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Design of a Flight Control System (FCS) applying airworthiness requirements of MIL-STD-1797A

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Abstract

This article shows a brief and practical application of control techniques to improve the stability and behavior of an aircraft whose dynamic modes do not meet the airworthiness requirements referred to Flying Qualities. Specifically, some control laws will be designed in order to change the closed-loop poles of a DC-8 and make them satisfy the standards of the American standards MIL-STD-1797A, which have remained almost the same since the 1980s and also have become a basis to other normatives.

1. Introduction

Dynamic stability and maneuverability of an aircraft are some of its most important features, not only for safety reasons and the structural loads it has influence in, but also due to the accelerations and frequencies passengers have to deal with and therefore affect their comfort. Both qualities are closely related to the aerodynamic, geometric and inertial characteristics of the airplane, which are responsible for the kind of response to external perturbations such as vertical or lateral gusts (stability) and deflections of the control surfaces (maneuverability).

Despite that dependence between the dynamic response and the the aircraft properties, it can be improved by the use of Stability Augmentation Systems (SAS). As it will be explained in detail later, the dynamic behavior of the airplane is determined by the roots of its characteristic equation, which define the evolution of the translation and rotation degrees of freedom during the time and, consequently, the stable or unstable nature of the dynamic modes. By using control laws that apply forces and moments proportional to and against the motion variables, the coefficients of the stability quartic change from their open-loop values to new ones, so the roots are modified as well.

2. Mathematical model

2.1 Euler's equations

Assuming the aircraft behaves as a perfectly rigid body, without any kind of strain and ignoring aeroelastic effects, the physic problem is reduced to a system of six degrees of freedom, correspondent to the three translations of the center of gravity and the three rotations around it. So, the resulting system of six differential equations are compound by the three scalar components of the linear momentum equation and the angular momentum equation.

Although the mass varies with time due to the fuel ejection in the engines, its variation during the time the in which the transient response is acting is so low that we can neglect it and consider the mass constant in our study. For the same reason, the tensor of inertia in body axes will be considered constant too. The equations of motion in inertial axes and vectorial form are:

$$\vec{F} = m\frac{d\vec{V}}{dt} \tag{1}$$

$$\vec{G} = \frac{d\vec{h}}{dt} \ , \ \ \vec{h} = I\vec{w} \tag{2}$$

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The motion variables have a physical meaning easier to understand in body-fixed axes, as well as a more simplified form because of the invariability of the moments and products of inertia. Since the previous equations are only valid for inertial axes with no angular velocity, those have to be corrected to take into account the inertia forces and moments by using the Coriolis theorem, which correlates a derivative in inertial axes with the derivative in non inertial ones:

$$\dot{\vec{A}}\Big|_1 = \dot{\vec{A}}\Big|_0 + \vec{w}_{01} \times \vec{A} \tag{3}$$

Hereafter, the forces, linear velocity, angular moment around the center of gravity and angular velocity are expressed in body-fixed axes and designated as it follows:

$$\vec{F} = (F_x, F_y, F_z)^T \quad , \quad \vec{V} = (u, v, w)^T \quad , \quad \vec{G} = (L, M, N)^T \quad , \quad \vec{w} = (p, q, r)^T \tag{4}$$

By applying the Coriolis theorem to the previous equations and developing them scalarly, the Euler's equations are obtained:

$$F_x = m(\dot{u} - rv + qw) \tag{5}$$

$$F_y = m(\dot{v} + ru - pw) \tag{6}$$

$$F_z = m(\dot{w} - qu + pv) \tag{7}$$

$$L = I_x\dot{p} - J_{xz}\dot{r} + (I_z - I_y)qr - J_{xz}pq \tag{8}$$

$$M = I_y\dot{q} + (I_x - I_z)pr + J_{xz}(p^2 - r^2) \tag{9}$$

$$N = I_z\dot{r} - J_{xz}\dot{p} + (I_y - I_x)pq + J_{xz}qr \tag{10}$$

2.2 Stability derivatives

The incremental aerodynamic forces and moments that are experienced by the airplane due to its perturbed movements can be expressed, according to the unsteady aerodynamics, as the sum of the contributions of the instantaneous values of the motion variables and their evolution along time since the start of the perturbed movement. This fact allows them to be defined through a Taylor expansion, and be linear if only the first derivatives are considered.

$$\Delta F = F_u\Delta u + F_{\dot{u}}\Delta\dot{u} + F_v\Delta v + F_{\dot{v}}\Delta\dot{v} + ... \tag{11}$$

As a result, each force and moment are function of the degrees of freedom and their first derivatives, and the constants of proportionality are called stability coefficients:

$$F_{x_i} = \frac{\partial F}{\partial x_i}\Big|_s \quad , \quad F_{\dot{x}_i} = \frac{\partial F}{\partial \dot{x}_i}\Big|_s \tag{12}$$

Although it might seem that each force depends on every degree of freedom, the reference condition of study of longitudinal movement ensures that the derivatives of the longitudinal forces and moments respect to the lateral-directional degrees of freedom are null. Analogously, the derivatives of the lateral-directional terms respect to the longitudinal variables are zero.

