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In-flight transition of a Dual-Bell nozzle - transonic vs. supersonic transition

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Abstract

The interaction between an external flow with a Dual-Bell nozzle flow is investigated experimentally with particle image velocimetry and schlieren shadowgraphy. Previous results showed that the interaction of a supersonic external flow and the jet plume triggers the flip-flop effect. The current results at transonic free-stream conditions show that transition occurs without retransition, eliminating flip-flop. However, in sea level mode at transonic free-stream conditions, nozzle screeching is identified to occur at $Sr_s \approx 0.3$. This is most likely triggered by the separated flow in the nozzle extension, which has to be addressed in future research on Dual-Bell nozzles.

1. Introduction

A Dual-Bell nozzle, such as proposed by Foster and Cowles[7] in 1949, is an adaptive nozzle which increases the integral of the thrust over a space launcher's trajectory. This type of nozzle's contour is characteristic through its geometric discontinuity, or inflection, between the throat and the exit. This splits the nozzle into two separate bells, hence a Dual-Bell. The first Bell is termed the base and the second bell is termed the extension. In contrast to a conventional Bell nozzle, a Dual-Bell nozzle has two operating modes; the sea level mode and the altitude mode (refer to figure 1). In the sea level mode the flow expands into the base nozzle, where it separates at the contour inflection in a spatially and temporally steady manner. In this state, the nozzle is overexpanded, creating a low pressure jet plume and a favorable pressure gradient from the outside of the nozzle into the nozzle extension. However, even at takeoff the sea level mode's overexpansion is not as extreme as it is the case for a conventional rocket nozzle. This increases the thrust integral in the troposphere while avoiding the risk of high side loads due to large separated flow regions during the start-up of the engine[15] . As the launcher ascends and the pressure in the atmosphere decreases below a certain level, the flow naturally expands into the nozzle extension, filling it entirely. This operating state is defined as the altitude mode. In the altitude mode, the flow expands to a much lower pressure than with a conventional nozzle, leading to a comparatively increased thrust from the stratosphere to the main engine's shutdown. The recently published work of Stark et al.[25] showed how an Ariane 5 could expect a 490 kg, or approximately 5 %, increase in its payload on a typical geostationary transfer orbit (GTO) mission with a change of its conventional nozzle to a Dual-Bell. However, the natural transition point is dependent on many geometrical parameters of the nozzle.

Horn and Fisher[12] investigated various contour profiles of the second divergent section of Dual-Bell nozzles. The extensions included a conical contour, a thrust-optimized contour[20] (TOC), a constant pressure contour, and an overturned positive pressure gradient contour. All of the contours had identical expansion ratios. The transition stability proved to be a function of the pressure gradient about the contour inflection. A highly favorable pressure gradient about the inflection provides for the quickest and most repeatable transition. This was achieved with the overturned contour, which extremely expands the flow about the inflection point prior to compressing it again. The drawback of this type of contour is that the thrust comparatively decreases due to the non-optimal contour. Therefore, the constant pressure extension, which keeps a constant pressure along the nozzle wall while gradually turning the flow axially, was considered the best trade-off. This contour provided repeatable transitions below 30 ms, while nearly matching the thrust coefficient of a TOC contour with the same expansion ratio. Next to this, Horn and Fisher stated that the low-pressure recirculation region present during sea level mode causes an early transition, since this pressure is below ambient conditions. Additionally, this causes aspiration drag, which has to be taken into account when evaluating the thrust performance of a Dual-Bell nozzle over the mission's trajectory.

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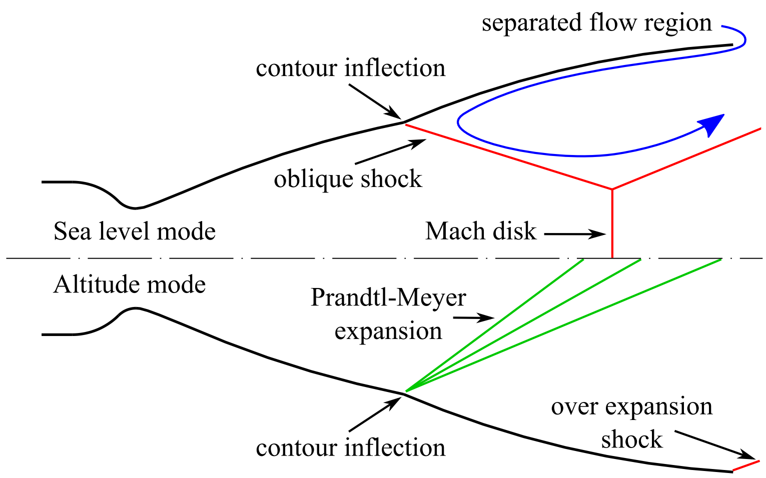

Figure 1: 2D sketch of the two Dual-Bell operating modes: Sea level mode is illustrated above the axis of symmetry. Altitude mode is illustrated below the axis of symmetry.

