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Optimal 4D Trajectory Planning for Multiple Aircraft in Continuous Descent Operations

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Abstract

This paper deals with trajectory planning for multiple aircraft in continuous descent operations using logical constraints in disjunctive form which arise in modeling passage through waypoints and time-based separation constraints. In the literature, logical constraints are modelled by means of the introduction of auxiliary integer variables. Furthermore, for the passage through waypoints usually a multiphase optimal control approach is considered. In this paper, an embedding technique is employed to transform logical constraints in disjunctive form into inequality and equality constraints which involve only continuous auxiliary variables, thus avoiding the computational complexity of multi-phase mixed-integer optimal control approaches.

1. Introduction

In Air Traffic Management (ATM), the flight of several aircraft can be modeled as an hybrid dynamical system, which can be regarded as a set of interacting continuous dynamical systems. A number of frameworks have been proposed to model hybrid dynamical systems, in which, in general, differential equations describe the dynamics of each system, whereas logical constrains describe the behavior of the systems during the interactions among them and the interaction with the environment in which they operate. In the ATM context, logical constraints describe, for instance, policies to apply in conflict detection and resolution and operational constraints to be fulfilled during flight. The main operational constraints to be fulfilled during flight are separation constraints, keep-out constraints to avoid no-fly zones and passage constraints through or by waypoints [1].

Given a set of aircraft, separation constraints between them can be expressed as follows: pairwise, they must keep a vertical distance greater than a minimum vertical safety distance or an horizontal distance greater than a minimum horizontal safety distance. The minimum horizontal separation distance can be fixed or variable. In the latter case, it can be established based on the turbulence generated by the preceding aircraft and the ability of the following aircraft to resist turbulence [2]. Obstacles and no-fly zones are in general polyhedral regions of air space. However, the corresponding keep-out constraints are usually introduced by bounding ellipsoids around obstacles. In some cases, this is a coarse approximation. Keep-out constraints from a polyhedral region of air space can be expressed as follows: each aircraft must stay outside one of the half-spaces defined by the planes that supports the faces of the polyhedron. This method for modeling keep-out constraints from obstacles can be extended to model passage constraints through windows or waypoints in the air space. In this manner, the multiphase modeling of the problem is avoided. This is an interesting possibility, since multiphase optimal control models imply the introduction of additional variables to the problem, such as the duration of the phases and additional constraints such as, the linkage constraints between them to enforce continuity of the state variables between contiguous phases [3].

It is easy to see that all the constraints mentioned above are expressed in disjunctive form. Standard modeling techniques are able to tackle constraints in disjunctive form using binary variables. The trajectory planning problem for multiple aircraft with logical constraints in disjunctive form can be solved as an Optimal Control Problem (OCP) for an hybrid dynamical system and a common approach for solving this class of problems is to formulate them as a mixed-integer programming problems. In [4], the optimal cooperative three-dimensional conflict resolution problem among multiple aircraft has been solved in which separation constraints among aircraft expressed in disjunctive form have been included in the model using continuous auxiliary variables. In [5], the optimal path planning problem for multiple UAVs in the horizontal plane with collision avoidance has been studied, in which constraints for collision avoidance

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with rectangular obstacles expressed in disjunctive form are included in the model using continuous auxiliary variables. In [6], the trajectory optimization problem for multiple aircraft landing on a single runway in the presence of constraints on the air space has been treated. The constraints considered are passage constraints through windows in the air space and optimal trajectories have been determined by solving a non-sequential constrained multiple-phase optimal control problem.

In this paper, the embedding technique proposed in [5] to model rectangular obstacle avoidance in the horizontal plane has been extended to model time-based separation constraints and passage through waypoints constraints in trajectory optimization for multiple aircraft. The modeling of passage constraints through waypoints has been done by defining vertical walls in the air space with a cuboidal window around the waypoint. In this way, introducing multiple phases in the model to enforce passage through waypoints is avoided. Moreover, the dimensions of the windows can be easily calibrated to induce a fly-by or a fly-through the waypoint. This study can be classified into the category of Continuous Descent Operations (CDO) [7]. During CDO, aircraft descend from the cruise altitude to the final approach fix at or near idle thrust without level segments at low altitude minimizing the need for high thrust levels to remain at a constant altitude and reducing the environmental impact. Actually, the term CDO makes reference to the different techniques to maximize operational efficiency and, at the same time, fulfilling local air space requirements and constraints. These operations are known as Continuous Descent Arrivals, Optimized Profile Descents (OPD), Tailored Arrivals, 3D Path Arrival Management and Continuous Descent Approaches (CDA). In particular, an OPD is a descent profile normally associated with a Standard Terminal Arrival Route (STAR) and designed to allow maximum use of a CDO. Planning CDO is one of the functions of the so called Arrival Managers (AMAN) whose purpose is to ensure an optimal sequencing and spacing of arrival traffic [8].

Most of the previous research on CDO based on optimal control theory focused on the trajectory optimization of a single aircraft. In [9] a multi-phase optimal control method based on the pseudospectral technique has been employed to optimize vertical trajectories for individual aircraft in CDA. Since the lateral path is assumed to be given by a STAR procedure, this work focused on optimizing vertical profile only using time and fuel consumption as performance indices. All the phases are formulated based on operational constraints and flap/gear schedules. The initial along track distance is free. Hence, it is possible to calculate both the optimal top-of-descent and CDA trajectory. The optimal trajectories have been computed for two aircraft types: a Boeing 737-500 and a Boeing 767-400. In [10], the vertical trajectory optimization for the en route descent phase of an aircraft has been studied in the presence of both along-track and cross winds, which are both modeled as functions of altitude. Flight idle thrust has been assumed during the entire descent phase. The problem has been formulated as an optimal control problem. The flight range has been specified from a point during the latter stages of the cruise to the meter fix. Calibrated airspeed (CAS) and Mach constraints, which were the state path constraints, have been considered, along with flight path angle constraints, and a maximum descent rate limit, which was a mixed input and state path constraint. The descent trajectory has been optimized with respect to two cost functionals: fuel and emissions. The effects of wind speed, windshear, and cross-wind on the optimal trajectory have been analyzed using the models of two types of aircraft, Boeing 737-500 and Boeing 767-400.