$$\Delta Y, \Delta L, \Delta N = f(\Delta v, \Delta p, \Delta r, \Delta\dot{v}, \Delta\dot{p}, \Delta\dot{r}, \Delta\delta_a, \Delta\delta_r, \Delta\dot{\delta}_a, \Delta\dot{\delta}_r) \tag{13}$$

$$\Delta X, \Delta Z, \Delta M = f(\Delta u, \Delta w, \Delta q, \Delta\dot{u}, \Delta\dot{w}, \Delta\dot{q}, \Delta\delta_e, \Delta\dot{\delta}_e) \tag{14}$$

In addition, some stability derivatives are usually insignificant in most commercial airplanes and can be ignored. To sum up, the forces and moments of perturbation can be expressed as it follows:

$$\Delta X = X_u\Delta u + X_w\Delta w + X_{\delta_e}\Delta\delta_e \tag{15}$$

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$$\Delta Y = Y_v \Delta v + Y_p \Delta p + Y_r \Delta r + Y_{\delta_r} \Delta \delta_r \tag{16}$$

$$\Delta Z = Z_u \Delta u + Z_w \Delta w + Z_q \Delta q + Z_{\dot{w}} \Delta \dot{w} + Z_{\delta_e} \Delta \delta_e \tag{17}$$

$$\Delta L = L_v \Delta v + L_p \Delta p + L_r \Delta r + L_{\delta_a} \delta_a + L_{\dot{\delta}_a} \Delta \dot{\delta}_a + L_{\delta_r} \Delta \delta_r \tag{18}$$

$$\Delta M = M_u \Delta u + M_w \Delta w + M_q \Delta q + M_{\dot{w}} \Delta \dot{w} + M_{\delta_e} \Delta \delta_e + M_{\dot{\delta}_e} \Delta \dot{\delta}_e \tag{19}$$

$$\Delta N = N_v \Delta v + N_p \Delta p + N_r \Delta r + N_{\delta_a} \Delta \delta_a + N_{\delta_r} \Delta \delta_r + N_{\dot{\delta}_r} \Delta \dot{\delta}_r \tag{20}$$

2.3 Linearization of the dynamic equations

The problem studied is the evolution of the movement of an airplane during the time after a perturbation, such as a gust or the pilot's action, that moves it away from its steady state to a transient movement. The subsequent motion can be divided into the steady state, which dos not depend on the time, and and a perturbation whose value is small in comparison to the steady value:

$$A(t) = A_s + \Delta A(t)\ ,\ \frac{\Delta A}{A} << 1 \tag{21}$$

This division can be used as a basis to linearize the equations if the terms of order ΔA become the variables of study and the terms of order $(\Delta A)^2$ and superior are not taken into account. Since the reference condition of symmetrical, steady rectilinear flight that we are analyzing is characterized by the lack of accelerations ($\dot{u_s} = \dot{v_s} = \dot{w_s} = \dot{p_s} = \dot{q_s} = \dot{r_s} = 0$) and angular velocities ($p_s = q_s = r_s = 0$), as well as a null roll and yaw angle ($\beta = \phi = 0$).Finally, by introducing the perturbed variables into the equations, developing the trigonometric functions taking into account that $\cos(A + \Delta A) \simeq \cos A - \sin A \Delta A$ y $\sin(A + \Delta A) \simeq \sin A + \cos A \Delta A$, subtracting the equations in the steady state and retaining only terms of order ΔA, the final system of differential equations is obtained:

$$-mg \cos\theta_s \Delta\theta + \Delta X = m(\Delta\dot{u} + w_s \Delta q) \tag{22}$$

$$mg \cos\theta_s \Delta\phi + \Delta Y = m(\Delta\dot{v} + u_s \Delta r - w_s \Delta p) \tag{23}$$

$$-mg \sin\theta_s + \Delta Z = m(\Delta\dot{w} - u_s \Delta q) \tag{24}$$

$$\Delta L = I_x \Delta\dot{p} - J_{xz} \Delta\dot{r} \tag{25}$$

$$\Delta M = I_y \Delta\dot{q} \tag{26}$$

$$\Delta N = I_z \Delta\dot{r} - J_x z \Delta\dot{p} \tag{27}$$

$$\Delta p = \Delta\dot{\phi} - \sin\theta_s \Delta\dot{\psi} \tag{28}$$

$$\Delta q = \Delta\dot{\theta} \tag{29}$$

$$\Delta r = \cos\theta_s \Delta\dot{\psi} \tag{30}$$

As the simplified equations show, there are two sets of equations which are decoupled one to another: the longitudinal equations and the lateral-directional ones.

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2.4 Longitudinal modes

The system of equations that describe the longitudinal dynamics of an aircraft after a perturbation is:

$$\begin{bmatrix} m & 0 & 0 & 0 \\ 0 & m - Z_{\dot{w}} & 0 & 0 \\ 0 & -M_{\dot{w}} & I_y & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} \Delta\dot{u} \\ \Delta\dot{w} \\ \Delta\dot{q} \\ \Delta\dot{\theta} \end{Bmatrix} = \begin{bmatrix} X_u & X_w & -mw_s & -mg\cos\theta_s \\ Z_u & Z_w & Z_q + mu_s & -mg\sin\theta_s \\ M_u & M_w & M_q & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{Bmatrix} \Delta u \\ \Delta w \\ \Delta q \\ \Delta\theta \end{Bmatrix} + \begin{bmatrix} X_{\delta_e} \\ Z_{\delta_e} \\ M_{\delta_e} \\ 0 \end{bmatrix} \left\{\delta_e\right\} \tag{31}$$