Following Horn and Fisher's results, the Dual-Bell concept started to receive serious consideration from Germany. Immich and Caporicci[13] compared the performance of a generic nozzle to a Dual-Bell for a polar mission. Both nozzles were optimized using the ALTOS trajectory optimization software[5] . The Dual-Bell nozzle yielded a 33 % payload increase over the generic nozzle. Thus, the performance advantage of this type of nozzle is significant, but its application can only be feasible if the transition event happens predictably.

For this reason the research interest on the transition event of Dual-Bell nozzles increased from the late 20th century. Frey and Hagemann[8] critically assessed the effect of the overall contour of a Dual-Bell on the transition and conducted a parametric study on the performance. The assessment was based on analytical solutions and empirical data from conventional Bell nozzles. With respect to the transition, the conclusions were similar to those of Horn and Fisher, stating that a constant pressure or overturned contour would be the most advantageous. Using Schmucker's[22] separation criterion, it was shown that the side loads of the constant pressure contour would be high, however the transition would be so quick that side loads may not even play a role. The performance evaluation showed that the vacuum thrust of Dual-Bell contours suffer minor deficits, on the order of 1 %, compared to an optimal Bell contour with the same expansion ratio. In sea level operation the performance deficit was less than 3 % due to aspiration drag. This value is linearly dependent on the ambient pressure and therefore decreases with increasing altitude.

Consequently, Hagemenn et al.[10] conducted complementary experimental investigations based on their previous findings. The results confirmed the analytical predictions, showing that the constant pressure and the overturned extensions require short time scales of around 10 ms for a complete transition. In addition, a strong hysteresis was observed with these contours, meaning that a retransitioning back to sea level mode would require a much higher ambient back pressure than the initial transition event. This is an important criterion for the feasibility of a Dual-Bell nozzle, as multiple 'flip-flops' between the modes would cause excessive side loads.

In the aftermath of these continuously positive results, the German Aerospace Center (DLR) carried out multiple investigations on transitioning, trying to characterize the most influential parameters for a controlled event with minimal side loads. For example Génin and Stark[15,16] investigated the influence of the length of the extension on the transition. Next to determining the transition nozzle pressure ratio NPR_{tr}, they showed that a longer extension length provides for a more stable and faster transition. The results also showed increasing hysteresis with increasing nozzle extension lengths. Furthermore, an intermediate state defined as sneak transition was found between the sea level and the altitude mode. This third mode is characterized by a stable separation shortly after the inflection and only occurs for certain wall contours.

Génin and Stark[9] also investigated the problematic regarding side loads during transition. The results showed that a Dual-Bell nozzle has lower side loads than a TIC nozzle during steady operation in either of its operating modes. During transition however, a brief peak side load was observed, which was up to a factor of four higher than the peak side loads of a TIC nozzle.

A shortcoming of the previous work carried out on transition is that the transition is triggered by either an increase in the thrust chamber pressure or a decrease of the external pressure. This can be achieved numerically[14] as well as experimentally in a high-altitude simulation chamber[28] , for instance. Even though the transition event is triggered

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somewhat realistically, the interaction of the nozzle flow with the external flow is completely neglected. Investigations in the past have tried to compensate this by imposing external pressure fluctuations as a boundary condition either numerically[19] or experimentally[27] . Regardless, this neglects many effects from the oncoming flow, which interact with the nozzle flow.

Recently, Bolgar et al.[4] discovered that the transition during supersonic flight conditions (Ma_∞ = 1.6) excites the so-called flip-flop effect. This is a quasi-periodic series of transitions from the sea level mode into the altitude mode, followed by retransitions back into the sea level mode. This is a phenomenon that should be avoided at all cost, due to the high dynamic side loads it exerts onto the nozzle structure. The authors suspected the flip-flop phenomenon to be caused by the interaction of the nozzle flow with a supersonic expansion around the nozzle's lip and consequently suggested for designing a nozzle with a transition point that occurs during transonic flight. Therefore, it is the aim of the current research to characterize the transition of a Dual-Bell nozzle flow with a transonic external flow at Ma_∞ = 0.8. In order to do so, experimental investigations with schlieren shadowgraphy are carried out.