Less research efforts have been devoted to combine optimization of trajectories of multiple aircraft and sequencing for approaching a Terminal Manoeuvring Area (TMA) in which all aircraft follow CDA, whilst satisfying the operational requirements. This fact motivated the study presented in this paper. Two types of CDA exist depending on the lateral path followed, generally referred to as CDA under vectoring and advanced CDA. In the first case, the lateral path followed by the aircraft is assumed to be specified through instructions provided by the Air Traffic Control (ATC). In the second case, the lateral path of the aircraft is pre-defined and is based on a STAR. The problem that has been solved to validate the method proposed in this paper can be stated as follows. Given the dynamic models of a set of aircraft, their initial and final states, and a set of operational constraints, find the optimal trajectories that steer the aircraft from the initial to the final states, fulfilling all the constraints and optimizing an objective functional. In particular, the optimal trajectories of multiple aircraft in converging arrival routes are computed taking into account time separation constraints and their optimized profile descent along a STAR lateral profile. The problem has been solved using optimal control techniques. In particular, the OCP is transcribed using a Hermite-Simpson collocation method [11]. The resulting Non Linear Programming (NLP) problem has been solved using the NLP solver IPOPT [12].

The paper is structured as follows. In Sec. 2, the general optimal control problem for multiple dynamical systems is stated and the direct collocation approach for its resolution is described. In Sec. 3, the aircraft equation of motion and the flight envelope constraints are stated. In Sec. 4, the general approach to model logical constraints is presented, which is then particularized to model time-based separation constraints between aircraft and waypoint constraints. In Sec. 5, the results of the application of the proposed method to solve a trajectory optimization problem for multiple aircraft with logical constraints are reported and discussed. Finally, in Sec. 6, some conclusions are drawn.

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2. Optimal Control Approach

2.1 Statement of the Optimal Control Problem

The multi-aircraft flight-planning problem considered in this paper can be regarded as a multi-trajectory optimization problem in which the motion of each aircraft has been modeled as a differential-algebraic dynamic system

$$\Sigma^p = \{f^p : \mathcal{X}^p \times \mathcal{U}^p \times \mathbb{R}^{n_{sp}} \to \mathbb{R}^{n_{xp}},\ g^p : \mathcal{X}^p \times \mathcal{U}^p \times \mathbb{R}^{n_{sp}} \to \mathbb{R}^{n_{zp}}\}$$

for $p = 1, 2, \ldots, N_p$, where f^p describes the right-hand side of the differential equation

$$\dot{x}^p(t) = f^p(x^p(t), u^p(t), s^p)$$

and g^p describes the algebraic constraints

$$0 = g^p(x^p(t), u^p(t), s^p)$$

where $\mathcal{X}^p \subseteq \mathbb{R}^{n_x^p}$ and $\mathcal{U}^p \subseteq \mathbb{R}^{n_u^p}$ are the state and control sets, respectively, $x^p(t) \in \mathbb{R}^{n_x^p}$ is a n_x^p-dimensional state variable, $u^p(t) \in \mathbb{R}^{n_u^p}$ is a n_u^p-dimensional control input, and $s^p \in \mathbb{R}^{n_s^p}$ is a vector of parameters.

Since this multi-aircraft flight-planning problem also involves operative performances and flight envelope conditions for multiple aircraft, as well as the optimization of a specified performance index, the multi-trajectory optimization problem can be formulated as an OCP of a set of dynamic systems in which the goal is to find the trajectories and the corresponding control inputs that steer the states of the systems between two configurations, satisfying a set of constraints on the state and/or control variables while minimizing an objective functional. Therefore, the optimal control problem considered in this work can be stated as follows:

$$\min J(x(t), u(t), s, t) = \sum_{p=1}^{N_p} \Phi(t_F^p, x^p(t_F^p)) + \sum_{p=1}^{N_p} \int_{t_I^p}^{t_F^p} L^p(x^p(t), u^p(t), s^p, t)dt \tag{1a}$$

Subject to:

$$\dot{x}(t) = f(x(t), u(t), s, t) \tag{1b}$$

$$0 = g(x(t), u(t), s, t) \tag{1c}$$

$$\phi_l \le \phi(x(t), u(t), s, t) \le \phi_u \tag{1d}$$

$$x(t_I) = x_I \tag{1e}$$

$$\psi(x(t_F)) = 0 \tag{1f}$$

where

$$x = [x^1, x^2, \ldots, x^{N_p}]^T, u = [u^1, u^2, \ldots, u^{N_p}]^T, s = [s^1, s^2, \ldots, s^{N_p}]^T$$

and t_F^p denote de final time for aircraft $p = 1, 2, \ldots, N_p$. The objective function

$$J : \mathbb{R}^{n_x} \times \mathbb{R}^{n_u} \times \mathbb{R}^{n_s} \times [t_I, t_F] \to \mathbb{R}$$

is given in Bolza form. It is expressed as a combination of a Mayer term

$$\sum_{p=1}^{N_p} \Phi(t_F^p, x^p(t_F^p))$$

and a Lagrange term

$$\sum_{p=1}^{N_p} \int_{t_I^p}^{t_F^p} L^p(x^p(t), u^p(t), s^p, t)dt$$

Functions

$$\Phi^p : [t_I^p, t_F^p] \times \mathbb{R}^{n_{xp}} \to \mathbb{R}$$

and

$$L^p : \mathbb{R}^{n_{xp}} \times \mathbb{R}^{n_{up}} \times \mathbb{R}^{n_{zp}} \times [t_I^p, t_F^p] \to \mathbb{R}$$

are assumed to be twice differentiable. Function f is assumed to be piecewise Lipschitz continuous within the time interval $[t_I, t_F]$, and the derivative of the algebraic right-hand side function g with respect to z, that is,

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$$\frac{\partial g}{\partial z} \in \mathbb{R}^{n_z \times n_z}$$

is assumed to be regular within the time interval $[t_I, t_F]$. Vector $x_I \in \mathbb{R}^{n_x}$ represents the initial conditions given at the initial time t_I and function

$$\psi : \mathbb{R}^{n_x} \rightarrow \mathbb{R}^{n_\psi}$$

provides the terminal conditions at the final time t_F, and it is assumed to be twice differentiable. The system must also satisfy algebraic path constraints within the time interval $[t_I, t_F]$ given by the vector function

$$\phi : \mathbb{R}^{n_x} \times \mathbb{R}^{n_u} \times \mathbb{R}^{n_z} \rightarrow \mathbb{R}^{n_\phi}$$

with lower bound $\phi_l \in \mathbb{R}^{n_\phi}$ and upper bound $\phi_u \in \mathbb{R}^{n_\phi}$. Function ϕ is assumed to be twice differentiable.

