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Karhunen-Loève Expansion of Ensemble Weather Forecasts for Aircraft Trajectory Planning

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Abstract

In this paper, the Karhunen-Loève expansion of ensemble weather forecasts is proposed to model meteorological uncertainties in aircraft trajectory planning based on stochastic optimal control techniques. Ensemble weather forecasts are sets of forecasts that represent a range of possible weather conditions together with the likelihood of their occurrence. Ensemble wind forecasts are treated as realizations of a stochastic processes and represented using the Karhunen-Loève approach. The obtained results show the ability of the Karhunen-Loève approach to provide suitable representations of ensemble wind forecasts to be included into aircraft trajectory planning problems.

1. Introduction

The Next Generation Air Transportation System (NextGen) and the Single European Sky ATM Research (SESAR), are currently ongoing programs to support the future air transportation system in the United States and Europe, respectively. These new Air Traffic Management (ATM) programs are based on the concept of Trajectory-Based Operations (TBO). A 4D aircraft trajectory consists of the 3D aircraft path plus time. In TBO, aircraft will be able to follow optimized 4D trajectories based on the preferences of the airlines. The planned trajectories must be precisely followed by the aircraft to avoid conflicts with other aircraft and ensure the safety of the flight and an efficient exploitation of the airspace. However, this accuracy is difficult to achieve mainly because of weather uncertainty. Among the most effective trajectory optimization techniques are the stochastic optimal control methods, which are able to take into account the inherent uncertainty of the problem and give robust solutions in the sense that probabilistic constraints must be fulfilled.

Uncertainties in parameters are usually represented as random variables, whereas uncertainties in functions of space or time are modeled as random processes. When uncertainties can be characterized by means of probability distribution functions, in the first case, uncertainty quantification can be directly addressed using Generalized Polynomial Chaos (gPC) expansion whereas, in the second case, it is usually based on a combination of Karhunen-Loève (KL) expansion and gPC [1]. In data-driven applications, the gPC technique can be generalized by the Arbitrary Polynomial Chaos (aPC) expansion, which can tackle raw data sets, and only demands the existence of a finite number of moments and does not require the complete knowledge of a probability distribution function [2].

The European Centre for Medium-Range Weather Forecast (ECMWF) developed a system called Ensemble Prediction System (EPS) to reflect uncertainty in weather forecast models. In this system, besides the forecast based on the best estimate of the model equations and the best estimate of the initial state of the atmosphere, which is called control, other 50 forecasts are generated using slightly perturbed initial conditions and also using different models which are close to, but not identical to the best estimate of the model equations. The combination of the control plus the other forecasts is called an ensemble. The spread of the ensemble prediction gives an estimate of the uncertainty of the prediction [3, 4].

In [5], an approach for dynamic optimization considering uncertainties is developed and applied to robust aircraft trajectory optimization. The nonintrusive polynomial chaos expansion scheme is employed to convert a robust trajectory optimization problem with stochastic ordinary differential equations into an equivalent deterministic trajectory optimization problem with deterministic ordinary differential equations. In [6], a stochastic optimal control method is developed for determining three-dimensional conflict-free aircraft trajectories under wind uncertainty considering a idealized Gaussian correlated wind model and a probabilistic conflict detection algorithm using the gPC method.

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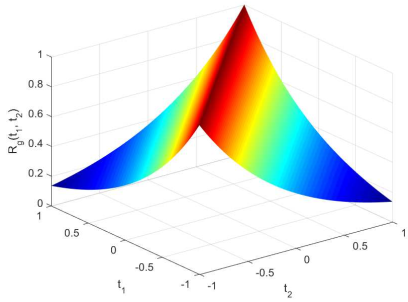

Figure 1: Plot of the autocorrelation function given by Eq. (1).

A spatially correlated wind model is considered because the wind correlation has a significant effect on the trajectories of aircraft when they are close to each other. In [7], a practical approach to generalize the KL expansion, called muKL, is proposed to model multiple-correlated non-stationary random processes, based on the spectral analysis of a suitable assembled random process which yields a series expansion in terms of an identical set of uncorrelated random variables.

Since there is no conclusive experimental evidence to support the assumption of the Gaussian process model for wind uncertainty, other wind models with data-driven probability distributions are needed for specific weather conditions. In this paper, the muKL expansion is applied to EPS wind data considering the wind module in the eastward and northward directions as correlated processes and the members of the EPS as realizations. The results of the numerical experiments show that the fist term of the muKL expansion of the considered EPS explains almost all the variability of the ensemble, and that this is a practical representation of the EPS to be used in stochastic optimal control problems for aircraft trajectory planning.

This paper is organized as follows. Stochastic processes are introduced in Sect. 2. The KL expansion for stochastic processes is explained in Sect. 3. The muKL expansion is presented in Sect. 4. Results obtained applying the muKL expansion to an EPS are reported in Sect. 5. Finally, Sect. 6 contains the conclusions.

2. Stochastic Processes

2.1 Definition of a stochastic process

Consider a random experiment specified by the outcomes ω from a sample space S, by the events defined on S, and by their probabilities. Suppose to associate to every outcome ω a function of a parameter t, which usually represents space or time,

$$f(t, \omega), t \in \mathcal{I}.$$

The graph of the function $f(t, \omega)$, for a fixed value of ω, is called a realization, sample path, or sample function of the random process. Thus, in this case, the outcome of the random experiment is a function of the parameter t. Fixing a value t_k from the index set $\mathcal{I}$, then $f(t_k, \omega)$ becomes a random variable since it is actually a mapping between outcomes $\omega \in S$ and real numbers. In this way, a family (or ensemble) of random variables indexed by the parameter t has been created, that is, $\{f(t, \omega), t \in \mathcal{I}\}$. This family is called a random process. Random processes are also referred to as stochastic processes [8].

