kg m^2, I_z=3356 kg m^2.
2. Case 2: the minimum mass m = 870 kg and the position of the center of gravity is r_x = 16% of the chord, I_x= 1185kg m^2, I_y=2195kg m^2, I_z=2636kg m^2.

2.2 Augmentation of the aerodynamic model by uncertainties

There is a number of sources of uncertainties of the aerodynamic model: influence of mounting devices used in wind tunnel tests, errors caused by extrapolation of aerodynamic data obtained for a scaled model on a real aircraft, a high level of aerodynamic uncertainties caused by flow separation at moderate-high angles of attack in the vicinity of stall, etc. To analyse the influence of uncertainties of the aerodynamic characteristics determined by experiments on calculation the aircraft's dynamic behaviour in the stalling and spin areas, in ONERA a large number of experiments was conducted, and a special technique for reliable averaging of various data was developed. Repeated wind tunnel measurements of the static characteristics of the aircraft under consideration were performed with various combinations of values of angle of attack and sideslip shown in Figure 4, with different methods of model suspension, different flow rates in wind tunnel, etc. Averaged aerodynamic characteristics were obtained by the Locally Weighted Projection Regression (LWPR) method [11-13], which is an incremental algorithm for approximation of functions. This allows to take into account a variety of results when testing in wind tunnels due to different experimental conditions (Re-effects, installation effects, etc.), and calculate the estimate of the reliability of the calculation. The final model is a combination of linear models weighted by Gaussian functions.
An example of the averaged static torque aerodynamic characteristics and the obtained confidence intervals is shown in Figure 5a). Figure 5b) shows the dependence of the aerodynamic coefficients on angle of attack and sideslip when averaging the static characteristics, the dotted lines show confidence bounds. Thin lines show one particular realization of measurements of the aerodynamic coefficients (curve A) used for analysis in this paper along with the average coefficients.

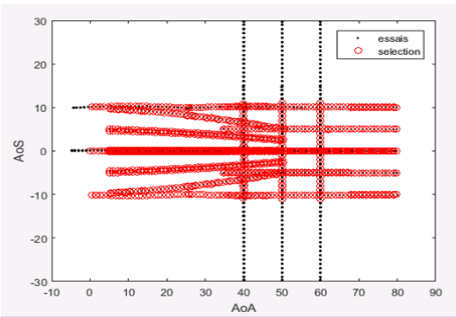

Figure 4: Training data: angle of attack and sideslip

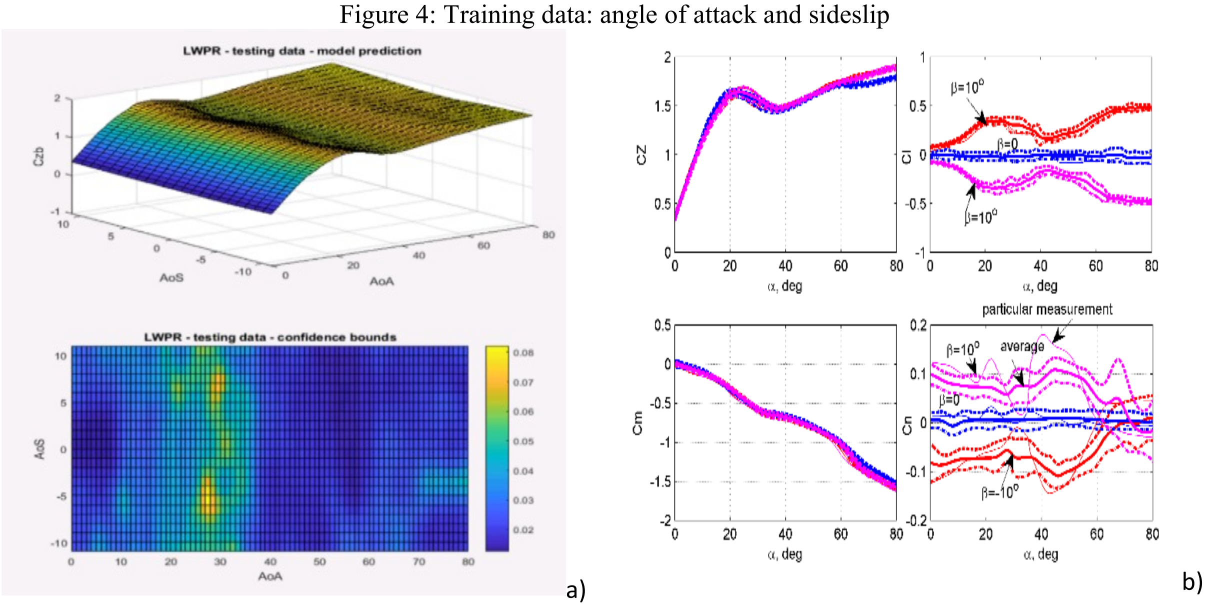

Figure 5: a) Nonlinear uncertainties modelling; b) average aerodynamic coefficients with confidence bounds.

DOI: 10.13009/EUCASS2019-537

3. Spin calculation

3.1 Steady-state spin parameters calculation. Comparison with the approximate approach

A traditional technique for computational investigation of spin is continuation on a parameter. The continuation and bifurcation analysis methodologies were used effectively in flight dynamics during last four decades [2-9]. A usual way to analyse the aircraft spin dynamics is considering the 8-th order autonomous system of motion equations obtained from the full six-degree of freedom motion equations in an assumption of fixed altitude [8] using the aerodynamic model (1)–(2):

$$\dot{} \qquad\qquad ,\delta) \tag{3}$$

where the state vector is $x_8 = (V, \alpha, \beta, p, q, r, \theta, \phi)' \in R^8$, and the control vector is $\delta = (\delta_e, \delta_a, \delta_r)' \in R^3$. The equilibrium states are defined by the system of algebraic equations:

$$F_8(x_8, \delta) = 0 \tag{4}$$

The equilibrium solutions of this system define an aircraft motion along a helical trajectory with the vertical axis of rotation. At high angles of attack and fast rotation such solutions correspond to equilibrium spin modes. During continuation of the equilibrium solutions of system (3) their local stability analysis using the linearized system of equations is also performed. Markers for qualitatively different types of instability are listed in Figure 5.
Often spin parameters are calculated according to the approximate 3-rd order equations expressing the balance of aerodynamic and inertial moments acting on an aircraft:

$$\begin{aligned}
&\frac{(I_y - I_z)}{1/2\rho S l^3}\Omega^{*2}\sin\alpha\sin\beta\cos\beta + C_l = 0\\
&\frac{(I_z - I_x)}{1/2\rho S l^3}\Omega^{*2}\sin\alpha\cos\alpha\cos^2\beta + C_m = 0\\
&\frac{(I_x - I_y)}{1/2\rho S l^3}\Omega^{*2}\cos\alpha\sin\beta\cos\beta + C_n = 0
\end{aligned} \tag{5}$$

The solutions of system (5) are used also as initial approximations when calculating spin equilibria using the full-order equations. The accuracy of approximate solutions is estimated comparing the equilibria of the 3rd order and 8th order systems calculated using the continuation technique. The results of such a comparison (for α<40°) calculated for the aerodynamic data represented by curve A (Fig.5) without damping terms are shown in Figures 6–7. Figures 6–7a) show the results of continuation on elevator deflection, Figure 7b) presents the results of continuation on rudder deflection. Here, the solutions of the approximate system (5) are marked in cyan, and all other colours, depending on the type of stability/instability (the symbols are shown in Figure 5) describe different stationary solutions of the full system (3), from normal flight regimes to spins. Figure 7 shows only part of the calculated eight variables describing the equilibria of system (3). As can be seen from the figures, for most of the deflections of the control surface in which there are stationary spins, the parameters calculated using the approximate and full equations coincide well. However, the range of existence of spins can differ significantly. Moreover, bifurcation diagrams according to the approximate and complete equations can be very similar for one altitude, H=500m, and differ essentially for another altitude, H=3000m, as seen from Figure 6. Therefore, the calculation of spin equilibria using the complete 8th order equations of motion is essentially more accurate.