In the objective function (1a), the Lagrange term represents a running cost, whereas the Mayer terms represent a terminal cost. A usual Lagrange objective function is to minimize the total amount of energy during the maneuver. A typical Mayer objective function is to minimize the final time. Equations (1b) and (1c) represent the differential-algebraic equation system that governs the motion of the dynamical system, e.g., the aircraft. Equation (1d) models the physical limits of performance of the dynamical system, typically expressed as upper and lower bounds on both states and control variables. Equations (1e) and (1f) denote the boundary (initial and final, respectively) conditions of the process in which the system is involved. Note that Equations (1c) and (1d) will also include the logical constraints that model conflict detection and resolution, and operational constraints as described in Sec. 4, which are the main interest of our study.

Hence, the optimal control problem (1) consists in finding an admissible control $u^*(t)$ such that the set of differential-algebraic subsystems follows an admissible trajectory $(x^*(t), u^*(t), s^*)$ between the initial and final state that minimizes the performance index $J(t, x(t), u(t), s, t)$. The final time, t_F, may be fixed or free.

2.2 Direct Collocation Transcription of the Optimal Control Problem

A direct numerical method has been employed to transcribe the OCP into a NLP problem. More specifically, a Hermite-Simpson direct collocation method [11] has been used. The time interval $[t_I, t_F]$ has been subdivided into N subintervals of equal length, whose endpoints are

$$\{t_0, t_1, \ldots, t_N\} \tag{2}$$

with $t_0 = t_I$ and $t_N = t_F$. In each time subinterval $[t_i, t_{i+1}]$, $i = 0, \ldots, N-1$, the Hermite-Simpson numerical integration scheme has been used.

The set of constraints of the resulting NLP problem includes the Hermite-Simpson system constraints that correspond to the differential constraint (1b) and the discretized versions of the other constraints of the optimal control problem. They include the algebraic constraints (1c), the state and control envelope constraints (1d), and the boundary conditions (1e) and (1f). The unknowns of the NLP problem are the values of the state and the control variables at the endpoints of each subinterval $[t_i, t_{i+1}]$, $i = 0, \ldots, N-1$.

For the NLP problem to be solved, the NLP interior point nonlinear solver IPOPT is one of the most suitable ones because it handles properly large-scale sparse nonconvex problems, with a large number of equality and inequality constraints. It implements an interior point line search filter method and can be used to solve general NLP problems. Moreover, it is open source. The mathematical details of the IPOPT algorithm can be found in [12]. Source and binary files are available at the Computational Infrastructure for Operations Research web site[1].

3. Aircraft Model Description

Following [13], a common three-degree-of-freedom dynamic model has been used which describes the point variable-mass motion of the aircraft over a spherical Earth model. In particular, a symmetric flight has been considered. Thus, it has been assumed that there is no sideslip and all forces lie in the plane of symmetry of aircraft.

[1] https://www.coin-or.org/

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3.1 Equations of Motion

The following equations of motion of the aircraft have been considered:

$$\begin{aligned}
\dot{V}(t) &= \frac{T(t) - D(h_e(t), V(t), C_L(t)) - m(t) \cdot g \cdot \sin\gamma(t)}{m(t)} \\
\dot{\chi}(t) &= \frac{L(h_e(t), V(t), C_L(t)) \cdot \sin\mu(t)}{m(t) \cdot V(t) \cdot \cos\gamma(t)} \\
\dot{\gamma}(t) &= \frac{L(h_e(t), V(t), C_L(t)) \cdot \cos\mu(t) - m(t) \cdot g \cdot \cos\gamma(t)}{m(t) \cdot V(t)} \\
\dot{\lambda}_e(t) &= \frac{V(t) \cdot \cos\gamma(t) \cdot \cos\chi(t)}{R \cdot \cos\theta_e(t)} \\
\dot{\theta}_e(t) &= \frac{V(t) \cdot \cos\gamma(t) \cdot \sin\chi(t)}{R} \\
\dot{h}_e(t) &= V(t) \cdot \sin\gamma(t) \\
\dot{m}(t) &= -T(t) \cdot \eta(V(t))
\end{aligned} \tag{3}$$

The three dynamic equations in (3) are expressed in an aircraft-attached reference frame (x_w, y_w, z_w) and the three kinematic equations are expressed in a ground based reference frame (x_e, y_e, z_e). The states of the system (3) are $V, \chi, \gamma, \lambda_e, \theta_e, h_e$ and m. Thus, $x(t) = (V(t), \chi(t), \gamma(t), \lambda_e(t), \theta_e(t), h_e(t), m(t))$. State variables V, χ and γ refer to the true airspeed, heading angle, and flight path angle, respectively. State variables λ_e, θ_e and h_e refer to the aircraft three-dimensional (3D) position, longitude, latitude and altitude, respectively. The aircraft position in two dimensions (x_e, y_e) is approximated as $x_e = \lambda_e \cdot (R + h_e) \cdot \cos\theta_e$ and $y_e = \theta_e \cdot (R + h_e)$. Finally, state variable m refers to the aircraft mass. The controls inputs are the bank angle μ, the engine thrust T, and the lift coefficient C_L. Thus, $u(t) = (T(t), \mu(t), C_L(t))$.