Let $f_1(t, \omega)$ and $f_2(t, \omega)$ be two stochastic processes. To specify their joint behavior, the joint distribution of all possible choices of t samples of all processes have to be specified. The cross-correlation $R_{f_1,f_2}(t_1, t_2)$ of $f_1(t, \omega)$ and $f_2(t, \omega)$ is defined as

$$R_{f_1,f_2}(t_1, t_2) = E[f_1(t_1) f_2(t_2)],$$

where $E[\cdot]$ denotes the statistical expectation operator, and the cross-covariance $C_{f_1,f_2}(t_1, t_2)$ of $f_1(t, \omega)$ and $f_2(t, \omega)$ as

$$C_{f_1,f_2}(t_1, t_2) = R_{f_1,f_2}(t_1, t_2) - m_{f_1}(t_1) \cdot m_{f_2}(t_2),$$

where $m_{f_1}(t_1)$ and $m_{f_2}(t_2)$ are the mean functions of each process at t_1 and t_2, respectively. In particular, if $f_1(t, \omega) = f_2(t, \omega) = f(t, \omega)$, $R_f(t_1, t_2)$ and $C_f(t_1, t_2)$ are called autocorrelation and autocovariance of $f(t, \omega)$ respectively. Without loss of generality, in this article zero-mean random processes will be considered. Therefore, $C_{f_1,f_2}(t_1, t_2) = R_{f_1,f_2}(t_1, t_2)$ [8].

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A random process of second order, also called a square integrable process, can be defined as a family of complex random variables which have finite second moments $E[|f(t,\omega)|^2] < \infty$ for all $t \in \mathcal{I}$ [9, 10].

A typical example of a stochastic process is the Gaussian random field $g(t,\omega)$, characterized by its variance σ^2 and autocorrelation function $R_g(t_1,t_2)$ given by

$$R_g(t_1,t_2) = \sigma^2 \exp\left(-\frac{|t_1 - t_2|}{L_c}\right), \tag{1}$$

where L_c is the correlation length. This autocorrelation function is represented in Fig. 1 in the interval $(t_1,t_2) \in [-1,1] \times [-1,1]$ for $\sigma^2 = 1$ and $L_c = 1$.

3. Karhunen-Loève Expansion for Stochastic Processes

This section introduces the KL expansion, which allows (possibly nonstationary) random process $f(t,\omega)$ to be expanded in a series

$$f(t,\omega) = \sum_{k=1}^{\infty} \sqrt{\lambda_k}\phi_k(t)\xi_k(\omega), \tag{2}$$

where $\xi_k(\omega)$ are uncorrelated random variables

$$\xi_k(\omega) = \frac{1}{\sqrt{\lambda_k}} \int_{\mathcal{I}} f(t,\omega)\phi_k(t)dt,$$

and the eigenvalues λ_k and the eigenfunctions $\phi_k(t)$ fulfill the Mercer's theorem

$$\lambda_k\phi_k(t_1) = \int_{\mathcal{I}} K(t_1,t_2)\phi_k(t_2)dt_2, \tag{3}$$

where

$$K(t_1,t_2) = \sum_{k=1}^{\infty} \lambda_k\phi_k(t_1)\phi_k(t_2).$$

Equation (3) is known as homogeneous Fredholm integral equation of the second kind and it is usually solved numerically using the Nyström method. An interesting property of the KL expansion (2) is that, since the random variables $\xi_k(\omega)$ are uncorrelated, it actually decorrelates the random process $f(t,\omega)$. Additionally, it can be used to reduce the dimensionality of the representation by simple truncation of the sum.

Eigenvalues contain the information about the percentage of variability explained by the corresponding term of the KL expansion. In Table 1, the first six eigenvalues of the KL expansion obtained applying the Nyström method to the Fredholm integral equation for the Gaussian process whose autocorrelation function is given by Eq. (1) are listed, along with the percentage of variability explained by each of them. Overall, they explain 92.70720% of the variability. The first six eigenfunctions and the histograms of the corresponding random variables are represented in Fig. 2.

Table 1: The first six eigenvalues of the KL expansion of the Gaussian process whose autocorrelation function is given by Eq. (1) and the percentage of variability explained by the first six terms of the expansion.

	λ_1	λ_2	λ_3	λ_4	λ_5	λ_6
Eigenvalue	23.30359	8.09230	3.28548	1.67425	0.99671	0.65760
% of variability explained	56.83802	19.73732	8.01338	4.08355	2.43102	1.60391

4. Karhunen-Loève Expansion for Multi-Correlated Processes

As said before, the eastward and northward components of the wind, which will be referred to as the X and Y components respectively, are correlated stochastic processes. Thus, to carry out their KL expansions, a special technique for correlated processes must be employed. In this paper, the results of the application of the muKL method proposed in [7] will be described. In this section, the muKL method is reported for convenience. The original paper contains a more comprehensive treatment of the topic and some illustrative examples.