3.2 Influence of damping terms on spin parameters

Analysis show that all spin eqilibria calculated according to aerodynamic model (1) are unstable. Comparison of bifurcations diagrams with and without damping terms is shown in Figure 6 where (and further) only four variables (*α, β, p, r*) from a full set of eight variables of system (3) are presented. Equilibrium parameters calculated without damping are shown by thin lines, and with damping by thick lines. With adding the damping terms a range of stable spins (red colour) appear, and bifurcations diagrams further change in comparison with the 3rd order approximate results. At the same time highly unstable spins at α=35° are practically invariable as to elevator deflection, as to adding damping terms.
To analyze the sensitivity of spin parameters and stability properties to variation of the damping value, the damping terms were varied by multiplying by a certain factor $k \in [0,1]$, the same for all aerodynamic derivatives and all angles

of attack. The calculation results for $k = 0,\ 0.5,\ 1$ are shown in Figure 9, demonstrating a change in the range of existence of the spin.
Another example of changing spin properties is presented in Figure 10, where continuation of spin equilibria on rudder deflection is performed. Here, at high angles of attack, spin parameters are practically invariable, but stability properties change significantly when adding the damping terms: stable spins appear which are the most dangerous.

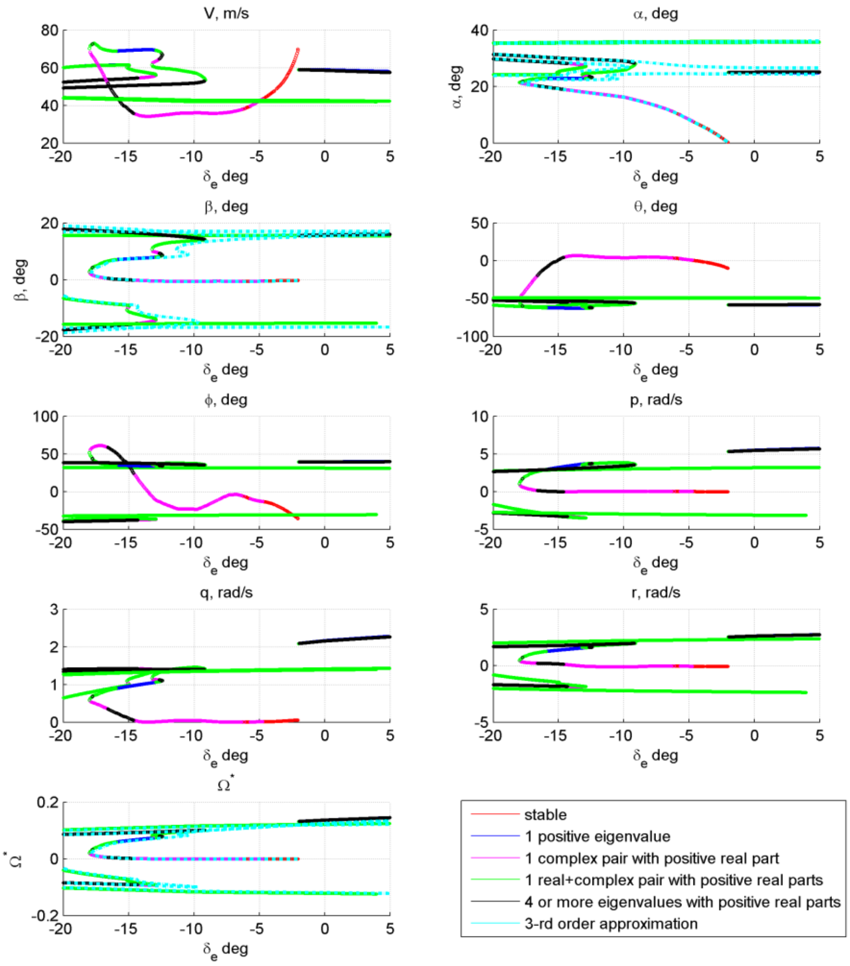

Figure 6: Comparison of equilibria according to the full and approximate equations, H=3000m, $\delta_a = 0, \delta_r = 0$.

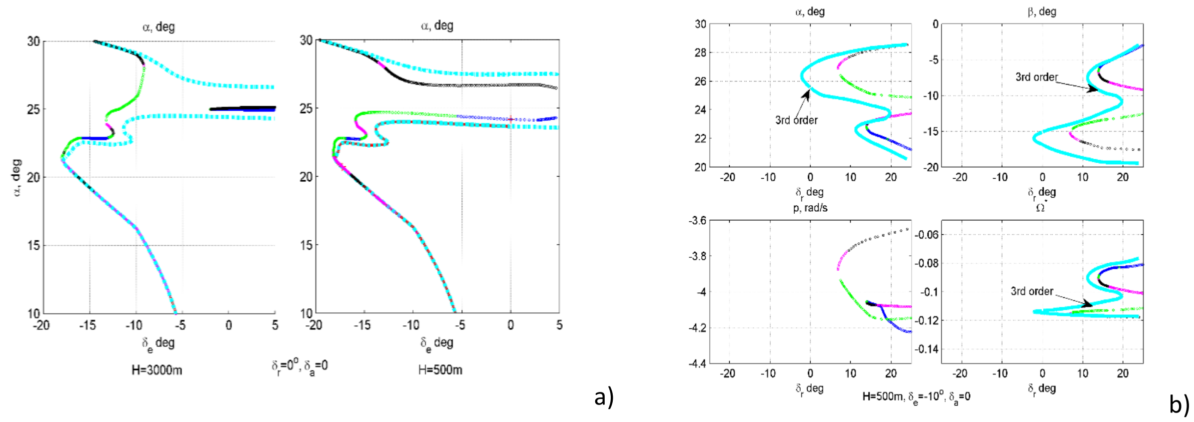

Figure 7. Comparison of equilibria according to the full and approximate equations a) continuation on elevator deflection, different altitudes b) continuation on rudder deflection.