Parameter R is the radius of Earth and η is the speed-dependent fuel efficiency coefficient. Lift, $L = C_L S \hat{q}$, and drag, $D = C_D S \hat{q}$, are the components of the aerodynamic force. Parameter S is the reference wing surface area and $\hat{q} = \frac{1}{2}\rho V^2$ is the dynamic pressure. A parabolic drag polar $C_D = C_{D0} + K C_L^2$, and an International Standard Atmosphere model are assumed. The lift coefficient C_L is a known function of the angle of attack α and the Mach number.

Note that differential equations in (3) take the form of (1b) of the continuous optimal control problem stated in Sec. 2.1.

3.2 Flight Envelope Constraints

Flight envelope constraints are derived from the geometry of the aircraft, structural limitations, engine power, and aerodynamic characteristics. The performance limitations model and the parameters has been obtained from the Base of Aircraft Data (BADA), version 3.6 [14]:

$$\begin{aligned}
0 \le h_e(t) &\le \min[h_{M0}, h_u(t)], & \gamma_{min} &\le \gamma(t) \le \gamma_{max}, \\
M(t) &\le M_{M0}, & m_{min} &\le m(t) \le m_{max}, \\
\dot{V}(t) &\le \bar{a}_l, & C_v V_s(t) &\le V(t) \le V_{Mo}, \\
\dot{\gamma}(t) V(t) &\le \bar{a}_n, & 0.1 &\le C_L(t) \le C_{L_{max}}, \\
T_{min}(t) \le T(t) &\le T_{max}(t), & \mu(t) &\le \bar{\mu}
\end{aligned} \tag{4}$$

In (4), h_{M0} is the maximum reachable altitude and $h_u(t)$ is the maximum operative altitude at a given mass (it increases as fuel is burned). $M(t)$ is the Mach number and M_{M_0} is the maximum operating Mach number. C_v is the minimum speed coefficient, $V_s(t)$ is the stall speed, V_{M_0} is the maximum operating CAS and $\bar{a}_n$ and $\bar{a}_l$ are, respectively, the maximum normal and longitudinal accelerations for civilian aircraft. Finally, $T_{min}(t)$ and $T_{max}(t)$ correspond to the minimum and maximum available thrust, respectively, and $\bar{\mu}$ corresponds to the maximum bank angle due to structural limitations.

Note that inequality constraints in (4) take the form of (1d) of the continuous optimal control problem stated in Sec. 2.1.

DOI: 10.13009/EUCASS2019-544

4. Logical Constraints Modeling

In this section, the approach proposed in [5] has been followed in which an extension of the embedding optimal control technique stated in [15] and developed in [16] was proposed. The embedding technique in [15] and [16] introduced to transform hybrid optimal control problems into traditional smooth optimal control problems, in which the discrete aspect of the system arised only from switches in the dynamic equations, was adapted in [5] to deal also with the logical (discrete) components which also might appear as constraints.

It was shown in [17] that every Boolean expression can be transformed into Conjunctive Normal Form (CNF). Thus, it has been assumed that any logical constraint considered in this study can be written as a CNF expression

$$Q_1 \wedge Q_2 \wedge \ldots \wedge Q_n \tag{5}$$

where

$$Q_i = P_i^1 \vee P_i^2 \vee \ldots \vee P_i^{m_i} \tag{6}$$

Proposition P_i^j is either X_i^j or $\neg X_i^j$. Term X_i^j is a literal that can be either True or False and $\neg$ represents the negation or logical complement operator. Term X_i^j is used to represent statements such as "longitud $\lambda_e \leq 40$". Therefore, P_i^j takes the form

$$P_i^j \equiv \{g_i^j(x(t)) \leq 0\} \tag{7}$$

where $g_i^j : \mathbb{R}^{n_x} \rightarrow \mathbb{R}$ is assumed to be a C^1 function.

In order to include the logical constraint (5) into a smooth continuous optimal control problem formulation, it must be converted into a set of equality or inequality constraints in which binary variables are not considered. In this way, the combinatorial complexity of integer programming is eluded. Transcribing the conjunction in (5) is straightforward since it is equivalent to the following expression

$$\forall i \in \{1, 2, \ldots, n\} : \quad Q_i \tag{8}$$

Thus, taking into account (6), the logical expression (5) can be represented as

$$\forall i \in \{1, 2, \ldots, n\} : \quad P_i^1 \vee P_i^2 \vee \ldots \vee P_i^{m_i} \tag{9}$$

For the transcription of the disjunctions into a set of inequality constraints, a continuos variable $\beta_i^j \in [0, 1]$ is defined and related to each P_i^j in (7). Thus, (9) can be expressed as

$$\begin{aligned} \forall i \in \{1, 2, \ldots, n\} : \quad & \beta_i^j \cdot g_i^j(x(t)) \leq 0 \\ \text{and} \quad & 0 \leq \beta_i^j \leq 1 \\ \text{and} \quad & \sum_{j=1}^{m_i} \beta_i^j = 1 \end{aligned} \tag{10}$$

It is immediate to check that if $\beta_i^j = 0$ in the first term in (10), then constraint $g_i^j(x(t)) \leq 0$ is not fulfilled. On the other hand, if $0 < \beta_i^j \leq 1$ then $\beta_i^j \cdot g_i^j(x(t)) \leq 0$ is in fact $g_i^j(x(t)) \leq 0$, and thus constraint $g_i^j(x(t)) \leq 0$ is enforced. Finally, the last term in (10) guarantees that at least one of the propositions P_i^j holds.

Note that, as expected, equality and inequality constraints in (10) take the form of (1c) and (1d), respectively, of the continuous optimal control problem stated in Sec. 2.1. In the following subsections, the application of this technique to two instances of interest in the ATM context will be presented in detail.

4.1 Time-Based Separation Between Aircraft

One of the problems of most interest in ATM is the conflict detection and resolution problem [18]. An instance of this problem will be studied in this subsection. In particular, a collision avoidance model among different aircraft along routes converging at the same waypoint has been considered, in which a safety time-based separation is guaranteed at the merging waypoint.