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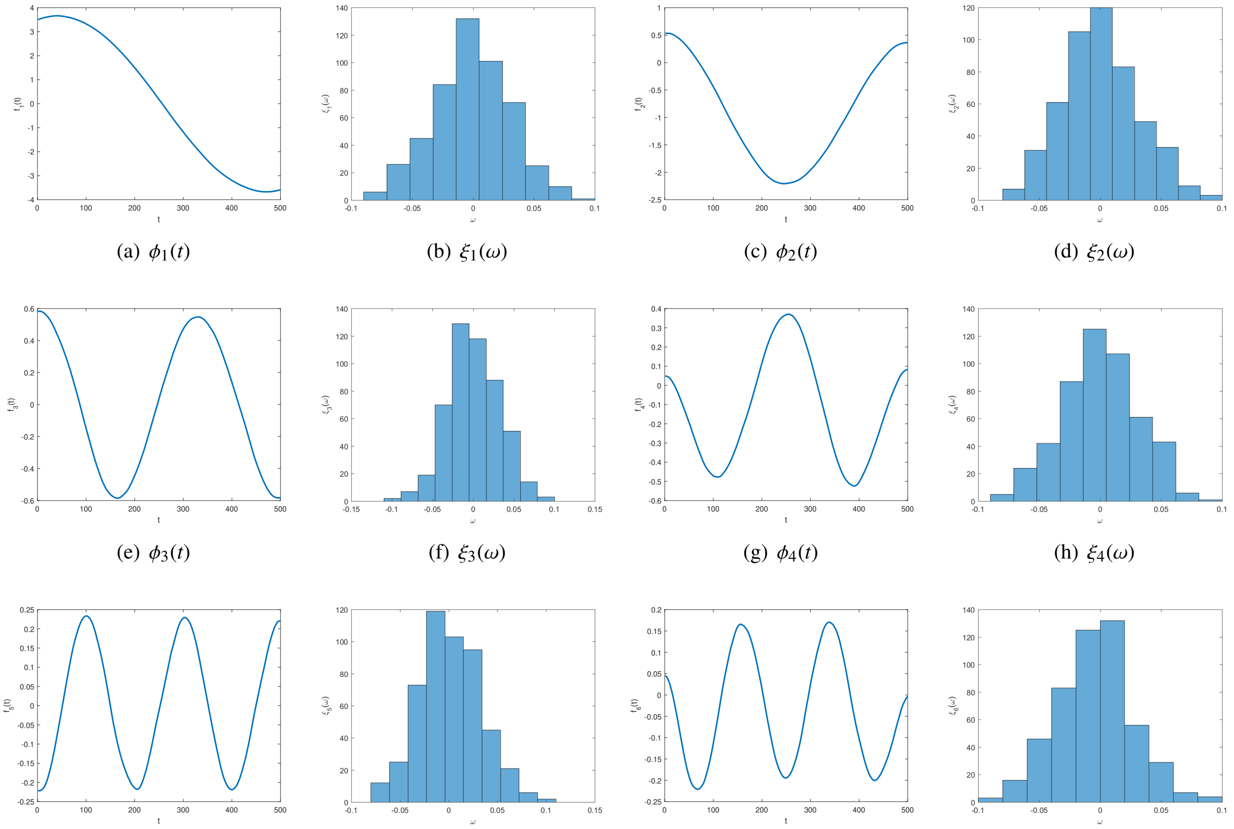

(a) $\phi_1(t)$ (b) $\xi_1(\omega)$ (c) $\phi_2(t)$ (d) $\xi_2(\omega)$

(e) $\phi_3(t)$ (f) $\xi_3(\omega)$ (g) $\phi_4(t)$ (h) $\xi_4(\omega)$

(i) $\phi_5(t)$ (j) $\xi_5(\omega)$ (k) $\phi_6(t)$ (l) $\xi_6(\omega)$

Figure 2: The first six eigenfunctions and the histograms of the corresponding random variables of the KL expansion of the Gaussian process whose autocorrelation function is given by Eq. (1).

Consider an ensemble of n zero-mean square integrable random processes

$$\{f_1(t;\omega),\ldots,f_n(t;\omega)\} \tag{4}$$

For the sake of simplicity, it is assumed that each process is defined in the same bounded interval $[0,T]$.

The correlation between the processes $\{f_1(t,\omega),\ldots,f_n(t,\omega)\}$ is described in terms of $n(n+1)/2$ covariance functions C_{ij},

$$C_{ij}(t_1,t_2) = E[f_i(t_1,\omega)f_j(t_2,\omega)], \quad 1 \ \le i \le j \le \ n. \tag{5}$$

The quantity $C_{ii}(t_1,t_2)$ is the auto-covariance function of the process $f_i(t,\omega)$, which will also be denoted as $C_i(t_1,t_2)$. If the processes $\{f_1(t,\omega),\ldots,f_n(t,\omega)\}$ are mutually independent, then the KL expansion (2) can be straightforwardly applied to each process $f_i(t,\omega)$ separately [11, 12]. As said before, if the cross-covariances $C_{ij}(s,t)$ are non zero, then it is not straightforward to obtain consistent expansions for all random processes, able to reflect both the autocorrelation and the cross covariance structure. The muKL method proposed in [7], overcomes this drawback by generating series expansions of all processes in terms of a single set of uncorrelated random variables. To construct such a series, an assembled process $\tilde{f}$ is considered, which is defined for each interval $\mathcal{I}_i$ as

$$\tilde{f}(t,\omega) = f_i(t-T_{i-1},\omega), \qquad t\in\mathcal{I}_i, 1\le i\le n, \tag{6}$$

where $T_i = iT$, $\mathcal{I}_1 = [0,T_1]$ and $\mathcal{I}_i = (T_{i-1},T_i]$. In this way, the restriction of the assembled process $\tilde{f}(t,\omega)$ to the interval $\mathcal{I}_i$ coincides with the process $f_i(t,\omega)$. Thus, $\tilde{f}(t,\omega)$ being the assembling of second order processes, is also a second order process which satisfies

$$E[\tilde{f}(t,\omega)] = 0, \qquad E[\tilde{f}(t_1,\omega)\tilde{f}(t_2,\omega)] = \tilde{C}(t_1,t_2),$$

where $\tilde{C}(t_1,t_2)$ is the assembled covariance function defined as

$$\tilde{C}(t_1,t_2) = C_{ij}(t_1-T_{i-1},t_2-T_{j-1}) \ \ t_1\in\mathcal{I}_i \ \ t_2\in\mathcal{I}_j. \tag{7}$$