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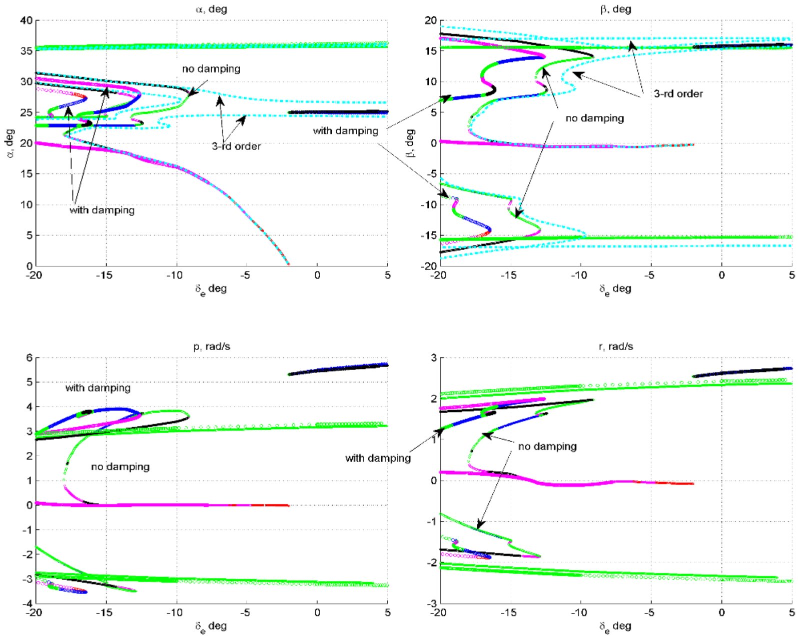


Figure 8. Comparison of equilibrium parameters with and without damping terms, H=3000m, δ_a=0, δ_r=0.

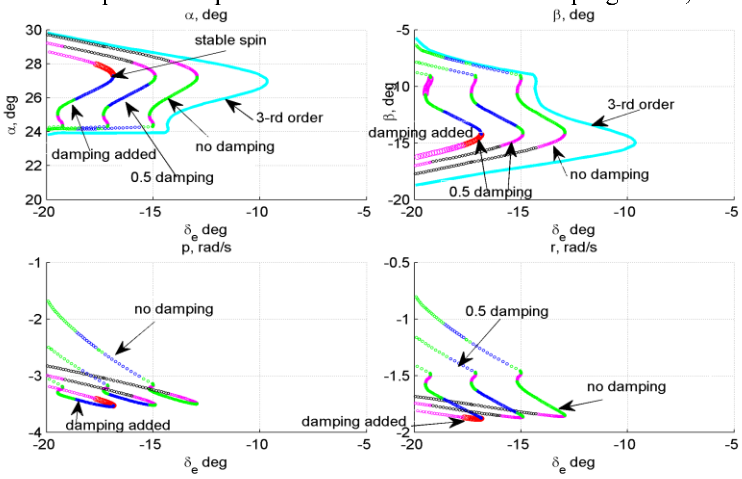


Figure 9. Change of spin parameters with adding the damping terms, H=3000m, δ_a=0, δ_r=0.

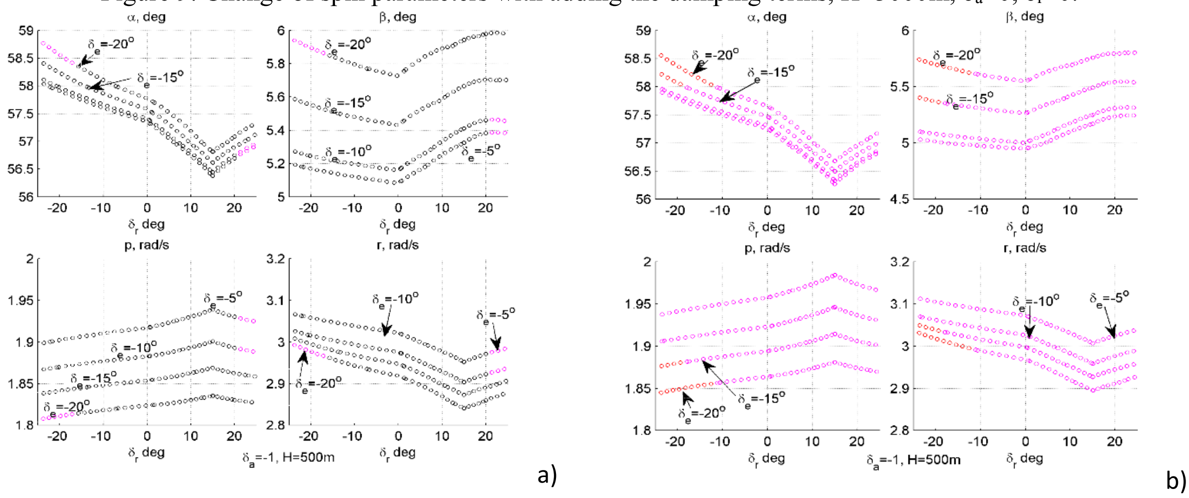


Figure 10. Change of stability properties of spins with damping adding, continuation on rudder deflection H=500m, δ_a=–1, a) without damping, b) with damping.

DOI: 10.13009/EUCASS2019-537

4. Sensitivity of spin parameters to uncertainties of the aerodynamic model

Two cases of static aerodynamic coefficient perturbations are considered:

- perturbations around the specific implementation of aerodynamic forces and moments, with a certain value of lateral aerodynamic asymmetry;
- perturbations around the averaged aerodynamic characteristics.

Various combinations of uncertainties of all aerodynamic coefficients within the obtained confidence bounds are considered. Calculations of stationary spin modes are carried out by continuation on three parameters: elevator deflection, ailerons, and rudder deflection. As calculations show, variations of coefficients of the aerodynamic loads ΔC_Y, ΔC_Z influence on the results of spin calculation very slightly. The scatter of the moment coefficients ΔC_l, ΔC_m, ΔC_n influences on the spin parameters much more strongly.

4.1 Variations of aerodynamic data with lateral asymmetry

The results below are obtained for the aerodynamic model (1)–(2) with the added damping terms (corresponding to Figure 10, case 1 of the aircraft mass/inertia parameters) at flight altitude H = 3000m. Figures 11–12 show the sensitivity of spin parameters to variation of static coefficients when all moment coefficients are increased inside the confidence limits: $+\Delta C_i$ $(i=l,m,n)$, or decreased $-\Delta C_i$., for equilibria with positive (Ω>0) and negative (Ω<0) rotation rates. Thin lines (or small markers) correspond to the perturbed coefficients, and thick lines correspond to the nominal coefficients. It can be seen that the bifurcation diagrams essentially depend on the perturbations of coefficients. Ranges of spin existence and stability properties also depend on the aerodynamic uncertainties. At the same time, unstable spins at α=35° are insensitive to both the elevator deflection and the uncertainty of the coefficients.