Let t_{p_F} and t_{q_F} be the unfixed time at the merging waypoint of aircraft p and q, respectively, and let d_t be the safety time difference between two consecutive aircraft. Since in terms of the discretization (2), $t_{p_F} = t_{p_N}$ and $t_{q_F} = t_{q_N}$, then multi-aircraft time-based separation constraints can be expressed as

$$\forall q > p : |t_{p_N} - t_{q_N}| \geq d_t \tag{11}$$

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where condition $q > p$ prevents unnecessary duplication of constraints. Constraints (11) can be rewritten as

$$\begin{aligned} \forall q > p: \quad & t_{p_N} - t_{q_N} \geq d_t \\ \text{or} \quad & t_{q_N} - t_{p_N} \geq d_t \end{aligned} \tag{12}$$

Following the technique described above, if we define new variables $\delta^1_{p_N,q_N}, \delta^2_{p_N,q_N} \in [0,1]$ satisfying condition

$$\delta^1_{p_N,q_N} + \delta^2_{p_N,q_N} = 1,$$

Eq. (12) can be transformed into

$$\begin{aligned} \forall q > p: \quad & \delta^1_{p_N,q_N}(t_{p_N} - t_{q_N} - d_t) \geq 0 \\ \text{and} \quad & \delta^2_{p_N,q_N}(t_{q_N} - t_{p_N} - d_t) \geq 0 \\ \text{and} \quad & 0 \leq \delta^j_{p_N,q_N} \leq 1,\ j = 1,2 \\ \text{and} \quad & \delta^1_{p_N,q_N} + \delta^2_{p_N,q_N} = 1 \end{aligned} \tag{13}$$

The last constraint in (13) ensures that at least one of the constraints in (12) is fulfilled, that is, the safety time-based separation between aircraft p and q is ensured.

4.2 Waypoint Constraints

In a second instance, the modeling of an aircraft flying through a waypoint has been considered. The design of the waypoints has been based on the use of cuboids. More specifically, a cuboid centered at each waypoint has been defined, in such a way that passage constraints through waypoints have been modeled as passage constraints through cuboids.

Let $(\lambda_{W_l}, \theta_{W_l}, h_{W_l})$ and $(\lambda_{W_u}, \theta_{W_u}, h_{W_u})$ be the positions of opposite corners of a cuboid surrounding a single waypoint. Flying by this waypoint (that is, passing through the related cuboid) involves that at every endpoint t_i of the time subintervals of the discretization (2), the position of the aircraft $(\lambda_i, \theta_i, h_i)$ must remain inside it. In terms of logical constraints, this condition can be expressed as

$$\begin{aligned} \forall i \in \{1,2,\ldots,N-1\}: \quad & \lambda_{W_l} - \lambda_i \leq 0 \\ \text{and} \quad & \lambda_i - \lambda_{W_u} \leq 0 \\ \text{and} \quad & \theta_{W_l} - \theta_i \leq 0 \\ \text{and} \quad & \theta_i - \theta_{W_u} \leq 0 \\ \text{and} \quad & h_{W_l} - h_i \leq 0 \\ \text{and} \quad & h_i - h_{W_u} \leq 0 \end{aligned} \tag{14}$$

Note that, for one hand, constraints (14) are enforced at every point of the discretization except for the initial and final points, t_0 and t_N, since there is no a priori knowledge about when the aircraft is going to fly by the waypoint. On the other hand, these constraints obviously make sense only when the aircraft is closed enough to the waypoint.

To overcome this drawback a second auxiliary cuboid is considered to modelled freeflight mode of the aircraft. Let $\lambda_{min}, \theta_{min}, h_{min}, \lambda_{max}, \theta_{max}$ and h_{max} be the minimum and maximum values of the state variables λ, θ and h, respectively. In terms of logical constraints, the freeflight mode condition can be expressed as

$$\begin{aligned} \forall i \in \{1,2,\ldots,N-1\}: \quad & \lambda_{min} - \lambda_i \leq 0 \\ \text{and} \quad & \lambda_i - \lambda_{max} \leq 0 \\ \text{and} \quad & \theta_{min} - \theta_i \leq 0 \\ \text{and} \quad & \theta_i - \theta_{max} \leq 0 \\ \text{and} \quad & h_{min} - h_i \leq 0 \\ \text{and} \quad & h_i - h_{max} \leq 0 \end{aligned} \tag{15}$$

Then, the transcription into a logical disjunction which allows to select along the whole trajectory between flying by the waypoint mode (WM) or freeflight mode (FM), namely WM $\vee$ FM, can be expressed as

$$\begin{aligned} \forall i \in \{1,2,\ldots,N-1\}: \quad & \mathsf{WM}_i \\ \text{or} \quad & \mathsf{FM}_i \end{aligned} \tag{16}$$

DOI: 10.13009/EUCASS2019-544

where WM_i and FM_i denote if at discretization instant i the aircraft is in waypoint mode or freeflight mode, respectively.

Once again, following the technique described above, if we define new variables $\kappa_i^1, \kappa_i^2 \in [0, 1]$ satisfying condition

$$\kappa_i^1 + \kappa_i^2 = 1, \quad \text{for all} \quad i = 1, 2, \dots, N-1,$$

Eq. (16) can be transformed into

$$\begin{array}{rlcl}
\forall i \in \{1, 2, \dots, N-1\}: & \kappa_i^1(\lambda_{W_l} - \lambda_i) \leq 0 & \text{and} & \kappa_i^2(\lambda_{min} - \lambda_i) \leq 0 \\
\text{and} & \kappa_i^1(\lambda_i - \lambda_{W_u}) \leq 0 & \text{and} & \kappa_i^2(\lambda_i - \lambda_{max}) \leq 0 \\
\text{and} & \kappa_i^1(\theta_{W_l} - \theta_i) \leq 0 & \text{and} & \kappa_i^2(\theta_{min} - \theta_i) \leq 0 \\
\text{and} & \kappa_i^1(\theta_i - \theta_{W_u}) \leq 0 & \text{and} & \kappa_i^2(\theta_i - \theta_{max}) \leq 0 \\
\text{and} & \kappa_i^1(h_{W_l} - h_i) \leq 0 & \text{and} & \kappa_i^2(h_{min} - h_i) \leq 0 \\
\text{and} & \kappa_i^1(h_i - h_{W_u}) \leq 0 & \text{and} & \kappa_i^2(h_i - h_{max}) \leq 0 \\
\text{and} & 0 \leq \kappa_i^j \leq 1, \, j = 1, 2 & & \\
\text{and} & \displaystyle\sum_{j=1}^{2} \kappa_i^j = 1 & &
\end{array} \tag{17}$$

The last constraint in (17) ensures that at least one of the conditions in (16) is fulfilled, which means that at each discretization instant i the aircraft flies in freeflight mode or waypoint mode. Note that, depending on the performance index (1a) considered in the optimal control problem, two related undesired issues could potentially arise.