2. Experimental setup

2.1 Flow conditions

Experimental measurements are carried out at the Trisonic Wind Tunnel Munich (TWM). For more details about the measurement facility, the reader is referred to Bolgar et al.[3] . A planar model with a BFS ahead of the nozzle fairing is used for the underlying experiments. The quasi-2D BFS model is symmetric about its horizontal plane and spans across the entire test section of the wind tunnel. It has a 150 mm long gently curved nose which smoothly transitions into a 102.5 mm long flat plate prior to the step. The nose's shape was carefully designed in order to ensure locally subsonic conditions (at Ma_∞ = 0.80) about the model's forebody[26] . The step is 5 mm in height and attaches to a splitter plate, or the nozzle fairing. The step's height-to-width ratio, or aspect ratio, is 1 : 60, which according provides an unaffected recirculation region due to sidewall effects[6] . The overall model's thickness is 25 mm, or 3.7 % of the test section's height. The nozzle fairing has a total length of 35 mm, or $7h$, allowing to keep the high dynamics of the shear layer close to the nozzle flow. The nozzle fairing has a height of 15 mm. The 2D Dual-Bell nozzle in its center has a nozzle exit height of 14 mm and spans 56 mm across the model. The nozzle contour was designed and provided by Chloé Génin from DLR Lampoldshausen. The base nozzle of the Dual-Bell has a truncated ideal contour (TIC) and the extension has a constant pressure contour. The nozzle throat is 3.26 mm in height, giving it an expansion ratio of ε = 4.29, resulting in a design exit Mach number of 2.73. Based on its throat height, the nozzle has a Reynolds number of Re^* = 485, 000 at $p_{n,0}$ = 10 bar. The nozzle was designed to transition at NPR_{tr} = 9.5. Figure 2 illustrates the Dual-Bell nozzle model and the field of view (FOV) under investigation. Table 1 provides the boundary conditions of the nozzle for the underlying experiments.

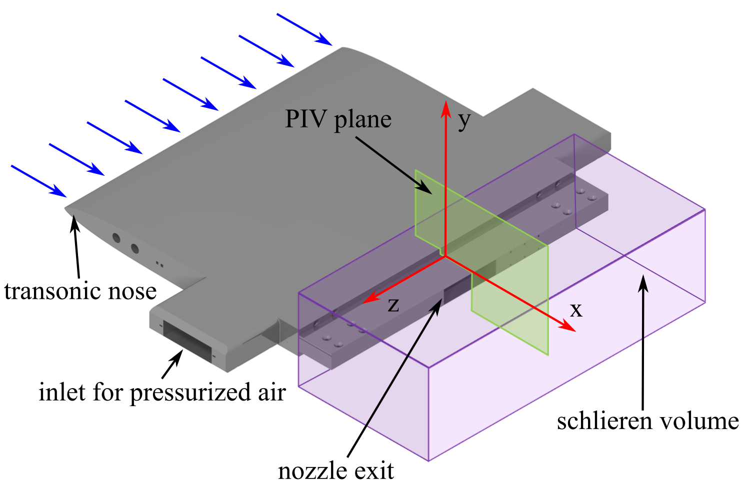

Figure 2: Illustration of the planar space launcher model with a 2D Dual-Bell nozzle and the measurement locations for PIV and schlieren

Table 2 provides an overview of the experimental free-stream conditions for the measurements. The ± values in the table indicate the standard deviation of each quantity during the measurements, while the measurement uncertainty

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Table 1: Nozzle flow conditions

Ma_∞	$p_{n,0}$ [bar]	NPR
0.80	10 ± 0.05	$\approx 6.9 - 10.5$

is within ± 1 %. In order to achieve transition of the nozzle at $Ma_\infty = 0.80$, the total pressure in the test section was linearly reduced from 2.00 bar to 1.30 bar in 8 s, held constant for 2 s, before it was linearly increased back to 2.00 bar in 8 s. At the same time, a constant thrust chamber pressure of 10 bar was maintained. This pressure profile also allows to investigate the hysteresis of the nozzle.

Table 2: Free-stream flow conditions

Ma_∞	p_0 [bar]	p_∞ [bar]	T_0 [K]	U_∞ [$\frac{m}{s}$]
0.80 ± 0.0010	2.00 – 1.30 – 2.00 in 20 s	1.30 – 0.85 – 1.30 in 20 s	287 ± 1	255

2.2 Schlieren recordings

The schlieren technology was used to qualitatively visualize the compressible flow effects. The light source of the schlieren system is 10 W (optical power) blue light emitting diode (LED). A condenser projects the light onto a vertical slit, visualizing the streamwise density gradients in the flow. The slit is placed in the focus of a concave mirror with a focal length of 4000 mm in a classical Z-setup, so that the light aft of the mirror travels through the side windows of the test section in parallel. On the other side of the test section, the changes in the parallelism of the light are detected. In order for this to work, the light is focused onto a so-called knife edge with a second concave mirror before being it gets projected onto a high-speed camera sensor. A Phantom v2640 camera was used to capture the images at 3,500 Hz during the experiments on transitioning, and at 50,000 Hz during the experiments on screeching. For a detailed description of the schlieren setup installed at the TWM facility, the reader is referred to Hampel[11] .