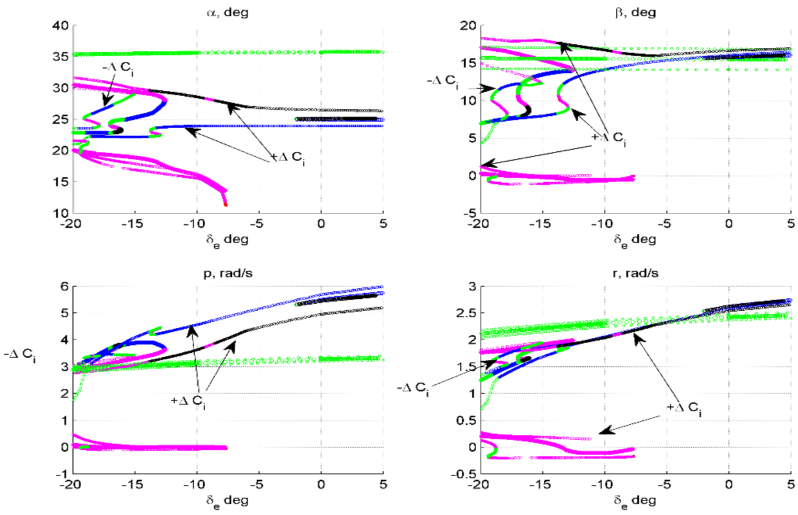


Figure 11. Sensitivity of spin parameters to variation of static coefficients, Ω>0, case 1.

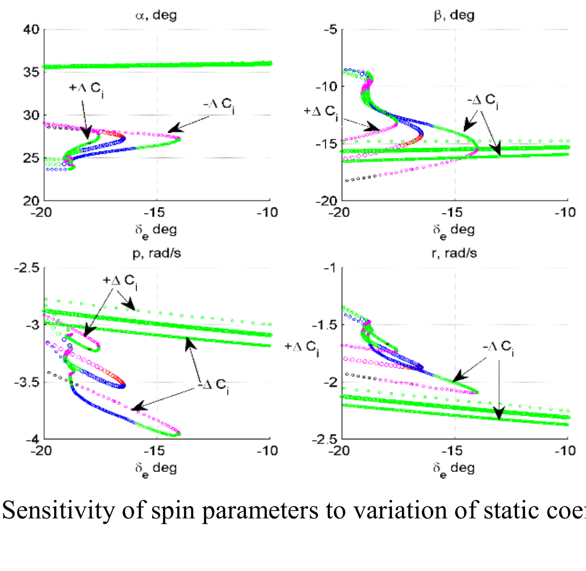


Figure 12: Sensitivity of spin parameters to variation of static coefficients, Ω<0, case 1.

DOI: 10.13009/EUCASS2019-537

Figure 13 shows the sensitivity of parameters of the same spins to separate variations of roll, pitch and yaw coefficients for the branch shown in Fig.11 (with Ω>0). It can be seen that for these spins the roll uncertainty has the largest influence on spin parameters. Figure 14 presents the worst-case combinations of uncertainties of aerodynamic coefficients, a) for spins with Ω>0 b) for spins Ω<0. The worst-case means in this case the largest difference with the nominal case: the roll and yaw uncertainties of different signs usually result in the largest difference in spin parameters compared with the nominal aerodynamic coefficients.

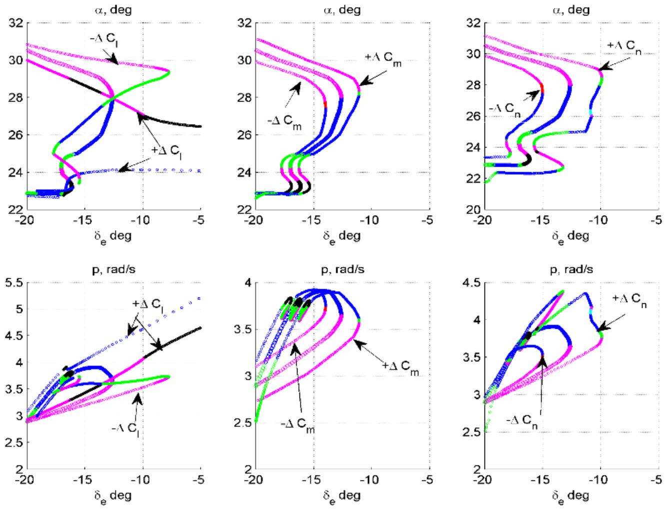
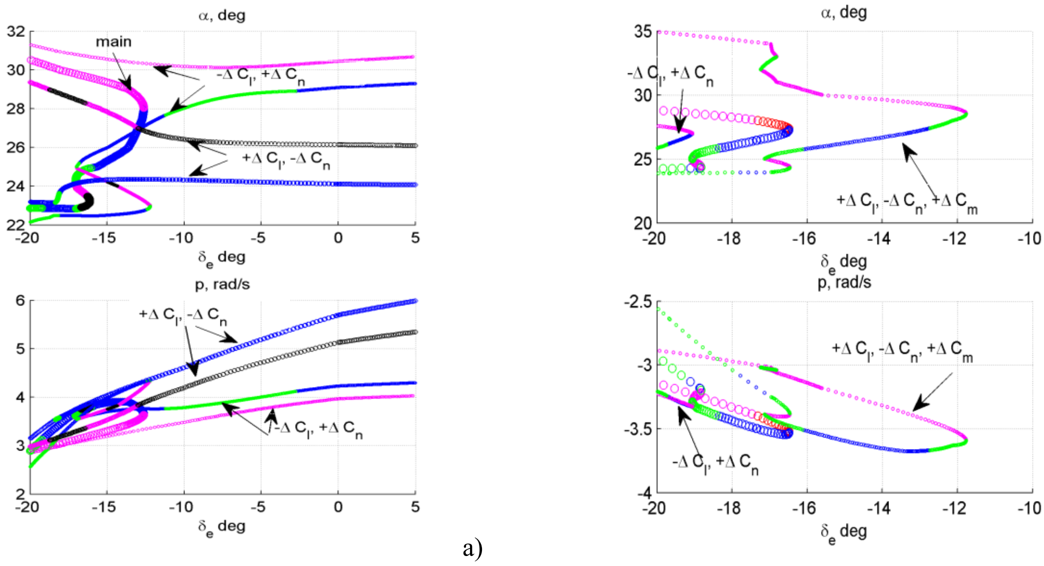


Figure.13: Sensitivity of spin parameters to separate variations of moment coefficients, case 1.



a) b)

Figure 14: The worst-case variations of aerodynamic coefficients, case 1 a) Ω>0, b) Ω<0.

Note that in all the considered cases there are no stable (red color) spins or its range is small. The search for periodic or other stable spin solutions near oscillatory unstable solutions gives only small ranges of δ_e where small-amplitude periodic solutions exist for some perturbations of aerodynamic coefficients. Thereby, stall into this kind of spin is unlikely, or the aircraft is recoverable from this spin despite the diversity of spin parameters under aerodynamic uncertainties, by moving the elevator to neutral position.

4.2 Variations around average aerodynamic coefficients

Another considered case of sensibility of spin parameters to the aerodynamic uncertainties is the case 2 of the aircraft mass/inertia parameters. The difference is that in this case, as calculations show, main spin regimes exist at angles of attack about 50°. The aerodynamic asymmetry is smaller in this case, and perturbations of the aerodynamic coefficients about the average values are considered.