On one hand, the optimal solution could provide a trajectory in which the aircraft flies in freeflight mode for each discretization instant i. Therefore, in order to enforce the aircraft to actually fly by the waypoint, a penalty term must also be added to the numerical transcription of the performance index (1a). The use of this penalty term, which is set forth to encode the desired control objectives, implies that the numerical solution of the optimal control problem must combine the usual collocation technique describe in Sec. 2.2 with a penalty function methodology. In particular, in this work, the well-known continuation method has been implemented following a similar approach to [19]. In this specific case of the waypoint modeling, the penalty term takes the form

$$c_1 \sum_{i=1}^{N-1} \kappa_i^1 + c_2 \sum_{i=1}^{N-1} \kappa_i^2 \tag{18}$$

where c_1 and c_2 are suitable constants determined by the continuation method. In the context of a minimization performance index such us (1a) a large enough value c_2 such that $c_2 \gg c_1 > 0$, which penalizes the freeflight mode, guarantees that the aircraft actually flies by the waypoint.

On the other hand, the optimal solution could provide a trajectory in which the aircraft flies by the waypoint more than once. This situation can be easily avoid introducing into the model the following simple constraint

$$\sum_{i=1}^{N-1} \kappa_i^1 \leq c_3 \tag{19}$$

where $c_3 \in \{1, 2, \dots, N-1\}$ is a suitable constant. In the context of the problem considered in this work, to include the penalty term (18) in the objective functional, a small value of c_3 is enough to avoid this potential undesired issue.

Note that this single-waypoint model for a single aircraft can be straightforwardly extended to a multi-waypoint model for multiple aircraft. Moreover, since the approach employed to model logical constraint in disjunctive form is general, it allows any other waypoint model described in terms of Eq. (6) to be considered.

5. Numerical Results

To show the effectiveness of the methodology describe in Sec. 4, a numerical experiment has been carried out. In particular, a minimum-time STAR-based continuous descent of three aircraft along converging routes has been studied. The considered STAR procedure includes sequencing the aircraft at a merging point and passing through two other waypoints. The numerical experiment involves Airbus A-320 BADA 3.6 aircraft models in which the performance index is the sum of the duration of the flights of the three aircraft. Constraints derived from current flight regulation have been introduced, namely, time-base separation operational constraints have been imposed. In particular, the

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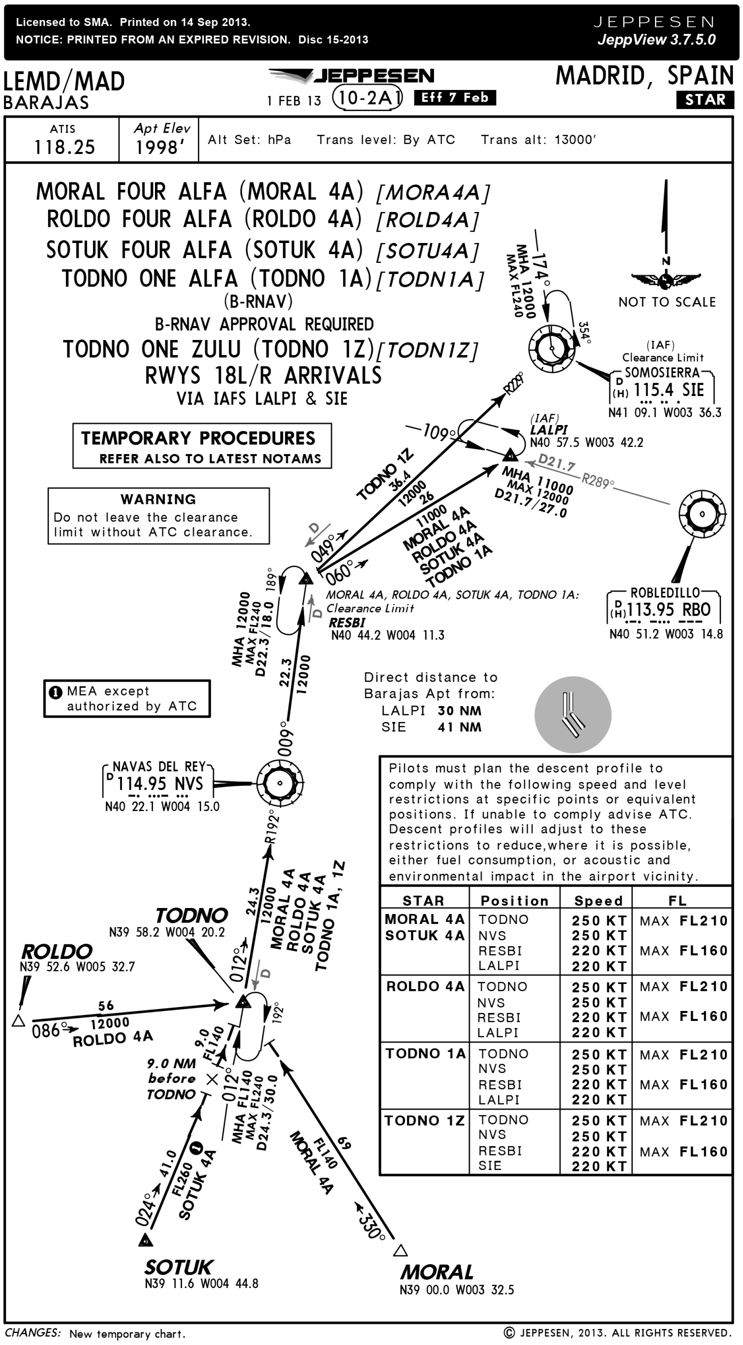

Figure 1: Chart of the Adolfo Suárez Madrid-Barajas (LEMD/MAD) STAR 10-2A1.

minimum time separation between aircraft has been 200 s. This specific value has been chosen taken as a reference the aviation regulation [20] in which, in general, aircraft have to be separated by at least 3 NM or 3 min in the TMA.