It is more common to express the system above in the form $\{\dot{X}\} = [A]\{X\} + [B]\{u\}$, where $\{X\}$ is the state vector, $\{u\}$ is the control vector, $[A]$ is the state matrix and $[B]$ is the control matrix. So, multiplying the previous matrices by the inverse of the inertia matrix we obtain:

$$\begin{Bmatrix} \Delta\dot{u} \\ \Delta\dot{w} \\ \Delta\dot{q} \\ \Delta\dot{\theta} \end{Bmatrix} = \begin{bmatrix} \frac{X_u}{m} & \frac{X_w}{m} & -w_s & -g\cos\theta_s \\ \frac{Z_u}{m-Z_{\dot{w}}} & \frac{Z_w}{m-Z_{\dot{w}}} & \frac{Z_q+mu_s}{m-Z_{\dot{w}}} & \frac{-mg\sin\theta_s}{m-Z_{\dot{w}}} \\ \frac{Z_u M_{\dot{w}}}{Iy(m-Z_{\dot{w}})} + \frac{M_u}{I_y} & \frac{Z_w M_{\dot{w}}}{Iy(m-Z_{\dot{w}})} + \frac{M_w}{I_y} & \frac{(Z_q+mu_s)M_{\dot{w}}}{Iy(m-Z_{\dot{w}})} + \frac{M_q}{I_y} & \frac{-mg\sin\theta_s M_{\dot{w}}}{Iy(m-Z_{\dot{w}})} \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{Bmatrix} \Delta u \\ \Delta w \\ \Delta q \\ \Delta\theta \end{Bmatrix} + \begin{bmatrix} \frac{X_{\delta e}}{m} \\ \frac{Z_{\delta e}}{m-Z_{\dot{w}}} \\ \frac{M_{\dot{w}} Z_{\delta e}}{I_y(m-Z_{\dot{w}})} + \frac{M_{\delta e}}{I_y} \\ 0 \end{bmatrix} \left\{\delta_e\right\} \tag{32}$$

This is a system of differential equations with constant coefficients, so the homogeneous solution of the motion variables has the form $\Delta u = u_0 e^{\lambda t}$, $\Delta w = w_0 e^{\lambda t}$, $\Delta q = q_0 e^{\lambda t}$, $\Delta\theta = \theta_0 e^{\lambda t}$, where λ are the eigenvalues of the state matrix. By calculating the determinant $|A - \lambda I| = 0$ we obtain the so-called stability quartic, whose roots are the eigenvalues of the longitudinal dynamics and determine the damping and frequency of its modes. The real part of every pole has to be negative in order to be dynamically stable. The eigenvectors $\{\xi_i\}$ of each mode are obtained by calculating $[A - \lambda_i I]\{\xi_i\} = 0$ for every eigenvalue, and they determine the relative amplitude and phase between every variable for each mode.

$$A\lambda^4 + B\lambda^3 + C\lambda^2 + D\lambda + E = 0 \tag{33}$$

Typically, most commercial airplanes flying at high altitude happen to have two pairs of conjugate complex roots.

$$\lambda_{1,2} = -\varepsilon_1 \pm iw_1 \tag{34}$$

$$\lambda_{3,4} = -\varepsilon_3 \pm iw_3 \tag{35}$$

$$t_{1/2} = \frac{\ln 2}{\varepsilon} \ , \ T_2 = -\frac{\ln 2}{\varepsilon} \tag{36}$$

$$\varepsilon = \zeta w_n \ , \ w = w_n\sqrt{1-\zeta^2} \ \rightarrow \ \lambda = -\zeta w_n \pm i w_n\sqrt{1-\zeta^2} \tag{37}$$

One pair of roots has both the real and the imaginary part significantly smaller that the other, and they are associated to the so-called phugoid mode. This oscillatory mode is characterized by a poor damping and a low frequency, with typical times to reduce a perturbation to its half and periods of roughly hundreds of seconds. This modes perturbs mainly the pitch angle $\Delta\theta$ and the longitudinal speed Δu.

The other pair is called the short period and it is described as a fast oscillatory mode, with high frequency and damping. Although it vanishes after a short time of approximately some seconds, it is very important not to exceed or lack some levels of damping and frequencies and therefore the normative sets a range of admissible values for both parameters. It affects mainly the pitch angle $\Delta\theta$ and the angle of attack $\Delta\alpha$.

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2.5 Lateral-directional modes

Analogously to the symmetric case, the system of equations that rules the lateral-directional dynamics is:

$$\begin{bmatrix} m & 0 & 0 & 0 \\ 0 & I_x & -J_{xz} & 0 \\ 0 & -J_{xz} & I_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} \Delta\dot{v} \\ \Delta\dot{p} \\ \Delta\dot{r} \\ \Delta\dot{\phi} \end{Bmatrix} = \begin{bmatrix} Y_v & Y_p + mw_s & Y_r - mu_s & mg\cos\theta_s \\ L_v & L_p & L_r & 0 \\ N_v & N_p & N_r & 0 \\ 0 & 1 & \tan\theta_s & 0 \end{bmatrix} \begin{Bmatrix} \Delta v \\ \Delta p \\ \Delta r \\ \Delta\phi \end{Bmatrix} + \begin{bmatrix} 0 & Y_{\delta_r} \\ L_{\delta_a} & L_{\delta_r} \\ N_{\delta_a} & N_{\delta_r} \\ 0 & 0 \end{bmatrix} \begin{Bmatrix} \Delta\delta_a \\ \Delta\delta_r \end{Bmatrix} \tag{38}$$