DOI: 10.13009/EUCASS2019-537

Figures 15–16 show the results of continuation on elevator deflection of some branches of spin, with different directions of rotation, for various perturbed (inside the confidence intervals) aerodynamic coefficients. The uncertainties leading to the largest ranges of spin existence are considered. For spins with $\Omega<0$ (Figure 15), such uncertainties in ascending order are $+\Delta C_m$, $+\Delta C_l$, $-\Delta C_n$. The branch corresponding to the worst combination of these uncertainties $(+\Delta C_m, +\Delta C_l, -\Delta C_n)$ is also shown. At the same time, for the nominal aerodynamic coefficients, this branch is outside the available range of elevator deflection, i.e. this type of spin does not exist. This example shows how strong can be influence of uncertainties on the spin parameters. Luckily, all these spins are unstable, and there are no any stable attractors in the vicinity. Therefore, they do not determine the aircraft behavior in the case of loss of control.

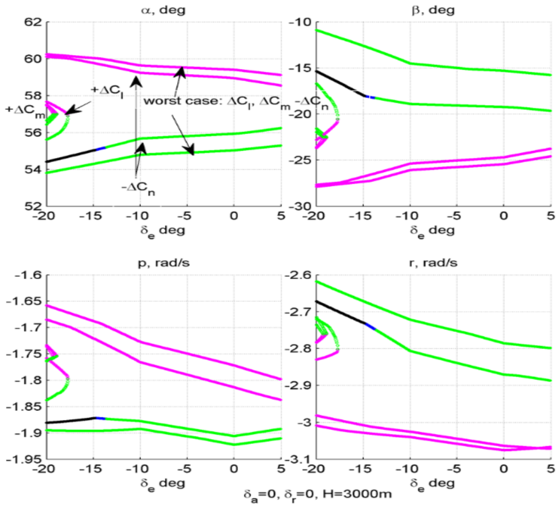


Figure 15: Spins for the perturbed roll, yaw, and pitch coefficients, the worst-case perturbation $(+\Delta C_m, +\Delta C_l, -\Delta C_n)$, case 2, $\Omega<0$.

Figure 16 shows the similar results for spins with $\Omega>0$. In this case the worst-case perturbations is $(-C_l, +\Delta C_m, +\Delta C_n)$. As in the previous case, for the nominal aerodynamic coefficients, the branch of spins is outside the range of elevator deflection, however, for a small damping (0.1 from the nominal value) it appears. All spins are unstable also.

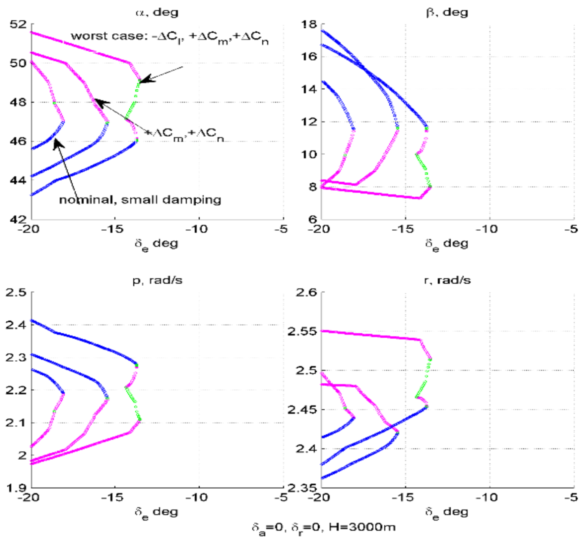


Figure 16: Spins for the perturbed aerodynamic coefficients, and the worst-case perturbation $(-C_l, +\Delta C_m, +\Delta C_n)$, case 2, $\Omega>0$.

For the nominal aerodynamic coefficients a small branch of spins appear at $\delta_e=-20°$, $\alpha=48°$ with aileron deflection $\delta_a=0.2$. With a further increase of aileron deflections to $\delta_a=0.5$, as shown in Figure 17a), the area of spins expands to elevator deflection range $\delta_e \in [-20, -16°]$. For the worst-case combination of uncertainties $(-C_l, +\Delta C_m, +\Delta C_n)$ it expands to the full the range of elevator deflections [–20, 5°], and a region [–20, -19°] of stable stationary spin at $\alpha=55°$ appears. The search for periodic solutions allows to detect stable periodic oscillatory spins resulting from the loss of stability of steady-state spins outside the allowed range of elevator deflections (Hopf bifurcation). At $\delta_e=-4°$ in a result of a series

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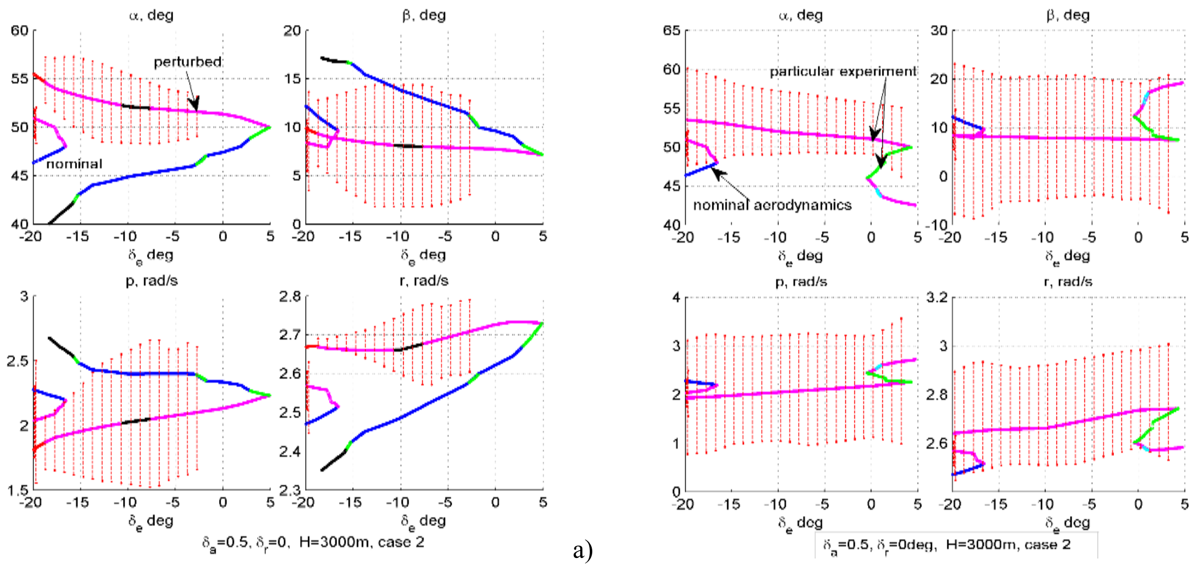
of further bifurcations, it transforms into a chaotic spin attractor. The range of existence of oscillatory spins for the perturbed aerodynamic model *(-ΔC$_l$,+ΔC$_m$,+ΔC$_n$)*, periodic or chaotic, is shown in Fig. 16a) (approximate amplitudes for chaotic spins). Examples of stable periodic and chaotic trajectories and their projections onto the plane (α, β) are shown in Figure 18. For the nominal model, the range of stable spins with respect to the parameter δ_e is very narrow: $\delta_e \in [-20, -19.5°]$, and these stable spins have chaotic trajectories.

For comparison, Figure 17 b) shows the parameters of stationary and stable oscillatory spins, periodic or chaotic, for the particular experimental curve A aerodynamic coefficients. For these aerodynamic coefficients stable oscillatory spins exist in almost the entire range of elevator deflections, on the contrary to the narrow region of stable spins for the average aerodynamic coefficients. The luck is that all these stable spins disappear when deflecting ailerons to zero.



a) b)

Figure 17: Stable oscillatory spins for average and a) perturbed aerodynamic coefficients, b) a particular realization of aerodynamic coefficients.