3. Results & discussion

3.1 Transition at transonic flight conditions

The transitioning of the nozzle is analyzed through the schlieren recordings. The light intensity of each schlieren image in a defined interrogation window in the jet plume region is evaluated (refer to white rectangle in the images of figure 5). As the average value of this interrogation window is usually close to either 0 or 1 (altitude or sea level mode, respectively) on a normalized scale after an intensity correction, it gives very reliable information about the nozzle mode. This value can then be quantified as the nozzle mode criterion, which provides the information about the nozzle's state. Figure 3 shows the run time of the wind tunnel vs. a nozzle mode criterion.

The plot in figure 3 shows that at transonic free-stream conditions only one transition from sea level to altitude mode occurs as NPR increases. This is in contrast to the previous findings of Bolgar et al.,[4] where at supersonic free-stream conditions, the flip-flopping with 41 transitions and retransitions of the two nozzle modes was excited as shown in figure 4. The schlieren images provided in figure 5 show the nozzle in its sea level and altitude modes, recorded at instances indicated by the asterisks in figure 3.

As NPR decreases from about 18 s in figure 3, the nozzle is forced to eventually retransitions back to sea level mode. The initial transition takes place around $NPR \approx 8.1$, whereas retransition occurs at $NPR \approx 7.9$. This indicates that the overturned contour of the nozzle extension provides a small hysteresis. More importantly however, the absence of the flip-flop effect proves Bolgar et al.'s[4] model, which states taht flip-flopping is excited with the presence of a supersonic external flow.

In more detail, it is triggered by an interaction of the jet plume with the external flow's Prandtl-Meyer expansion around the nozzle's lip, such as illustrated in figure 6. The portion below the axis of symmetry in this figure shows how the transition into altitude mode would cause an instant decrease in the Prandtl-Meyer expansion angle around the nozzle's lip, leading to an increased static pressure in the vicinity of the nozzle exit. This local increase in static

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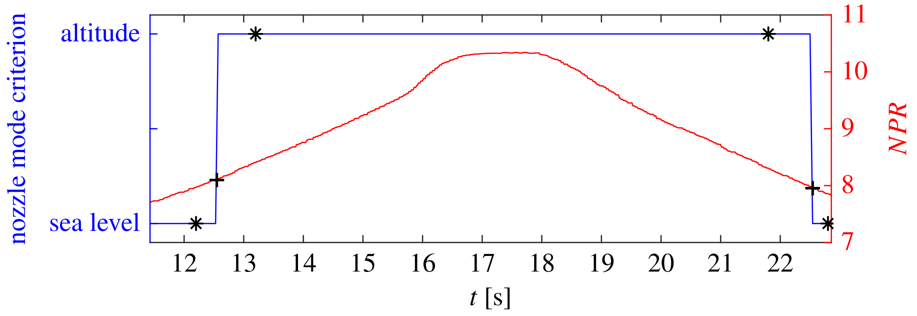

Figure 3: Time vs. nozzle mode criterion at $Ma_\infty = 0.80$. Asterisks provide the instance of the schlieren images provided in figure 5. The + signs indicate the moment of transition and retransition.

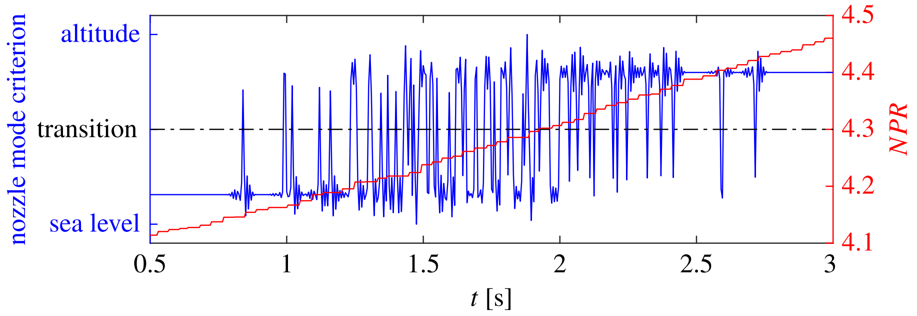

Figure 4: Time vs. nozzle mode criterion at $Ma_\infty = 1.60$. Originally published in Bolgar et al. 2019[4] .

pressure then decreases the effective nozzle pressure ratio (NPR_{eff}) acting on the jet plume below NPR_{tr}, forcing the nozzle flow to retransition back into its sea level mode. On the contrary, the portion above the axis of symmetry shows that when the nozzle flow retransitions back into sea level mode, the Prandtl-Meyer expansion angle around the nozzle's lip instantly increases again, decreasing the static pressure in the vicinity of the nozzle exit. This increases NPR_{eff} back to above NPR_{tr}, triggering the transition back into altitude mode again. The coupled phenomena of the Prandtl-Meyer expansion angle around the nozzle's lip and the nozzle's mode would also explain the reoccurring flip-flop phenomenon taking place.