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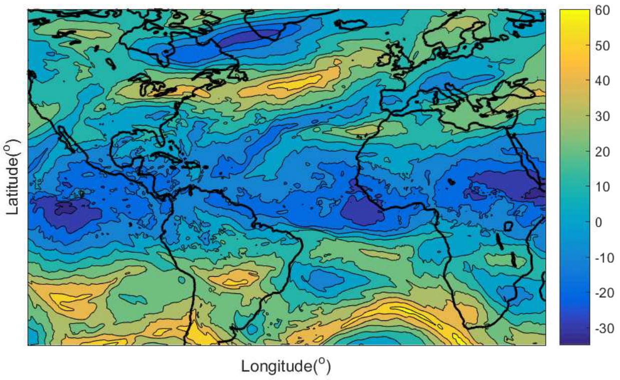
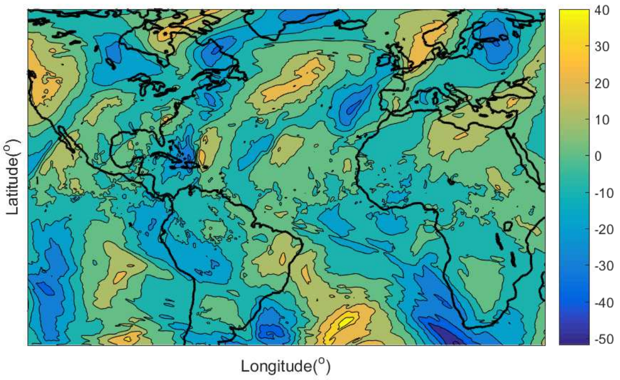
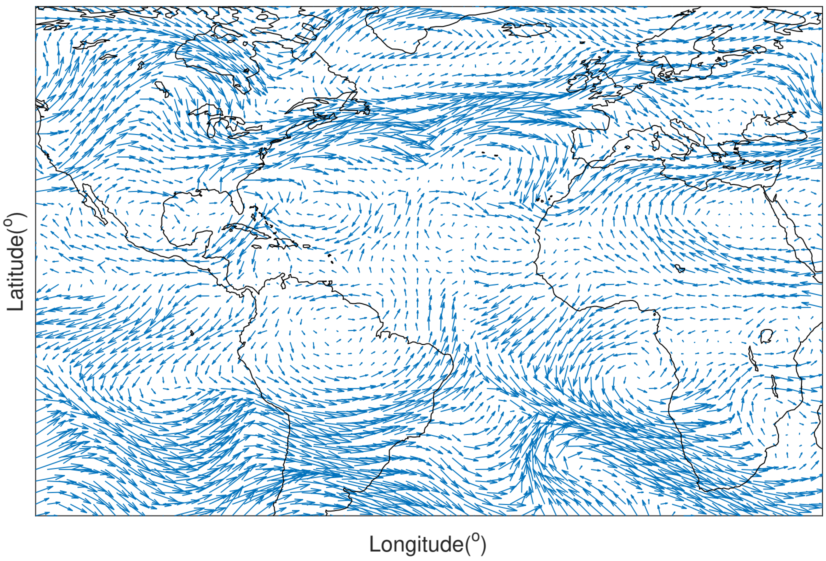


(a) Wind intensity in the X direction in [m/s]



(b) Wind intensity in the Y direction in [m/s]



(c) Wind field

Figure 3: First member of the EPS.

Now, the KL expansion (2) can be applied to the assembled process (6) obtaining

$$\tilde{f}(t,\omega) = \sum_{k=1}^{\infty} \sqrt{\lambda_k} \tilde{f}_k(t) \xi_k(\omega), \tag{8}$$

where $\xi_k(\omega)$ are uncorrelated random variables

$$\xi_k(\omega) = \frac{1}{\sqrt{\lambda_k}} \int_0^{T_n} \tilde{f}(t,\omega) \tilde{f}_k(t) dt,$$

where λ_k and $\tilde{f}_k(t)$ are, respectively eigenvalues and eigenfunctions solutions to the homogeneous Fredholm integral equation of the second kind

$$\lambda_k \tilde{f}_k(t_1) = \int_0^{T_n} \tilde{C}(t_1,t_2) \tilde{f}_k(t_2) dt_2. \tag{9}$$

In practical applications it is preferable to have a non negative covariance function. Unfortunately, the assembled covariance $\tilde{C}(t_1,t_2)$ could have negative eigenvalues since, in general, it is not positive semi definite, even if all the