Again, it is better to express it in the form $\{\dot{X}\} = [A]\{X\} + [B]\{u\}$:

$$\begin{Bmatrix} \Delta\dot{v} \\ \Delta\dot{p} \\ \Delta\dot{r} \\ \Delta\dot{\phi} \end{Bmatrix} = \begin{bmatrix} \frac{Y_v}{m} & \frac{Y_p}{m} + w_s & \frac{Y_r}{m} - u_s & g\cos\theta_s \\ \frac{I_zL_v+J_{xz}N_v}{I_xI_z-J_{xz}^2} & \frac{I_zL_p+J_{xz}N_p}{I_xI_z-J_{xz}^2} & \frac{I_zL_r+J_{xz}N_r}{I_xI_z-J_{xz}^2} & 0 \\ \frac{I_xN_v+J_{xz}L_v}{I_xI_z-J_{xz}^2} & \frac{I_xN_p+J_{xz}L_p}{I_xI_z-J_{xz}^2} & \frac{I_xN_r+J_{xz}L_r}{I_xI_z-J_{xz}^2} & 0 \\ 0 & 1 & \tan\theta_s & 0 \end{bmatrix} \begin{Bmatrix} \Delta v \\ \Delta p \\ \Delta r \\ \Delta\phi \end{Bmatrix} + \begin{bmatrix} 0 & \frac{Y_{\delta_r}}{m} \\ \frac{I_zL_{\delta_a}+J_{xz}N_{\delta_a}}{I_xI_z-J_{xz}^2} & \frac{I_zL_{\delta_r}+J_{xz}N_{\delta_r}}{I_xI_z-J_{xz}^2} \\ \frac{I_xN_{\delta_a}+J_{xz}L_{\delta_a}}{I_xI_z-J_{xz}^2} & \frac{I_xN_{\delta_r}+J_{xz}L_{\delta_r}}{I_xI_z-J_{xz}^2} \\ 0 & 0 \end{bmatrix} \begin{Bmatrix} \Delta\delta_a \\ \Delta\delta_r \end{Bmatrix} \tag{39}$$

Unlike the longitudinal case, most commercial airplanes have one pair of conjugate complex roots, which corresponds to an oscillatory mode, and two real roots, corresponding to two pure exponential modes:

$$\lambda_1 = -\varepsilon_1 \tag{40}$$

$$\lambda_2 = -\varepsilon_2 \tag{41}$$

$$\lambda_{3,4} = -\varepsilon_3 \pm iw_3 \tag{42}$$

Usually, one of the real roots has a module much higher than the other one. It is associated to the roll mode, which consists in a highly damped non-oscillatory mode that disturbs basically the roll angle $\Delta\phi$ and rolling angular velocity Δp, being quite decoupled from the other modes. Its characteristic time is smaller than a second.

The smallest real root is associated to the spiral mode, which consists in a short non-oscillatory mode with a very poor damping. Unlike the other modes, it is not strange to be divergent and even the normative allows it to be unstable as long as the module of its negative damping is under a maximum value. It affects the sideslip angle β, roll angle ϕ and yaw, and it is mainly excited by a perturbation in sideslip angle.

The pair of conjugate complex roots correspond to the Dutch roll mode, which can be considered as the lateral-directional homologous of the short-period mode. It is an oscillatory mode of high frequency, with periods of typically 7-10 seconds, but with a significantly lower damping, having $t_{1/2}$ of roughly 10 seconds. It presents a high coupling among roll and yaw, being these two perturbation delayed around 90º. It is a critical mode for safety reasons, and because of his lack of damping, the normative establishes minimum values for it, as well as its natural frequency and their product.

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3. Open-loop analysis

It will be calculated the damping and frequency of each longitudinal and lateral-directional mode of a Douglas DC-8 for two typical flight conditions: cruise and flight at maximum horizontal speed V_{NE}. Then, these values will be checked to see if they meet or not the requirements of MIL-STD-1797A. The evolution in the time of the motion variables will be simulated too with Matlab.