Overall, the findings of this section show that a natural transition of a Dual-Bell nozzle is possible. The transition in transonic flight conditions completely eliminates the flip-flopping of the nozzle. Therefore, a Dual-Bell nozzle should be designed for the transitioning to occur during the transonic phase of flight.

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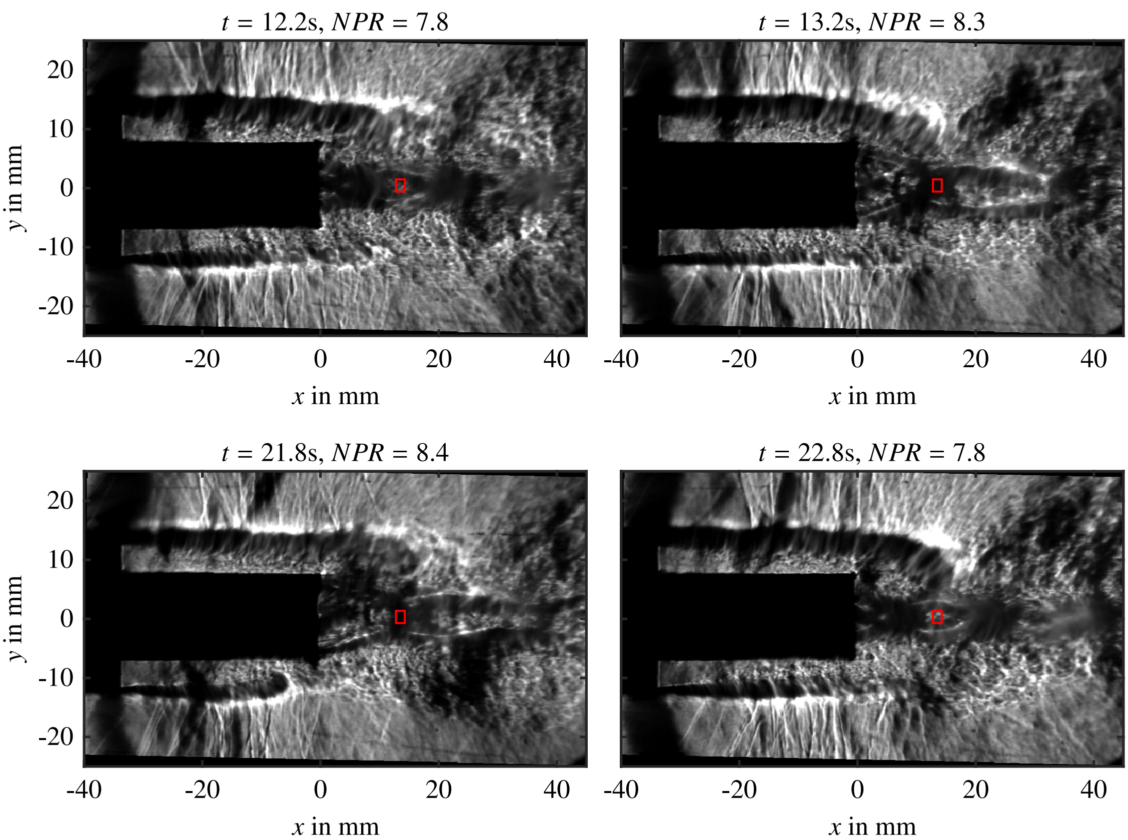

Figure 5: Schlieren recordings of the nozzle transitioning and retransitioning with increasing and decreasing *NPR*, respectively. The red box in the images illustrates the interrogation window for the nozzle mode criterion.