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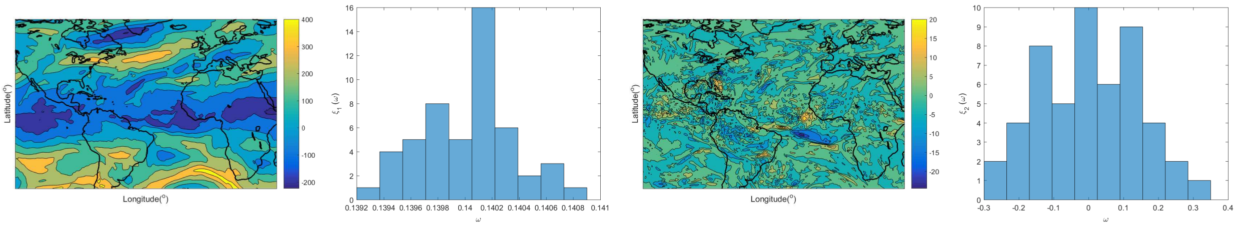
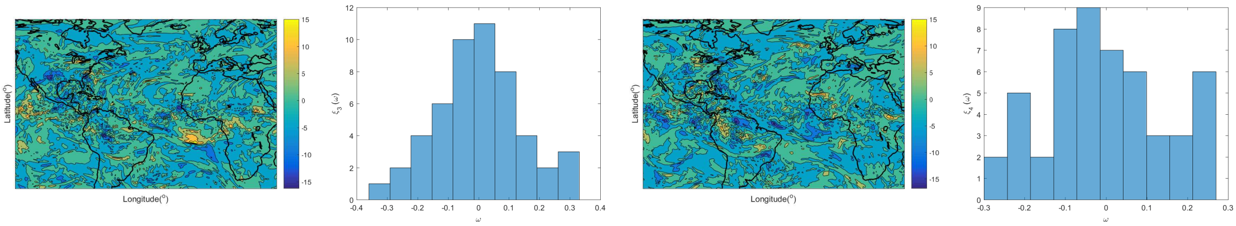
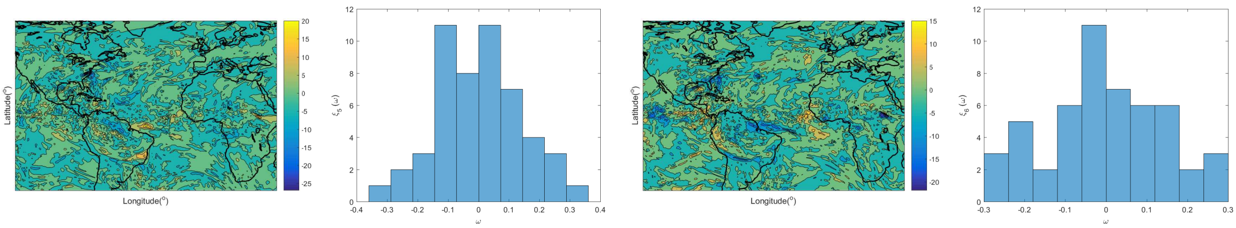


(a) $\phi_1^{(1)}(t)$ (b) $\xi_1(\omega)$ (c) $\phi_2^{(1)}(t)$ (d) $\xi_2(\omega)$



(e) $\phi_3^{(1)}(t)$ (f) $\xi_3(\omega)$ (g) $\phi_4^{(1)}(t)$ (h) $\xi_4(\omega)$



(i) $\phi_5^{(1)}(t)$ (j) $\xi_5(\omega)$ (k) $\phi_6^{(1)}(t)$ (l) $\xi_6(\omega)$

Figure 4: The first six eigenfunctions and the histograms of the corresponding random variables of the muKL expansion of the 51 members of the wind components in the X direction of the EPS.

covariances (5) are. Therefore, the following positivity condition for the assembled discretized covariance $\tilde{C}(t_i, t_j)$ is introduced

$$\sum_{j=1}^{m} \sum_{i=1}^{m} \tilde{C}(t_i, t_j) x_i x_j \geq 0,$$

for any finite sequence $\{t_1, \ldots, t_m\}$ and real numbers $\{x_1, \ldots, x_m\}$. In other words, the $m \times m$ matrix

$$\tilde{C} = \begin{bmatrix} \tilde{C}(t_1, t_1) & \tilde{C}(t_1, t_2) & \cdots & \tilde{C}(t_1, t_m) \\ \tilde{C}(t_2, t_1) & \tilde{C}(t_2, t_2) & \cdots & \tilde{C}(t_2, t_m) \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{C}(t_m, t_1) & \tilde{C}(t_m, t_2) & \cdots & \tilde{C}(t_m, t_m) \end{bmatrix}$$

must be positive definite for any set of m distinct values of the parameter t in $[0, T]$. An example of the positivity condition that must be introduced in case of exponential covariances is given in [7].

Suppose that the eigen-pairs $\{\lambda_k \tilde{f}_k(t)\}, k = 1, 2, \ldots,$ obtained solving Eq. (9) have been ordered according to the magnitudes of the eigenvalues λ_k. Then, each eigenfunction $\tilde{f}_k(t)$ can be represented in terms of n sub-components $\phi_k^{(i)}(t), i = 1, \ldots, n,$

$$\phi_k^{(i)}(t) = \tilde{f}_k(t + T_{i-1}) \mathcal{I}_{[0,T]}(t),$$

where $\mathcal{I}_{[0,T]}$ is the indicator function of the set $[0, T]$.

Finally, the i-th random process $f_i(t, \omega)$ of the original ensemble (4) is expanded as

$$f_i(t, \omega) = \sum_{k=1}^{\infty} \sqrt{\lambda_k} \phi_k^{(i)}(t) \xi_k(\omega). \tag{10}$$