3.1 Cruise

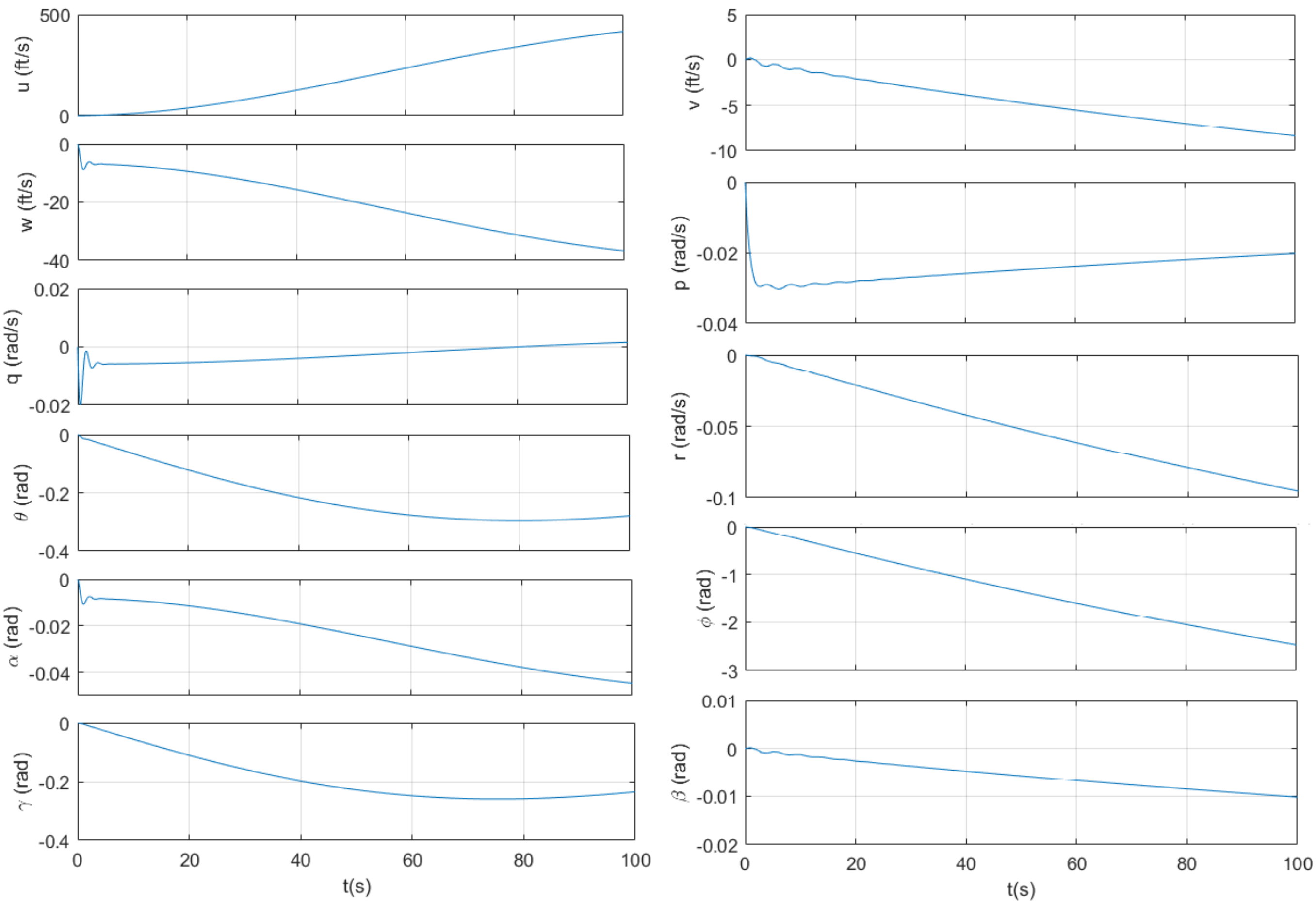

Figure 1: Response of the DC-8 to a deflection of 1° of the elevator and rudder in a flight condition of cruise

Table 1: Parameters of the dynamic modes of the DC-8 and adequacy with the normative in a flight condition of cruise

ζ_p	ζ_{sp}	CAP(s^{-2})	$\tau_r(s)$	$\tau_s(s)$	ζ_d	$w_d(s^{-1})$	$\zeta_d w_d(s^{-1})$
0.2410	0.3421	0.4520	0.7977	250	0.0793	1.4956	0.1186
√	√	√	√	√	X	√	X

As the table above shows, the damping of the Dutch roll is not enough as it would be expected and therefore the design of control laws in order to increase it is necessary.

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3.2 Maximum horizontal speed V_{NE}