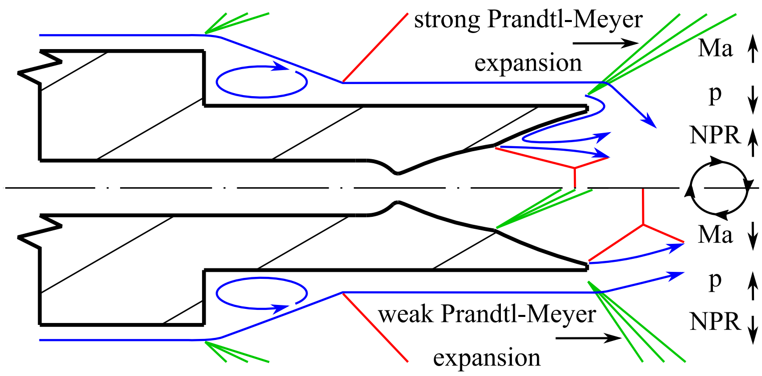

Figure 6: Sketch of the driving mechanism of the flip-flop effect at supersonic external conditions

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3.2 Screeching at transonic flight conditions

Even though the difficulties of the flip-flop effect can be controlled through designing the nozzle to transition in transonic flight, nozzle screeching was observed when the nozzle is in its sea level mode in transonic conditions. This was already assumed in the previous findings of Bolgar et al.[4] , however was not verified at the time. Back then, shock-like streaks were observed in each instantaneous vector field of the recorded data at $Ma_\infty = 0.80$ (refer to figure 7). Thus, even though the flow's local velocity was around $U = 250\,\mathrm{m/s}$, while the speed of sound was around $c \approx 320\,\mathrm{m/s}$, discontinuities, or shocks, were found in the vector fields. This is due to the fact that the acoustical waves, or screeching[1,18] , generated by the jet plume in sea level mode are superimposed onto the external flow. This causes a relative Mach number which is above the speed of sound, ultimately leading to the formation of shocks.

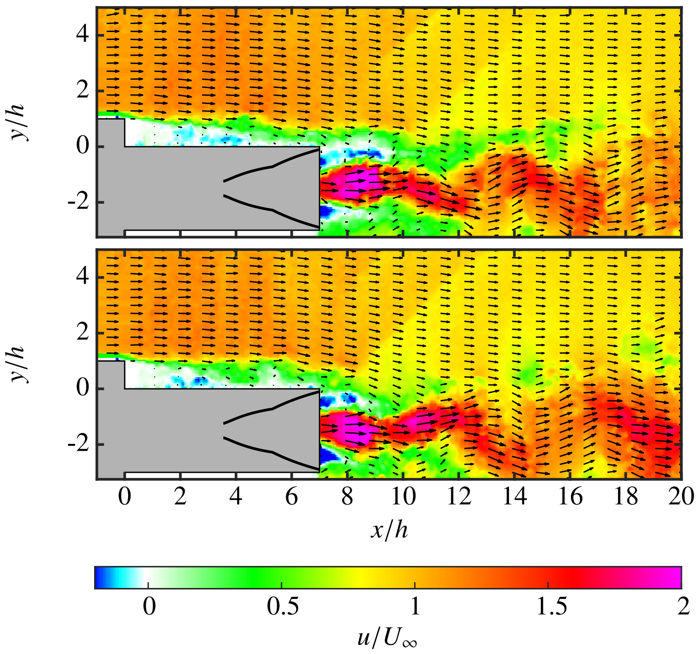

Figure 7: Illustration of the streamwise component of instantaneous vector fields at $Ma_\infty = 0.80$ with $NPR = 7$. Refer to Bolgar et al.[4] for more details on the measurements.

The frequency of these periodic shocks can be estimated by calculating the relative Mach number of the superimposed flows through the oblique shock wave theory (refer to the sketch in figure 8). This was done by approximating the sudden change in the flow direction across the shocks in an ensemble of vector fields. This yielded an average flow direction change of $\theta = 6.5°$. The shock angle relative to the incoming flow is $\beta = 50°$. By substituting these two values into the oblique shock wave theory equation for the shock angle[2] , it is possible to calculate the Mach number ahead of the shock:

$$\tan\theta = 2\cot\beta \frac{Ma^2 \sin^2\beta - 1}{Ma^2(\kappa + \cos 2\beta) + 2} \tag{1}$$

where κ is the ratio of specific heats.