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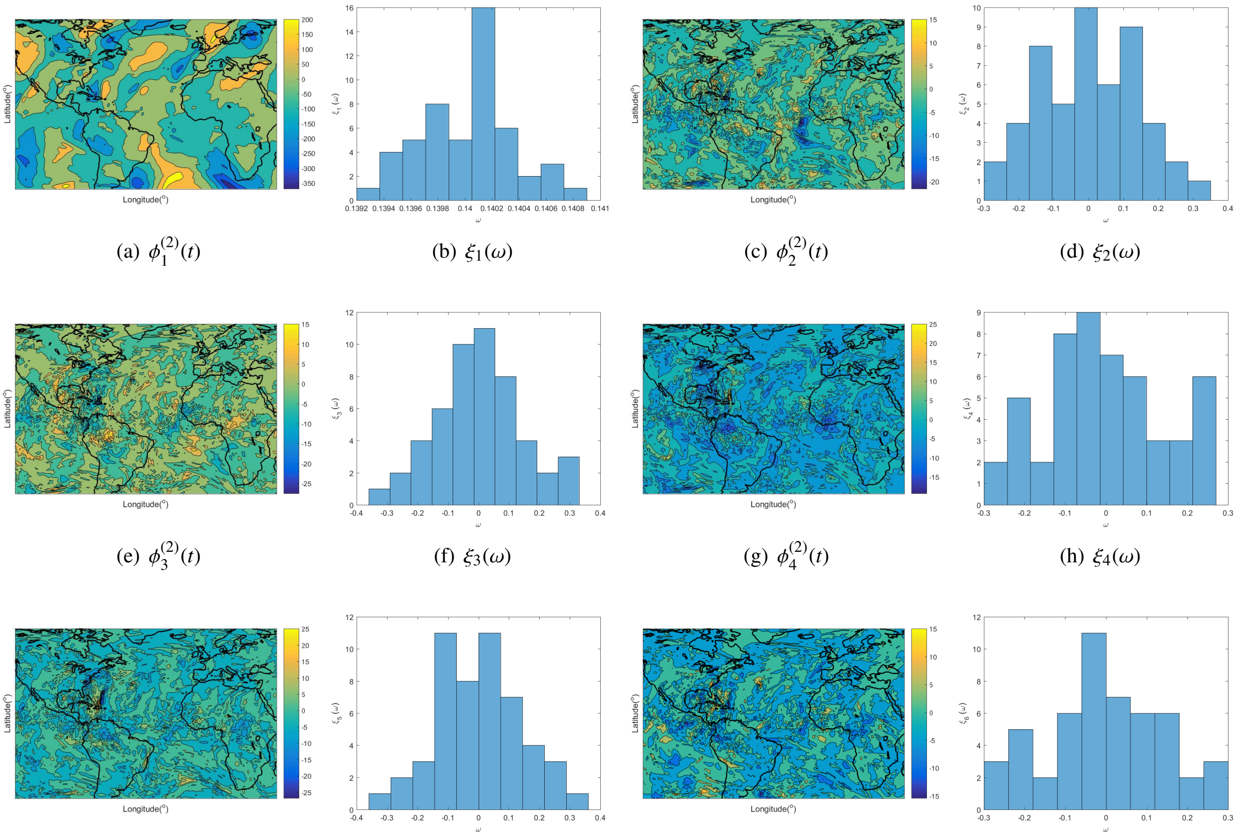

(a) $\phi_1^{(2)}(t)$ (b) $\xi_1(\omega)$ (c) $\phi_2^{(2)}(t)$ (d) $\xi_2(\omega)$

(e) $\phi_3^{(2)}(t)$ (f) $\xi_3(\omega)$ (g) $\phi_4^{(2)}(t)$ (h) $\xi_4(\omega)$

(i) $\phi_5^{(2)}(t)$ (j) $\xi_5(\omega)$ (k) $\phi_6^{(2)}(t)$ (l) $\xi_6(\omega)$

Figure 5: The first six eigenfunctions and the histograms of the corresponding random variables of the muKL expansion of the 51 members of the wind components in the Y direction of the EPS.

Remark 1. Eigenvalues λ_k and random variables $\xi_k(\omega)$ in (10) are the same as those appearing in the assembled process (8).

Remark 2. For each index i, the set of sub-components $\{\phi_k^i\}, k = 1, 2, \dots$ is neither orthogonal nor normalized in $t \in [0, T]$. Nevertheless, $\phi_k^{(i)}(t)$ can be normalized within the interval $[0, T]$ obtaining

$$f_i(t, \omega) = \sum_{k=1}^{\infty} \sqrt{\hat{\lambda}_k^{(i)}} \hat{\phi}_k^{(i)}(t) \xi_k(\omega), \tag{11}$$

where $\hat{\phi}_k^{(i)}(t) = \dfrac{\phi_k^{(i)}(t)}{\left\|\phi_k^{(i)}(t)\right\|_2}$ and $\hat{\lambda}_k^{(i)} = \lambda_k \left\|\phi_k^{(i)}(t)\right\|_2^2$.

Remark 3. Since random processes in (10) or (11) are represented in terms of the same set of random variables $\xi_k(\omega)$, the muKL method cannot be used to represent heterogeneous sets of processes.

In our application it is important to reduce the dimensionality of the expansion by truncation and calculate the associated mean-squared error. The truncated assembled process, in which the first M terms of the expansion (8) are considered, is defined as

$$S_M = \sum_{k=1}^{M} \sqrt{\lambda_k} \tilde{f}_k(t) \xi_k(\omega), \tag{12}$$

and the corresponding mean-squared error as

$$\varepsilon_M^2 = \int_0^{T_n} E[(\tilde{f}(t, \omega) - S_M(t, \omega))^2] dt.$$