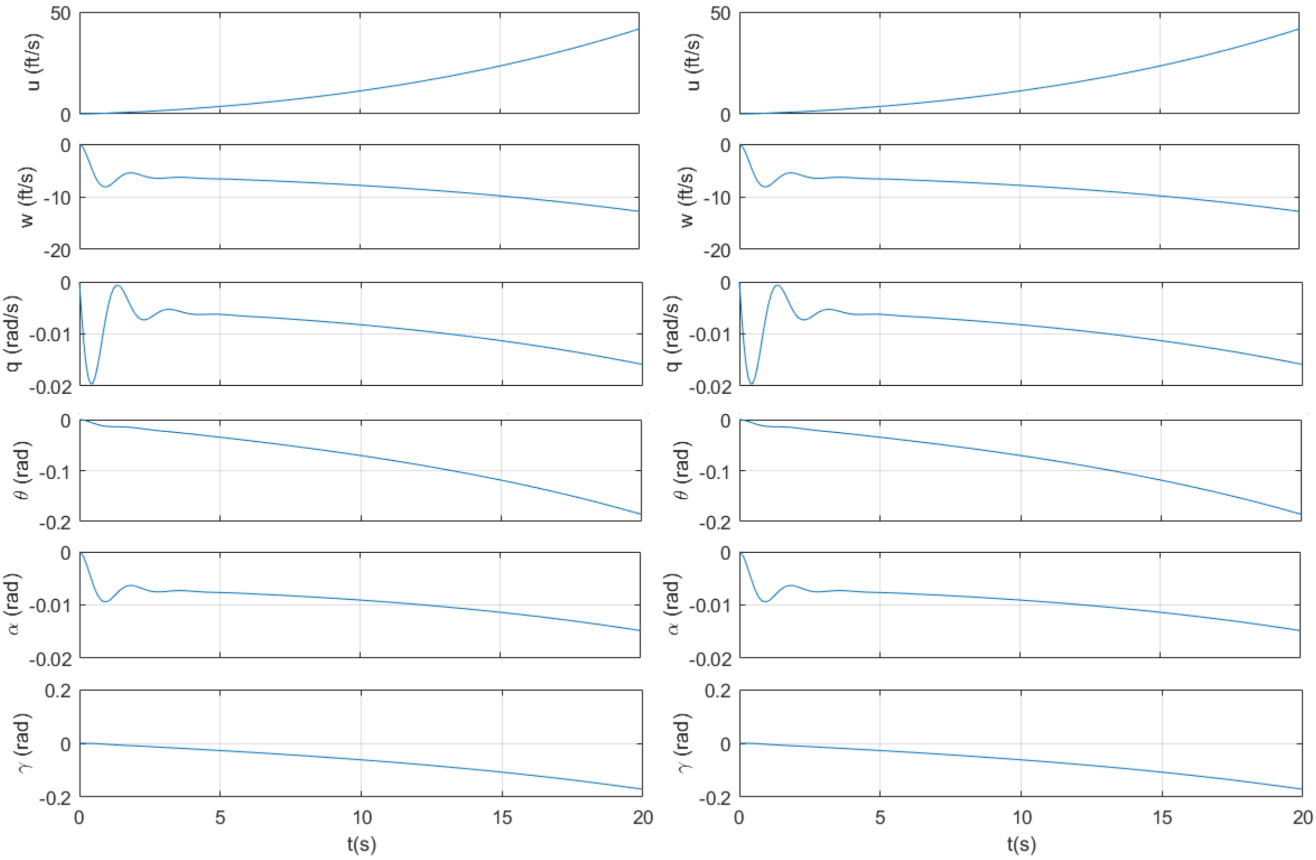

Figure 2: Response of the DC-8 to a deflection of 1° of the elevator and rudder in a flight condition of V_{NE}

Table 2: Parameters of the dynamic modes of the DC-8 and adequacy with the normative in a flight condition of V_{NE}

$\tau_{p1}(s)$	$\tau_{p2}(s)$	ζ_{sp}	CAP(s^{-2})	$\tau_r(s)$	$\tau_s(s)$	ζ_d	$w_d(s^{-1})$	$\zeta_d w_d(s^{-1})$
9.2764	-14.1043	0.3245	0.5219	0.7376	227.27	0.0854	1.5898	0.1358
-	X	X	√	√	√	X	√	X

The speed in that flight condition is so high that not only the Dutch roll is poorly damped, but the damping of the short-period is too low as well. Apart from that, it can bee seen in the plots how the classical phugoid mode disappears to originate two pure exponential modes, being one of them divergent and therefore inadmissible.

4. Control

Modify the aerodynamic properties and redesign the aircraft would be too costly at the stage of the project in which the deficiencies in the transient response are detected, since they are difficult to estimate through calculations. The use of stability augmentation systems (SAS) allows improving it once the aircraft is designed. These systems are based on the negative feedback of the state vector, thereby applying forces and moments proportionally to the motion variables.

$$\delta_s = \delta_{s\ pilot} \pm \Delta\delta_{s\ SAS} \tag{43}$$

$$\Delta\delta_{s\ SAS} = K_u \Delta u + K_w \Delta w + K_q \Delta q + ... \tag{44}$$

As a result, the closed-loop poles vary and the dynamic behaviour can be improved.

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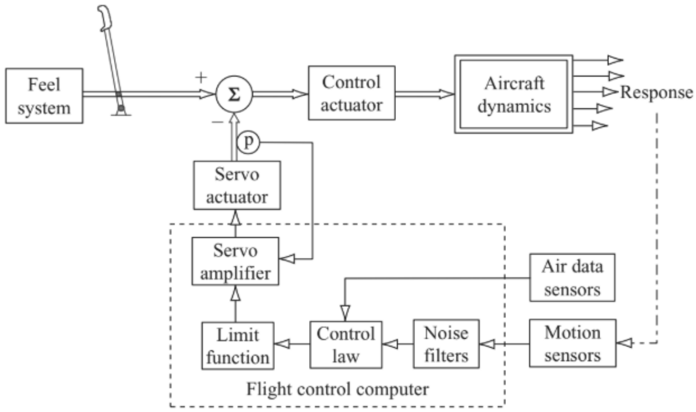

Figure 3: General scheme of a SAS. [1]