In this case, the weak solution of this equation yielded a relative Mach number of $Ma_{\mathrm{rel}} \approx 1.5$, which is equivalent of a relative velocity of $U_{\mathrm{rel}} \approx 480\,\mathrm{m/s}$ when assuming the speed of sound from the physical reference frame stated above. Since the free-stream's flow velocity is around 250 m/s at that location, the shock moves upstream with the velocity difference of $|\,U\,|_{\mathrm{shock}} = |\,U - U_{\mathrm{rel}}\,| = 230\,\mathrm{m/s}$. Due to the small change in the flow direction, the magnitude of the streamwise velocity and its vector are nearly identical ($\cos\theta \approx 1$). Similary, the average distance between shocks can also be approximated into the streamwise direction. This quantity was also extracted from the instantaneous vector fields, which is $d_{\mathrm{shock}} = 0.018\,\mathrm{m}$. Now, the screech frequency can be calculated by $f_s = U_{\mathrm{shock}}/d_{\mathrm{shock}} \approx 12,500\,\mathrm{Hz}$. When considering the nozzle exit velocity and the height of the base nozzle at the contour inflection, the result yields a dimensionless screech frequency of $Sr_s \approx 0.3$. This is in good agreement with common screech frequencies for overexpanded jets[21,24] . Even though other measurement methods need to be utilized to determine a precise screech frequency, this rough estimation allows to identify the previously observed shock-like structures as screeching.

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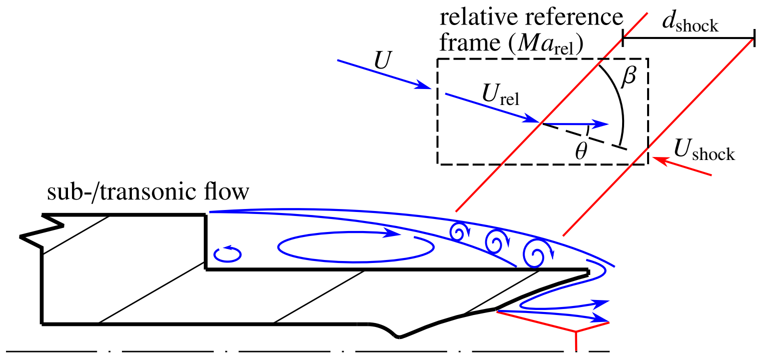

Figure 8: Graphical illustration of the calculation method of the screech frequency

Similar to the shock-like structures described above, coherent patches alternating in bright and dark colors are observed moving upstream in schlieren recordings (refer to figure 9). These data was recorded with the current nozzle without an external flow. The recordings were captured at a frame rate of 50 kHz, allowing to statistically resolve coherent phenomena with a frequency up to 25 kHz[17,23]. The light intensity of the schlieren recordings within the interrogation window is evaluated across 10,000 images. A fast Fourier transform (FFT) using Welch's method[29] yields a dominant frequency of $f_s = 11,200$ Hz. This is shown in the frequency vs. power spectral density (PSD) plot in figure 10. Since, the velocity of the jet plume is not available from the schlieren images, the Strouhal number cannot be determined at this point. However, the frequency closely matches the estimation from the vector fields described above. Therefore, screeching is an omnipresent phenomenon with Dual-Bell nozzles.

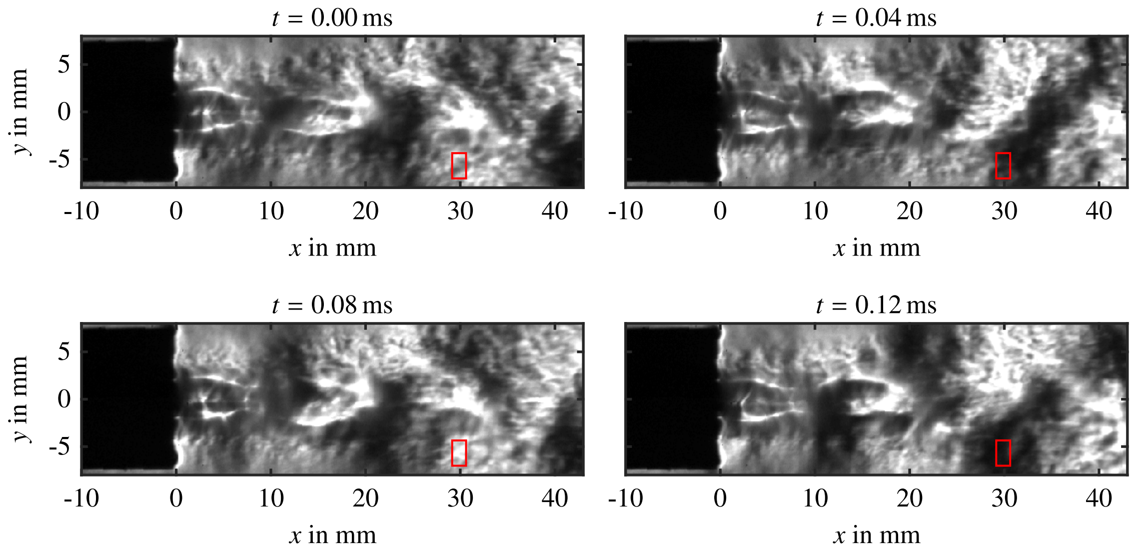

Figure 9: Schlieren recordings of the nozzle in sea level mode. The alternating bright and dark patches indicate coherent changes in the density gradient. The red box in the images illustrates the interrogation window for the FFT.