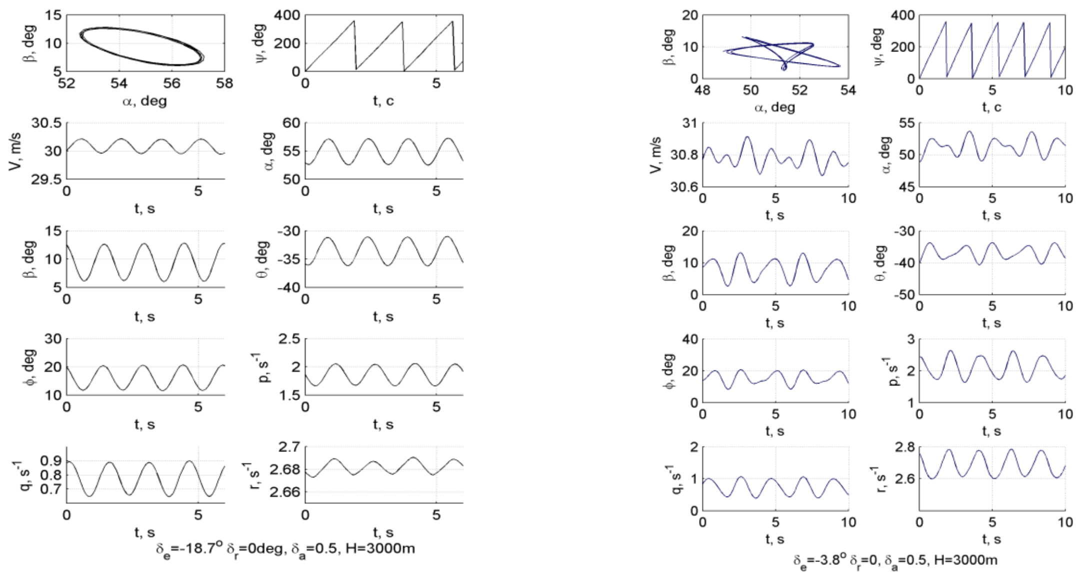


Figure 18. Examples of trajectories of stable oscillatory spins (case 2).

The following results are obtained considering the continuation of spin equilibria on another parameter – aileron deflection. Figure 19 shows the dependencies of spin parameters versus aileron deflections at δ_e=–15°, δ_r=0 for the nominal and perturbed moment coefficients: roll, pitch or yaw, respectively (branch with Ω<0). A significant dependence of the spin parameters and bifurcation diagrams on the uncertainty of the coefficients, especially on the uncertainty of yaw moment coefficient, is visible. However, the above stationary spin modes are unstable, and this does not allow to predict the behavior of the aircraft in the case of loss of control.

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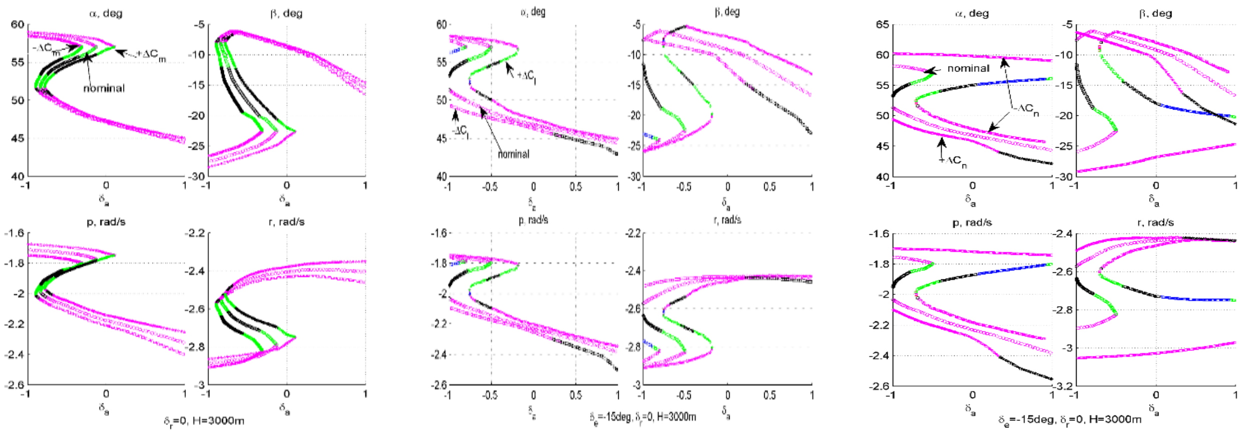

Figure 19: Continuation on aileron deflection: dependence of spin parameters on the uncertainties of roll, pitch, and yaw moment coefficients, case 2.

The result of search for stable oscillatory spins is shown Figure 20, dependence of the parameters of stationary and approximate amplitudes of oscillatory chaotic spins for the nominal and perturbed yaw coefficients *(–ΔC$_n$)* are shown in Figure 20a) for $\delta_e = -15°$, $\delta_r = 0$ ($\Omega<0$). It can be seen that the ranges of stable spins are quite different for the nominal and perturbed coefficients, and this makes a problem for an adequate prediction of the aircraft behavior after loss of control. A good opportunity is that there are no stable spins at δ_a=0 in both cases, that is in both cases spins are recoverable be setting aileron to neutral position.
Figure 20b) shows the branches of stationary spins and ranges of stable periodic and chaotic spins for the nominal and the worst-case uncertainties of the aerodynamic coefficients $(-\Delta C_l, +\Delta C_m, +\Delta C_n)$ for $\Omega>0$. For the perturbed aerodynamic coefficients the region of the stable oscillatory spins is much wider than for the nominal coefficients.

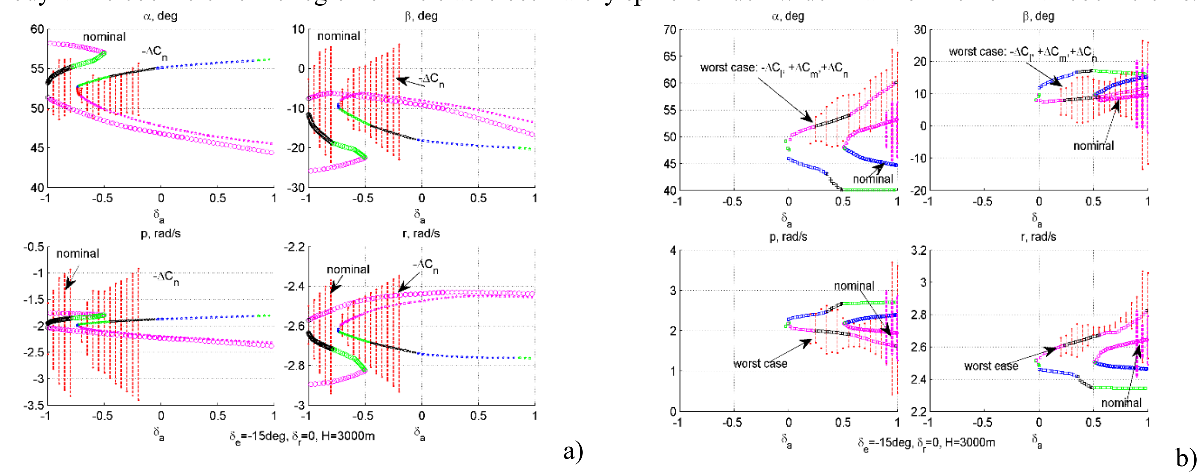

Fig.20: Parameters of stationary and oscillatory spins for the nominal and perturbed coefficients, case 2.