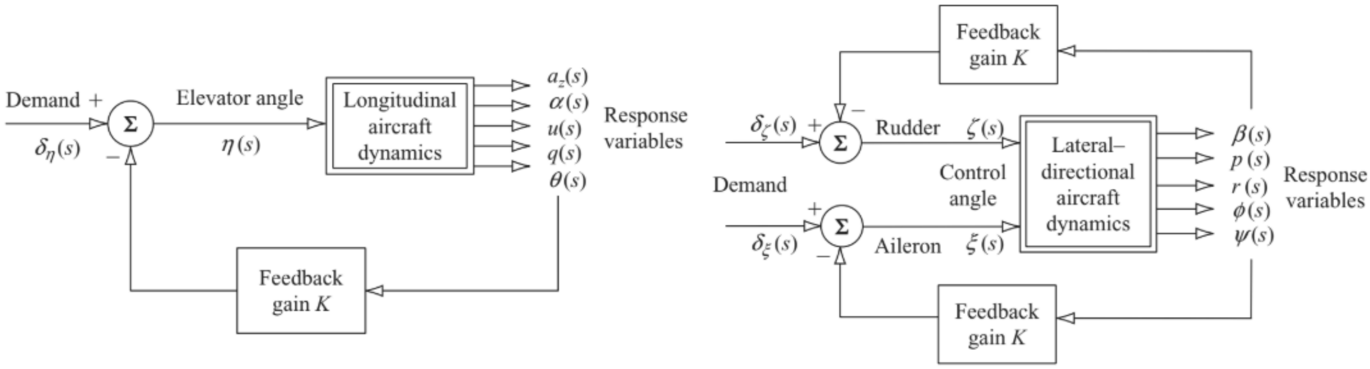

Figure 4: Possibilities of longitudinal and lateral-directional SAS. [1]

4.1 Pole-Placement Method

The Pole-Placement Method is a numerical method that lets the control designer select the desired closed-loop poles and, as a consequence, the parameters of the dynamic modes. If the control vector is expressed as:

$$\{u\} = \{v\} - [K]\{x\} \tag{45}$$

Where $\{v\}$ is the entry without feedback, then the system of equations in state form can be written as:

$$\{\dot{X}\} = [A - BK]\{X\} + [B]\{v\} \tag{46}$$

So the actual equation that gives the new poles is $|A - BK - \lambda I| = 0$. The method calculates the required gains to make the desired poles be the roots of the new stability quartic. For univariable entries such as the longitudinal case, the feedback matrix is unique and the four poles can be fixed only if all the motion variables are available to measure. For multivariable entries like the lateral-directional case, the feedback matrix is not unique if there are more possible feedbacks than variables.

5. Closed-loop analysis

In this section, the values of the parameters of the dynamic modes which did not satisfy the requirements will be selected to suit them. To avoid the problem of the multiple possible feedback matrices for the lateral-directional system, only the rudder deflection will be used for feedback and the obtained gains will be rounded to hundredths for practical reasons.

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5.1 Cruise

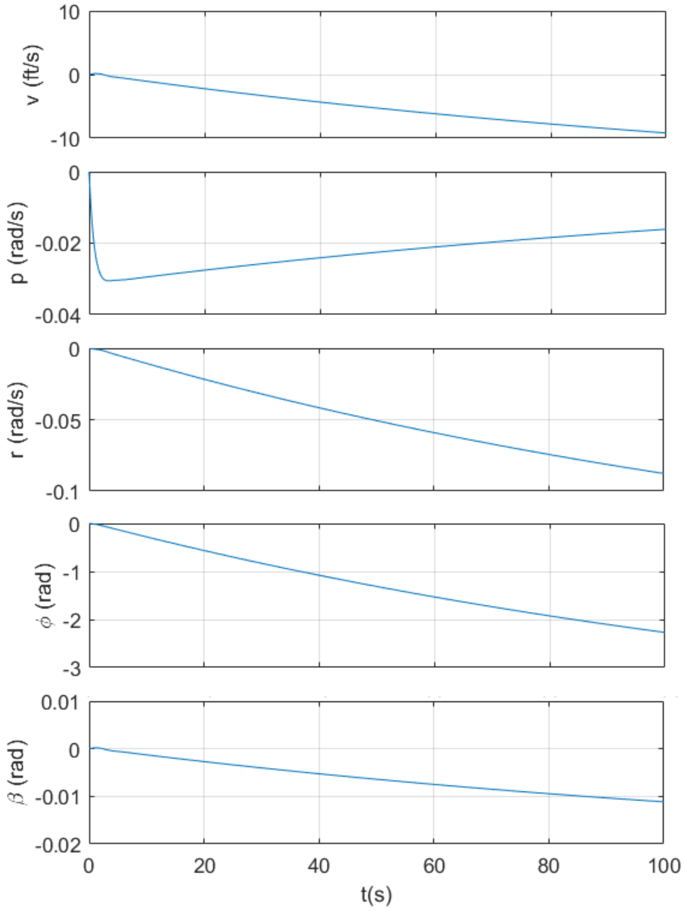

Figure 5: Response of the DC-8 in closed-loop to a deflection of 1° of the rudder in a flight condition of cruise

$$[K] = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0.02 & -0.82 & 0.03 \end{bmatrix} \tag{47}$$

The rudder deflection is used for feedback with the roll and yaw angular velocities and the roll angle. The new parameters turn out to be:

Table 3: Parameters of the dynamic modes of the DC-8 in closed-loop and adequacy with the normative in a flight condition of cruise

$\tau_r(s)$	$\tau_s(s)$	ζ_d	$w_d(s^{-1})$	$\zeta_d w_d(s^{-1})$
0.8029	149.25	0.4066	1.4861	0.6042

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5.2 Maximum horizontal speed V_{NE}