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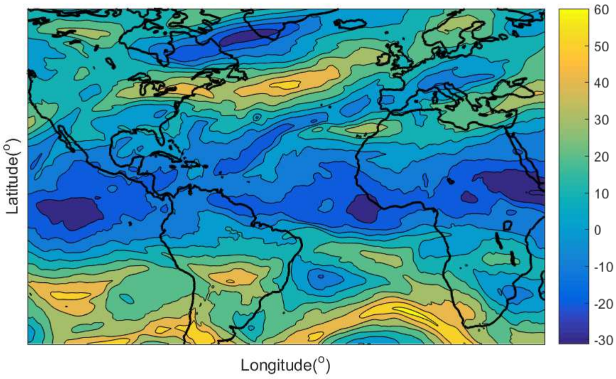
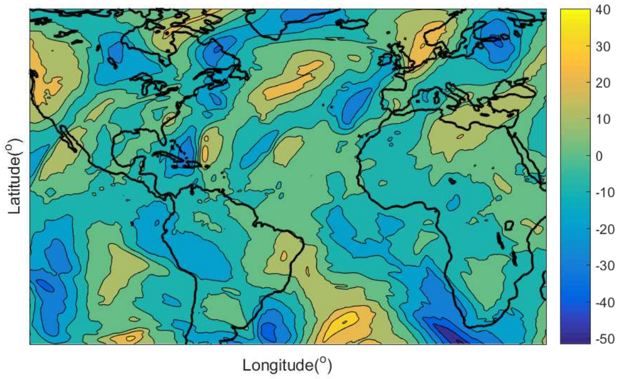
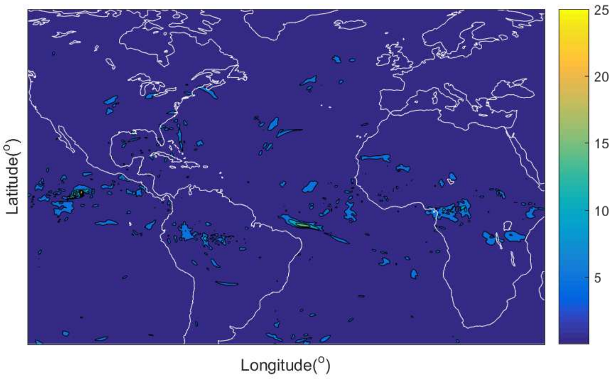


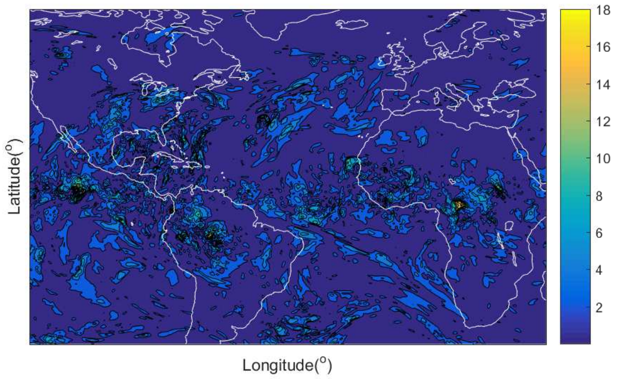
(a) Wind intensity in the X direction in [m/s]



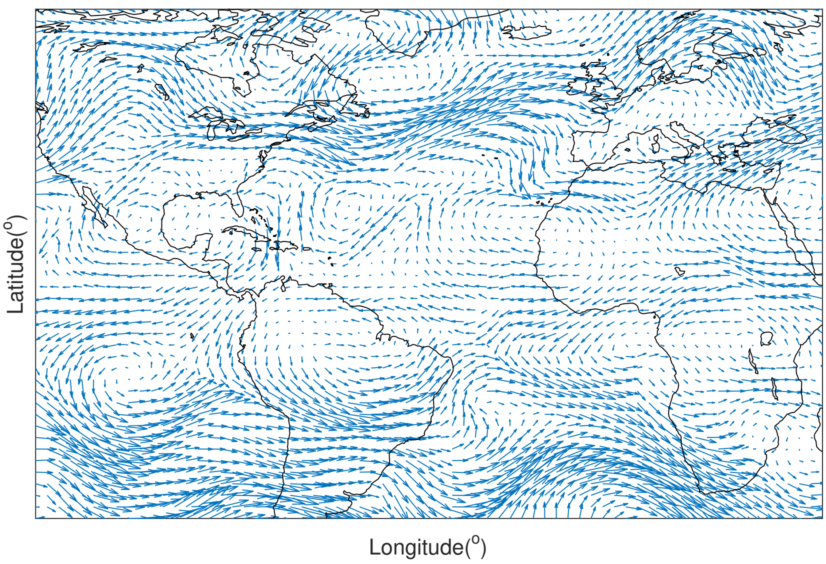
(b) Wind intensity in the Y direction in [m/s]



(c) Error wind intensity in the X direction in [m/s]



(d) Error wind intensity in the Y direction in [m/s]



(e) Wind field

Figure 6: Reconstructions of the first member of the EPS shown in Fig. 3 obtained using only the first component of the muKL expansion of the ensemble of the two random processes.

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Since $\xi_k(\omega)$ are uncorrelated random variables and $\tilde{f}_k(t)$ are orthonormal eigenfunctions, it follows that

$$\varepsilon_M^2 = \sum_{k=M+1}^{\infty} \lambda_k, \tag{13}$$

that is, the truncation error of the series (8) decreases with respect to the decay rate of the eigenvalues. Moreover, the errors of the cross-covariances $C_{ij}(t_1, t_2)$ are bounded by the error of the assembled covariance $\tilde{C}(t_1, t_2)$ in Eq. (7).

A key point of any KL expansion, is the choice of M. For the muKL expansion, the choice of M in (12) can be done by imposing a threshold for the error ε_M, which can be achieved by setting a threshold for the relative sum of the eigenvalues

$$\sum_{k=1}^{M} \lambda_k \geq \alpha \sum_{k=1}^{\infty} \lambda_k,$$

where $\alpha \in [0, 1]$ is chosen so that the accuracy of the approximation is satisfactory for a particular application.

To use the muKL in aircraft trajectory planning problems based on stochastic optimal control techniques, firstly the eigenfunctions $\phi_k^{(i)}(t), i = 1, 2,$ of the expansion of the eastward and northward components of the wind must be interpolated to be converted into analytic expressions. Secondly, the uncertainty of the random variables $\xi_k(\omega)$ must be quantified using an aPC expansion.