Thus, the ranges of the spin existence, and in particular, of the most dangerous, stable spin, can differ markedly with a slight variation, within the experimental errors of the parameters of the aerodynamic coefficients of the aircraft. It should be noted that the nature of the trajectories depends significantly on the values of the damping terms, which, like all quantities determined experimentally, have uncertainties. Figure 21 shows the change in the spin trajectories with an increase in damping by 15% compared to the nominal one: the trajectories from chaotic become periodic. When reducing the damping, the attracting set (consisting of chaotic or periodic trajectories) can turn into weakly unstable or "quasi-stable", which is much more difficult to calculate than stable stationary modes, but which can also be very dangerous. This fact should be taken into account when determining the most dangerous, stable spin modes.

4.3 Flat spin. Sensitivity to aerodynamic uncertainties

For the case 1 of the aircraft of mass/inertia parameters two separate branches of flat spins at high angles of attack (65-70°) were also found (an extension of Fig. 7). Figure 22 shows these branches of spins having negative rotation rate $\Omega<0$ (δ_a=0, δ_r=0, H=3000m) and their low sensitivity to yaw moment uncertainty. Sensitivity to uncertainties of other

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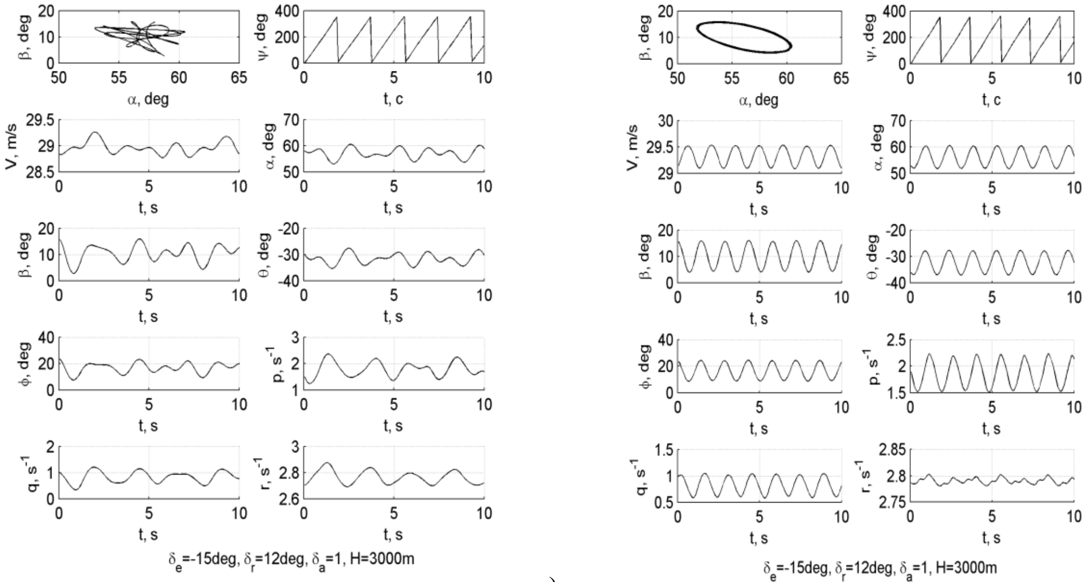
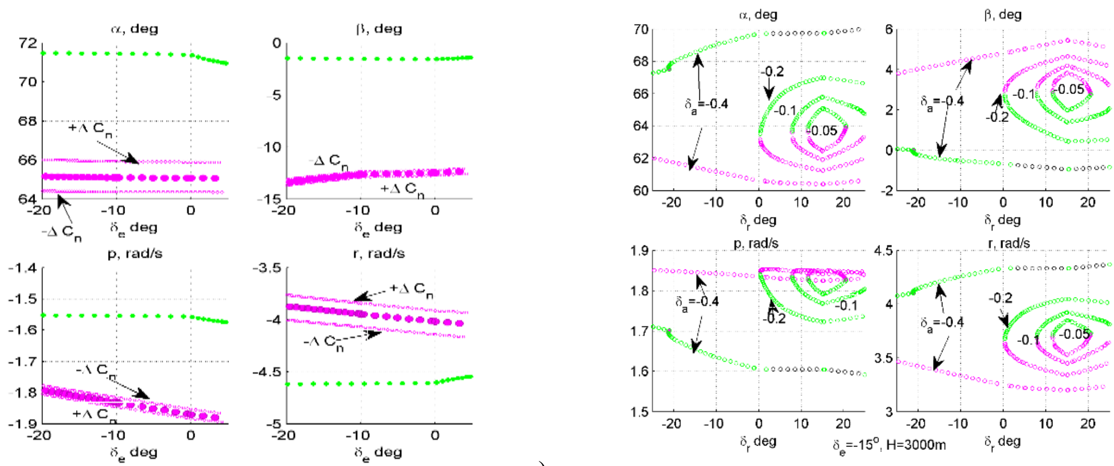
coefficients is essentially less. Branches of steady–state flat spins with Ω>0 appear only with deflection of ailerons. The dependence of flat spin parameters on aileron deflection is shown in Figure 21 b).



a) b)

Fig.21. Sensitivity of trajectories of stable spin to variation of damping terms of the aerodynamic model a) nominal damping: chaotic spin b)damping increased by 15%: periodic spin.



a) b)

Fig.22.Sensitivity of steady-state flat spin to yaw moment uncertainty (Ω<0), b) sensitivity of flat spin to aileron deflection (Ω>0).

The oscillatory instability of stationary spin modes (magenta color) indicates a possibility of existence of periodic, possibly stable, periodic solutions, i.e. stable oscillatory spins. The search for such solutions made it possible to detect such stable oscillatory spins both for Ω>0, and Ω<0 when δ_a=0, although there are no generative oscillatory instable equilibria when Ω>0. Figure 23 shows the amplitudes of such stable oscillatory periodic spins and the approximate amplitudes of stable chaotic spins depending on elevator deflection at δ_a=0, δ_r=0, along with other steady-state solutions (both, red and magenta vertical lines correspond to stable oscillatory spins). Oscillatory spins with Ω>0 are periodic, and with Ω<0 —periodic or chaotic, depending on elevator deflection, see trajectories in Fig. 24.
An important feature of these spin modes is their low sensitivity both to the amount of deflection of various controls (the effectiveness of controls at high angles of attack is small) and to uncertainties of the aerodynamic coefficients. The variation of roll and pitch moments within the confidence bounds practically does not change the character of the spin. With elevator deflection changing periodic spin trajectories can become chaotic, or ice-versa, that does not change the danger of the spin.
The analysis shows that the uncertainty of the yaw moment coefficient has the greatest effect on the spin parameters, and the bifurcation patterns can significantly depend on even small variations of the aerodynamic coefficients.