A STAR-based CDA has been considered, that is, the lateral path followed by the aircraft has been assumed to be specified in a navigation chart. In particular, the boundary conditions of the state variables have been selected from the chart of the Adolfo Suárez Madrid-Barajas (LEMD/MAD) TMA shown in Fig. 1. The initial position of Aircraft 1, Aircraft 2, and Aircraft 3 are supposed to be coincident with the ROLDO, SOTUK, and MORAL waypoints, respectively. Aircraft are constrained to pass through TODNO and RESBI waypoints and their common final position is assumed to be the LALPI waypoint. For the setting of the cuboids centered at the given waypoints the modeling defined in (14) has been considered. In particular, the cuboid centered at TODNO waypoint has been defined by the two corners $(39.560°, -4.24°, 5400\text{ m})$ and $(39.640°, -4.160°, 6000\text{ m})$ whereas the cuboid centered at RESBI waypoint has been defined by the two corners $(40.400°, -4.150°, 4200\text{ m})$ and $(40.480°, -4.070°, 4800\text{ m})$.

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As mentioned above, besides waypoint constraints, the model also includes time-based separation logical constraint of at least 200 s between aircraft as described in Sec. 4.1. The initial mass of the three aircraft has been assumed equal to the maximum landing weight of the aircraft. The specific boundary conditions of the state variables are given in Table 1. The final mass and time given by the solution of the corresponding OCP are reported for each aircraft in Table 2. In Fig. 2 the 3D view of the paths are represented whereas the horizontal and vertical profiles are depicted in Fig. 3 and Fig. 4, respectively. The mass consumption is shown in Fig. 5.

Table 1: Boundary conditions for the experiment

Symbol	Unit	Aircraft 1	Aircraft 2	Aircraft 3
h_I	m	7400	7000	7200
h_F	m	3350	3350	3350
θ_I	deg	39.526	39.116	39.000
θ_F	deg	40.575	40.575	40.575
λ_I	deg	-5.327	-4.448	-3.325
λ_F	deg	-3.422	-3.422	-3.422
V_I	m/s	130	130	130
V_F	m/s	110	110	110
μ_I	deg	0	0	0
γ_I	deg	0	0	0
χ_I	deg	356	294	240
m_I	kg	65000	65000	65000

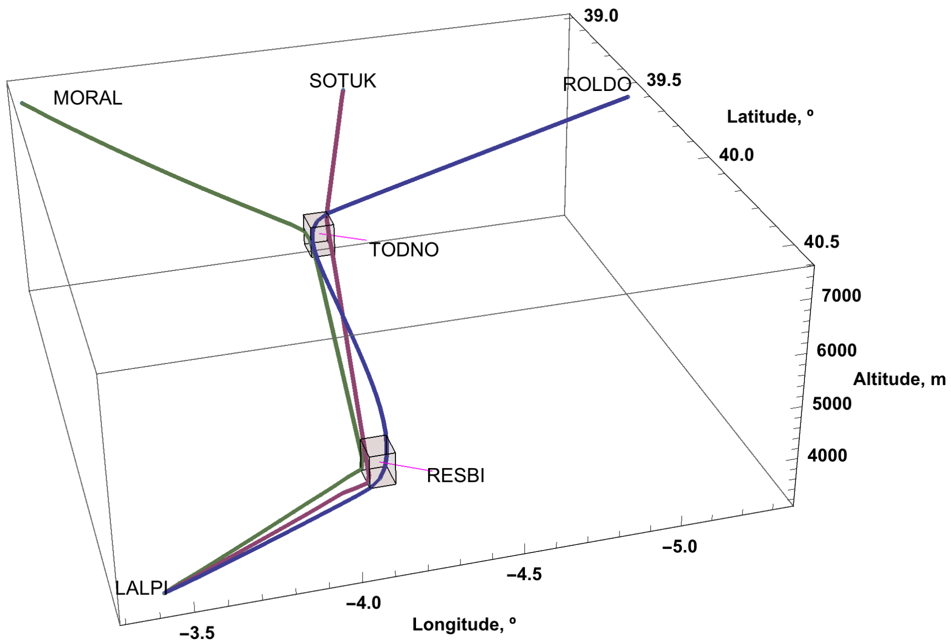

Figure 2: 3D view of the paths with waypoints and time-based separation constraints.

It can be seen that the three aircraft pass through both waypoints and, at the same time, they fulfil the time-based separation requirement at the merging LALPI waypoint. More specifically, the final time of the three aircraft are 2109 s, 1619 s and 1909 s, respectively. The difference between the final time of Aircraft 1 and 3 is 200 s whereas the difference between the final time of Aircraft 2 and 3 is 290 s. Notice that Aircraft 2 and 3 do not maintain exactly the minimum required time separation of 200 s. This is due to the fact that the performance index includes in this case, besides the minimization of the duration of the flights, the penalty term associated to the waypoints constraints. Therefore, the saturation of the time-based constraints may not happen in this setting.