5. Results

For the numerical experiments data from the ECMWF has been used. More precisely, a wind EPS forecast issued by the ECMWF on August 31, 2016 at 00:00 hours which prediction horizon goes from August 31, 2016 at 12:00 hours to September 01, 2016 at 18:00 hours. The prediction makes reference to an altitude of 200 [hPa] (11769.9 [m]) for a wide region of the Earth (between 70 [^{o}] N and 40 [^{o}] S, and between 125 [^{o}] W and 45 [^{o}] E) and contains 51 members. For the sake of illustration, only the first member of the ensemble is represented in Fig. 3. In particular, the wind intensity in the X direction is represented in Fig. 3.a, and the wind intensity in the Y direction is represented in Fig. 3.b. Fig. 3.c shows the corresponding wind field.

The first six eigenfunctions and the histograms of the corresponding random variables of the muKL expansion of the 51 members of the wind components in the X direction of the EPS are represented in Fig. 4, whereas the first six eigenfunctions and the histograms of the corresponding random variables of the muKL expansion of the 51 members of the wind components in the Y direction of the EPS are represented in Fig. 5. Notice that, since the muKL method generates series expansions of random processes in terms of a single set of uncorrelated random variables, the histograms of the random variables of the muKL expansions of the processes that represent the wind components in the X and Y directions coincide. The first six eigenvalues of the muKL expansion of the EPS and the percentage of variability explained by the first six terms of the expansion are reported in Table 2. Notice that, as explained in Sect. 4, the set of eigenvalues is the same for two correlated stochastic processes considered. It can be seen that the 98.80786% of the variability of the ensemble of the two random processes is explained by the first term of the muKL expansion. This result is reasonable, since in ensemble-based probabilistic forecasting, besides a main forecast, 50 additional forecasts are generated from slightly perturbed initial conditions and with a slightly perturbed model. Thus, only the first component of the muKL expansion is enough to get an accurate approximation of the ensemble of the two random processes.

The reconstructions of the wind intensity in the X and Y directions obtained using only the first component of the muKL expansion of the ensemble of the two random processes are reported in Fig. 6.a and Fig. 6.b, respectively. Fig. 6.e shows the corresponding vector field. The errors in reconstructions of the wind intensity in the X and Y directions are reported in Fig. 6.c Fig. 6.d, respectively.

Table 2: The first six eigenvalues of the muKL expansion of the EPS and the percentage of variability explained by the first six terms of the expansion.

	λ_1	λ_2	λ_3	λ_4	λ_5	λ_6
Eigenvalue	14769.01	8.35300	7.66338	7.41429	6.99105	6.71256
% of variability explained	98.80786	0.05588	0.05126	0.04960	0.04677	0.04490

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6. Conclusions

In this paper, a Karhunen-Loève expansion of ensemble weather forecasts has been proposed to model meteorological uncertainties in aircraft trajectory planning problems based on stochastic optimal control techniques. The eastward and northward components of the wind have been regarded as realizations of correlated stochastic processes and a special technique has been employed to compute their Karhunen-Loève expansions in terms of an identical set of uncorrelated random variables. The Karhunen-Loève expansion has the useful characteristic of allowing the dimension of the representation for a designated numerical precision to be chosen by a simple truncation. Numerical experiments have been conducted using an ensemble wind forecast with 51 members. The obtained results show that using a single term of the Karhunen-Loève expansion of an ensemble wind forecasts, 98.80786% of the variability can be explained, providing a practical representations of ensemble wind forecasts to be used in aircraft trajectory planning problems based on stochastic optimal control techniques.

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References

[1] D. Xiu, *Numerical Methods for Stochastic Computations: A Spectral Method Approach*. Princeton University Press, 2010.

[2] S. Oladyshkin and W. Nowak, "Data-driven uncertainty quantification using the arbitrary polynomial chaos expansion," *Reliability Engineering & System Safety*, vol. 106, pp. 179–190, 2012.

[3] F. Molteni, R. Buizza, T. N. Palmer, and T. Petroliagis, "The ECMWF ensemble prediction system: Methodology and validation," *Quarterly Journal of the Royal Meteorological Society*, vol. 122, no. 529, pp. 73–119, 1996.

[4] R. Buizza, P. Houtekamer, G. Pellerin, Z. Toth, Y. Zhu, and M. Wei, "A comparison of the ECMWF, MSC, and NCEP global ensemble prediction systems," *Monthly Weather Review*, vol. 133, no. 5, pp. 1076–1097, 2005.

[5] X. Li, P. B. Nair, Z. G. Zhang, L. Gao, and C. Gao, "Aircraft robust trajectory optimization using nonintrusive polynomial chaos," *Journal of Aircraft*, vol. 51, no. 5, pp. 1592–1603, 2014.

[6] Y. Matsuno, T. Tsuchiya, J. Wei, I. Hwang, and N. Matayoshi, "Stochastic optimal control for aircraft conflict resolution under wind uncertainty," *Aerospace Science and Technology*, vol. 43, pp. 77–88, 2015.

[7] H. Cho, D. Venturi, and G. Karniadakis, "Karhunen–Loève expansion for multi-correlated stochastic processes," *Probabilistic Engineering Mechanics*, vol. 34, pp. 157–167, 2013.

[8] A. Leon-Garcia, *Probability, Statistics, and Random Processes for Electrical Engineering*. Pearson-Prentice Hall, 2017.

[9] A. Makagon and A. Miamee, "Spectral representation of periodically correlated sequences," *Probabability and Mathematical Statistics*, vol. 33, no. 1, pp. 175–188, 2013.

[10] E. Wong and B. Hajek, *Stochastic Processes in Engineering Systems*. Springer-Verlag, 1985.

[11] S. U. Pillai and A. Papoulis, *Probability, Random Variables, and Stochastic Processes*. McGraw-Hill, 2002.

[12] S. Huang, S. Quek, and K. Phoon, "Convergence study of the truncated Karhunen–Loève expansion for simulation of stochastic processes," *International Journal for Numerical Methods in Engineering*, vol. 52, no. 9, pp. 1029–1043, 2001.