DOI: 10.13009/EUCASS2019-537

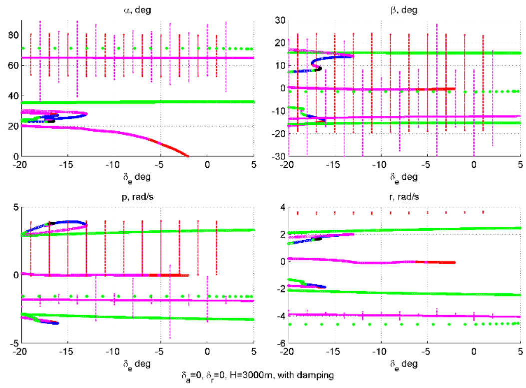
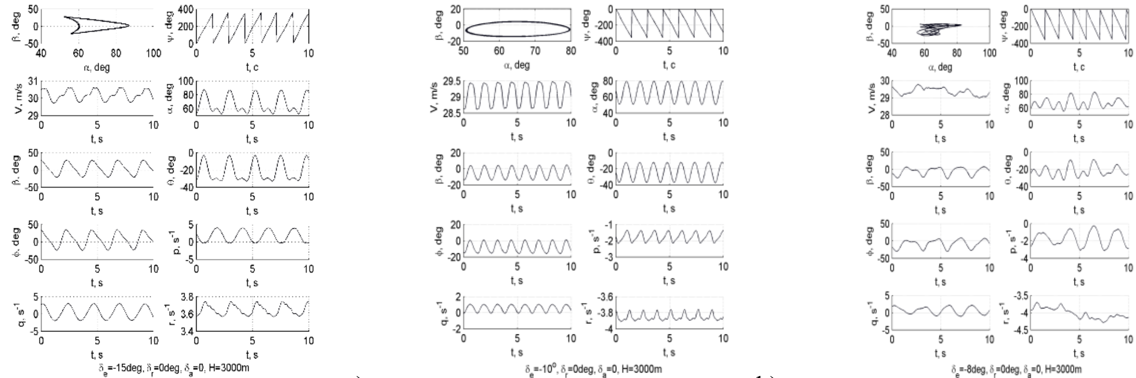


Fig. 23. Steady-state and stable oscillatory spins depending on elevator deflection, $\delta_a = 0$, $\delta_r = 0$.



a) b) c)

Fig. 24. Trajectories in flat spin a) Ω>0, b, c) Ω<0.

Another example of the high sensitivity of bifurcation diagrams to the uncertainty of the yaw coefficient is shown in Figure 25. The uncertainty varies from 20% to 70% of the maximum yaw moment uncertainty $+\Delta C_n$. It can be seen that the bifurcation pattern is changing very intricately. However, the most dangerous, stable spins with a large area of capture are substantially less sensitive to aerodynamic uncertainties. In may be useful to estimate the size of the region of attraction of a stable spin. An attempt of such estimation for stationary spins was undertaken in [14]. Unfortunately, here and in many other cases, the most dangerous stable spin modes are oscillatory, and estimation of their region of attraction, as well as understanding its correlation with sensitivity to uncertainties, is problematic.

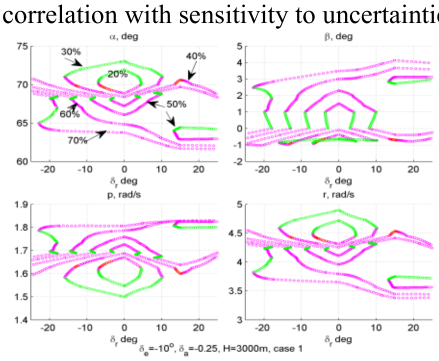


Figure 25: An example of sensitivity of bifurcation diagrams to uncertainties of yaw moment, δ_e =–10, δ_a =–0.25.

Spin entry/recovery: sensitivity to uncertainties. It is practically important to consider precisely those spin modes in which the aircraft can stall from normal flight. In this analysis spin entry is performed from horizontal flight at H=3000m by elevator deflection up to δ_e=–15–20° and rudder deflection up to $\delta_r = \pm$ 20, 25° (δ_a=0) for a few seconds. The pilot's quick response allows to recover the airplane from spin, setting the controls to neutral positions if angle of attack has not yet reached ~ 40° both, for the nominal, and for the disturbed aerodynamic model. Delayed setting of controls to the neutral positions results in flat spin does not allow recover the airplane from this spin. Similar behavior is observed for nominal and perturbed aerodynamic models. The use of aileron and rudder against rotation sometimes allows to recover the airplane from the flat spin, but not for all perturbations of the aerodynamic model, one example is shown in Figure 26. In any case, the pilot actions must be very accurate, otherwise the airplane may enter in another spin with opposite rotation.

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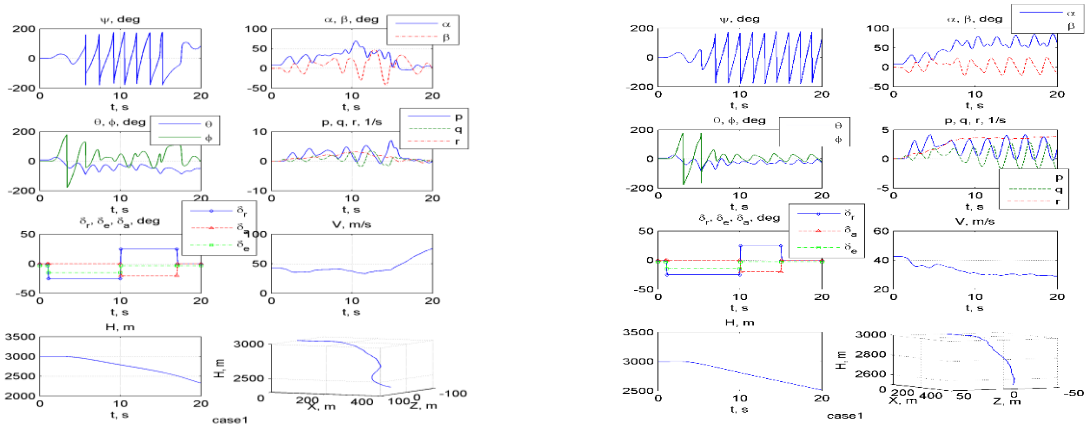
5. Conclusions

A representative aerodynamic model of a light airplane in a wide range of angles of attack, sideslip, angular rates, and controls was developed, and augmented by uncertainties based on a large number of experiments. A comparative analysis of spin parameters, and spin entry/recovery behavior for the nominal and perturbed aerodynamic models allowed to reveal high sensitivity of bifurcation diagrams to uncertainties of the aerodynamic coefficients, and lower sensitivity of the most dangerous stable flat spins to the aerodynamic uncertainties. The desirability of spin analysis has become clear not only for the nominal aerodynamic model, but also for the model taking into account aerodynamic uncertainties, especially the uncertainty of the yaw moment.



a) b)

Figure 26: Spin entry and recovery a) nominal aerodynamics: recovery b) perturbed $+\Delta C_n$: no recovery.

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