The strong density gradients present in figure 9, or the shocks seen in figure 7, indicate that the screeching of a Dual-Bell nozzle is rather intense. In conventional nozzles, screeching is generated by an interaction of instabilities with the shock cell structures of the overexpanded jet, stemming off of the nozzle's lip[24]. This interaction forms a resonant feedback mechanism in the vicinity of the nozzle exit, resulting in the propagation of acoustic waves. Thus, it is a possibility that the cavity, or reverse flow region, in the nozzle extension present during sea level mode excites screeching in a more pronounced manner than the interaction of instabilities with the shock cell structures of an overexpanded jet. The reverse flow region in the nozzle extension (refer to figure 7) creates instabilities on a much larger scale than the ones present in the boundary layer of a generic nozzle. This also results in the strong undulations of the jet plume observed by Bolgar et al.[4]. Since screeching in the sea level mode is present with a transonic external

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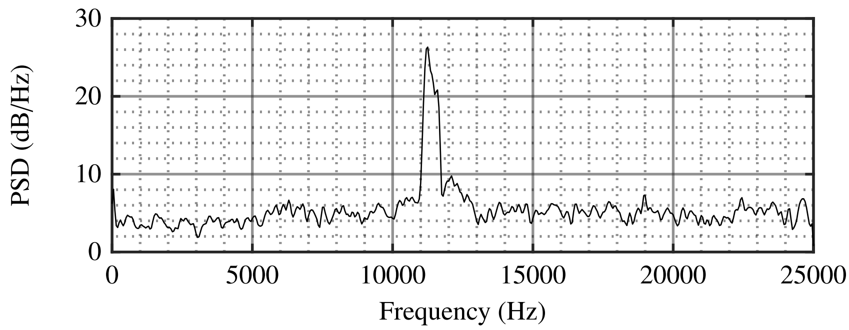

Figure 10: Frequency vs. PSD of the light intensity in the schlieren recordings in figure 9

flow and without an external flow, future investigation on Dual-Bell nozzles need to focus on the difficulties pertaining to this acoustic phenomenon.

4. Summary & conclusions

Experimental investigations of a Dual-Bell nozzle with a transonic external flow at $Ma_\infty = 0.80$ were conducted and compared to previous findings at $Ma_\infty = 1.60$. Schlieren recordings were used to capture the transition from sea level to altitude mode. The analysis of the data shows that the previously found flip-flop effect present at $Ma_\infty = 1.60$ is completely eliminated when the transition takes place below the sound barrier. Thus, at free-stream conditions below sonic free-stream velocities, a natural transition of the Dual-Bell nozzle is achieved. At the same time these results prove that above sonic conditions, the interaction between a Prandtl-Meyer expansion around the nozzle's lip and the jet plume causes the flip-flop effect, ultimately making a natural transition of the Dual-Bell nozzle excessively difficult. Therefore, a Dual-Bell nozzle should be designed to transition in the transonic phase of flight in order to avoid high side-loads and to maximize performance.

In the nozzle's sea level mode, screeching is observed. The findings of this phenomenon were analyzed via the calculation of the screech shock's present in previously recorded velocity vector fields at $Ma_\infty = 0.80$. This yielded a screech frequency of $Sr_s \approx 0.3$, which is in good agreement with other values found in literature. An analysis of current schlieren recordings without an external flow yielded a dominant frequency of $f_s = 11,200\,\text{Hz}$ versus the $f_s \approx 12,500\,\text{Hz}$ obtained from the vector fields. Therefore, the observed phenomenon can be characterized as screeching, however with a very high intensity, which can be deduced from the high density gradients in the schlieren recordings, or the shocks present in the vector fields. The high intensity of this phenomenon can be explained through the presence of the large cavity, or reverse flow region, in the nozzle extension. The interaction of this separated region with the shock cells of the jet plume excites screeching in a more pronounced manner than the instabilities in the boundary layer of a generic nozzle. Thus, nozzle screeching in sea level mode is an inherent difficulty of a Dual-Bell nozzle.

Overall, it can be concluded that the current results prove that a Dual-Bell nozzle can be designed for a natural transition to occur below sonic free-stream conditions. In supersonic free-stream conditions, a natural transition of the nozzle flow may be impossible. At this time, the presence of high-intensity screeching is identified as a challenge for the Dual-Bell nozzle concept. Therefore, future research on this type of nozzle should focus on controlling this acoustic phenomenon, or find solutions to manage the difficulties pertaining to the adverse effects of screeching.

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