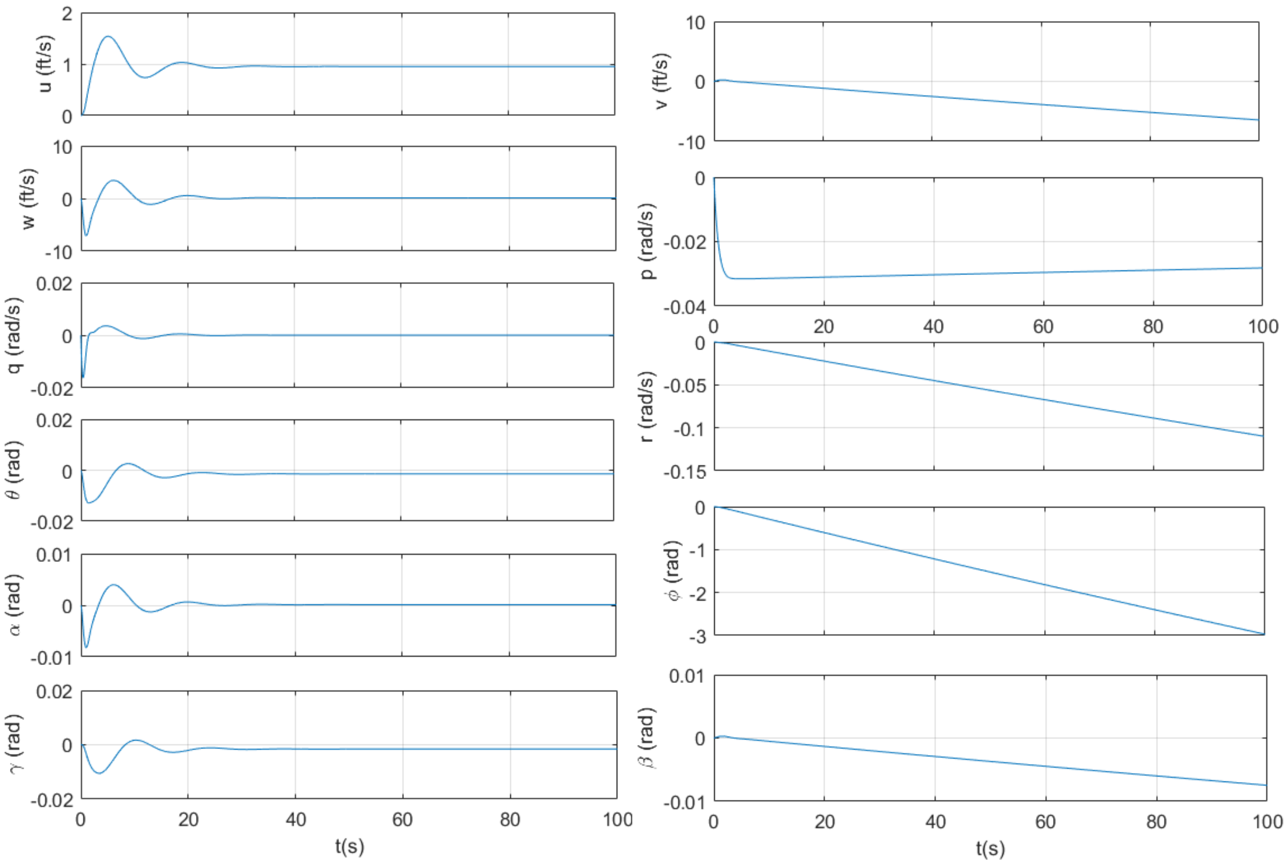

Figure 6: Response of the DC-8 in closed-loop to a deflection of 1° of the elevator and the rudder in a flight condition of V_{NE}

$$[K] = \begin{bmatrix} 0.02 & 0 & -0.3 & 0.57 \end{bmatrix} \tag{48}$$

The elevator deflection is used for feedback with the longitudinal speed Δu, the pitch angle $\Delta\theta$ and the pitch angular velocity Δq.

$$[K] = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0.02 & -1.02 & 0.04 \end{bmatrix} \tag{49}$$

All the lateral-directional variables are used for feedback except the sideslip angle, as well as in the cruise condition.

Table 4: Parameters of the dynamic modes of the DC-8 in closed-loop and adequacy with the normative in a flight condition of V_{NE}

ζ_p	$w_p(s^{-1})$	ζ_{sp}	$w_{sp}(s^{-1})$	$\tau_r(s)$	$\tau_s(s)$	ζ_d	$w_d(s^{-1})$	$\zeta_d w_d(s^{-1})$
0.3049	0.4782	0.5687	3.1708	0.7398	833.33	0.5024	1.5863	0.797

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6. Conclusions

Nowadays, the use of control systems in flight mechanics is mandatory as it solves the stability problems the aircraft face without having to redesign the whole aerodynamic configuration. Commonly, the chosen control systems are stability augmentation systems, which modify the transient response of the airplane to a perturbation in their steady state, either external or produced by the control surfaces. These systems are mainly based on the negative feedback of the motion variables, and the deflections of the control surfaces are proportional to them.

The most critical modes are the short-period and the Dutch roll. Both of them are oscillatory, have a high frequency and perturb the rotation degrees of freedom, making them hazardous. Therefore, they have strict requirements to meet regarding damping and frequency. At high speeds, they are likely to have an insufficient damping, specially the Dutch roll, which can be mitigated with the use of a yaw damper.

The Pole-Placement Method is a powerful tool to estimate the required gains to establish the values of the damping and frequencies of the dynamic modes, providing there are enough motion variables available to measure and use it for feedback. The typical longitudinal variables used for feedback are pitch angle and pitch angular velocity, although horizontal velocity Δu should be used to stabilize the phugoid mode at very high speed. The most used lateral-directional variables used for feedback are the roll and yaw angular velocities, since they increase the damping of the Dutch roll, usually the main problem.

7. References and bibliography

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