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Table 2: Results of the experiment

# Aircraft	Final Time, s	Final mass, kg
1	2109	63908
2	1619	64127
3	1909	63956

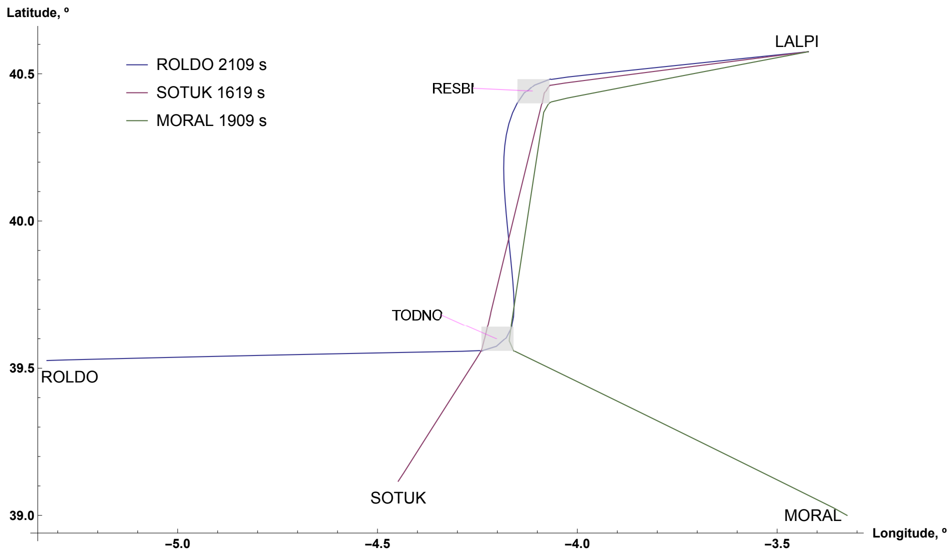

Figure 3: Horizontal profiles with waypoints and time-based separation constraints.

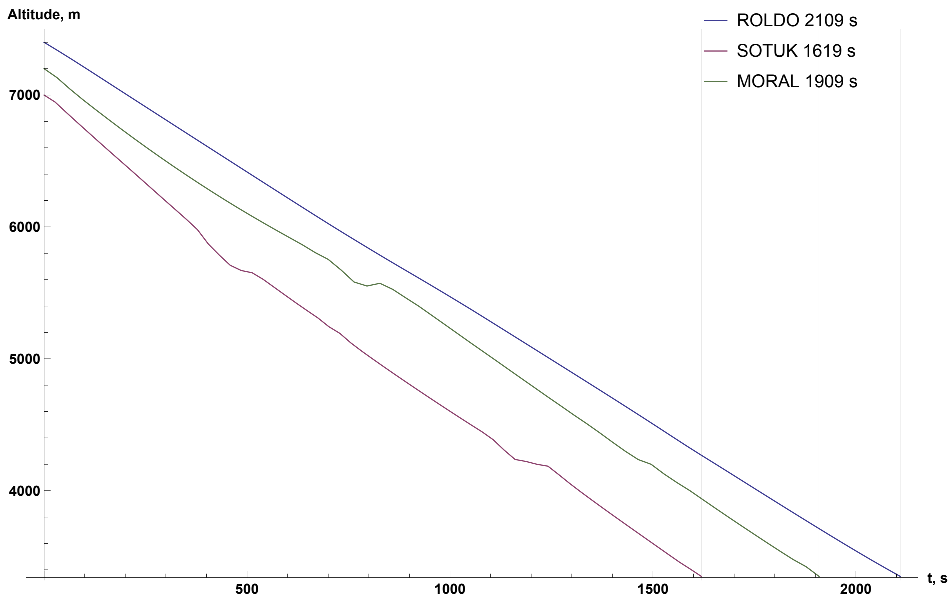

Figure 4: Vertical profiles with waypoints and time-based separation constraints.

6. Conclusions

In this paper, the trajectory planning problem for multiple aircraft has been studied in which logical constraints in disjunctive form are included in the model. The logical constraints in disjunctive form have been transformed into inequality and equality constraints, which involve only continuous auxiliary variables, by means of an embedding

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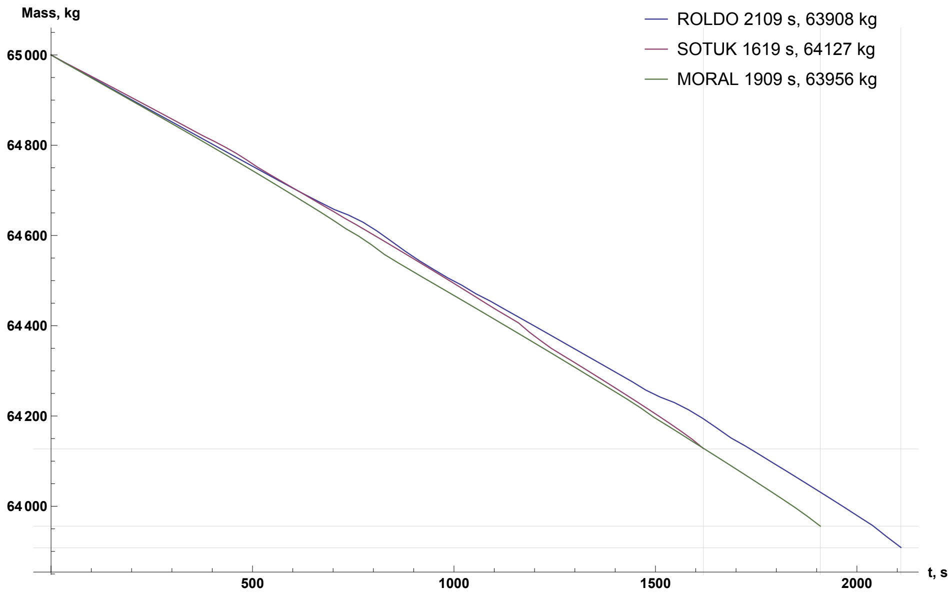

Figure 5: Mass consumption with waypoints and time-based separation constraints.

technique. In this way, the optimal control problem with logical constraints has been converted into a smooth optimal control problem which has been solved using standard techniques. This approach has been applied to the computation of the optimized profile descent of multiple aircraft in converging arrival routes within the Adolfo Suárez Madrid-Barajas (LEMD/MAD) TMA. The results show the effectiveness of the proposed technique.

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