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Investigation of uncertain parameters for characterizing welded structures using Vibration-Based Methods

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Abstract

Of relevance in many industrial applications, Vibration-based methods have been proved to be an effective non-destructive alternative for characterizing welded structures. Recent numerical investigations modeled the bond as a lumped mass connecting the (welded) portions of the structural element by linear springs, and found good agreement with experiments, providing a quantitative estimation of the welding rigidity. Here, we investigate the sensitivity of the proposed model to uncertain parameters, whose determination is not possible in any practical case, confirming its robustness and reliability for assessing the integrity of welded structures.

1. Introduction

Over the past decades, the application of Vibration-based methods (VBM) has demonstrated to be a powerful alternative for characterizing welded structures, of interest in many engineering and industrial applications, particularly, for permanent monitoring of the welding condition. VBM present the advantage of providing quantitative information of the welding, compared to other extended Non-Destructive Testing (NDT) techniques, like ultrasound scanning or radio-logical screening.[1,2]

Recent numerical investigations[3] modeled the bond as a lumped mass connecting the portions of the structural element by linear springs, which oppose to shear force and bending moment, describing the vibrational response under the effect of mechanical and inertia properties of the welding. Even more recent experiments[4] have examined its applicability to characterize simple welded structures, providing rigidities estimations that were compared to other NDT approaches. However, part of the key parameters used for determining the welding characteristics, like the rotational inertia resulting from the added mass during the welding process, were unknown to some extent. Although good agreement was found between numerical predictions and experiments, differences of about 10 % in the natural frequencies were often obtained. These discrepancies are attributed, at least in part, to the uncertainty of different welding parameters and the associated simplifications used for their estimation.

Throughout this work, we analyze the sensitivity of the afore-mentioned model to such uncertain parameters by applying inverse engineering techniques, and compare these results with general optimization procedures. In the former case, this is understood as the process by which a designed structure has a desired dynamic response under external loads or dynamic properties like natural frequencies, normal modes and damping coefficients. This process is commonly defined in the literature as an inverse eigenvalue problem.

Some of the early researches in this area were presented by Vanhonacker (1980),[5] Chen and Garba (1980)[6] and Belle (1982).[7] Other subsequent investigations, like the works of He (1997)[8] and Mottershead (1998),[9] used structural modifications, like adding point masses or stiffness modifications, to achieve the targeted natural frequencies. All these approaches were applied to discrete systems made up of simple linear spring and mass elements. The inverse vibration problem to continuous finite elements, on the other hand, was studied by Djoud and Bahai (2000, 2002),[10,11] and Bahai and Aryana (2002),[12] and later improved by using second order Taylor approximation in the inverse formulation.[13] This method modified the mass and stiffness matrices at a local level and showed a high computational efficiency.

Design sensitivity analysis of structures, in this sense, deals with the calculation of the response derivatives to the design variables. These derivatives, called the sensitivity coefficients, can be used in the solution of different problems and, in particular, to assess the effects of uncertainties in the system response. Developments in methods for sensitivity

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analysis were discussed, for example, by Haug et al. (1986)[14] and Haftka and Gurdal (1993).[15] Later, it was found that accurate results in terms of dynamic properties of the system can be achieved by the calculation of the analytical derivatives of approximate analysis models.[16]

In this context, the present study can be framed within the sensitivity analysis of discrete linear systems under dynamic loading to some physical or geometrical parameters. While some methods make use of finite differences with respect to the design parameters to determine the gradient of mode shapes and natural frequencies, see e.g. Adelman and Haftka (1986),[17] using exact derivatives reduces the round-off and truncation errors related to the step size used, and improves the method performance.

Here, the method based on the direct computation of the first derivative with respect to the design parameters is used. Parameters considered for the welding problem are the mass and moment of inertia of the added material during the welding process. In this sense, we propose a systematic investigation on how the afore-mentioned parameters, whose experimental determination in any practical case is difficult or even not possible, affect the vibrational response.

Arising from the impossibility of adjusting all frequencies, a Finite Element Method (FEM) model including the welded section is used. Based on measured data from different welded steel platens, which were welded with different conditions, the welding section rigidities are adjusted to reproduce experimental results.[4] Then, the sensitivity of the calculated natural frequencies to the inertia parameters is analyzed, providing a quantification of their influence on the welding characterization. This work confirms the applicability and reliability of the method for assessing the integrity of welded structures, and further demonstrates somewhat the robustness of the welding quality determination.

The paper is structured as follows. In Sec. 2, the mathematical formulation used for modelling the welding and the sensitivity analysis are reviewed. Then, in Sec. 3, the experimental setup and modal analysis procedure, and the optimization method applied for determining the welding rigidities are described. Results are presented and discussed in Sec. 4. Finally, conclusions are offered in Sec. 5.

2. Mathematical formulation

2.1 Welded beam model and Finite Element Method implementation

An Euler-Bernoulli beam of length L welded at $x = l_1$ is considered (see Fig. 1). The welding is represented by an added mass, of translational and rotational inertia m_w and I_w, that is attached to both halves of the beam by a set of massless springs, two translational and two torsional. These springs oppose to vertical displacement and rotation between both sides of the beam, and are assumed to have equal rigidities of values k and k_T at both sides of the added mass, respectively.[3,4]

Vibrational dynamics are given by[18]

$$EI\frac{\partial^4 w_1}{\partial x^4} + m\frac{\partial^2 w_1}{\partial t^2} = 0, \quad 0 \le x \le l_1^-; \tag{1a}$$

$$EI\frac{\partial^4 w_2}{\partial x^4} + m\frac{\partial^2 w_2}{\partial t^2} = 0, \quad l_1^+ \le x \le L; \tag{1b}$$

$$m_w\frac{\partial^2 z_w}{\partial t^2} + k\,(2\,z_w - w_1^- - w_2^+) = 0, \tag{1c}$$

$$I_w\frac{\partial^2 \theta_w}{\partial t^2} + k_T\left(2\,\theta_w - \frac{\partial w_1^-}{\partial x} - \frac{\partial w_2^+}{\partial x}\right) = 0, \tag{1d}$$

while the following equations apply at $x = l_1$

$$EI\,\frac{\partial^2 w_1^-}{\partial x^2} - k_T\left(\theta_w - \frac{\partial w_1^-}{\partial x}\right) = 0, \quad x = l_1^-; \tag{2a}$$

$$EI\frac{\partial^3 w_1^-}{\partial x^3} + k\,(z_w - w_1^-) = 0, \quad x = l_1^-; \tag{2b}$$

$$EI\frac{\partial^2 w_2^+}{\partial x^2} - k_T\left(\theta_w - \frac{\partial w_2^+}{\partial x}\right) = 0, \quad x = l_1^+; \tag{2c}$$

$$EI\frac{\partial^3 w_2^+}{\partial x^3} + k\,(z_w - w_2^+) = 0, \quad x = l_1^+. \tag{2d}$$

Here, z_w and θ_w are the generalized coordinates for the translation and rotation of the lumped inertia, w_1 and w_2 are the beam deformations on $0 \le x \le l_1^-$ and $l_1^+ \le x \le L$, and EI and m are the beam flexural stiffness and mass per unit length, respectively. Superscripts '$\mp$' refer to the left and right-sided limits as $x \to l_1$.

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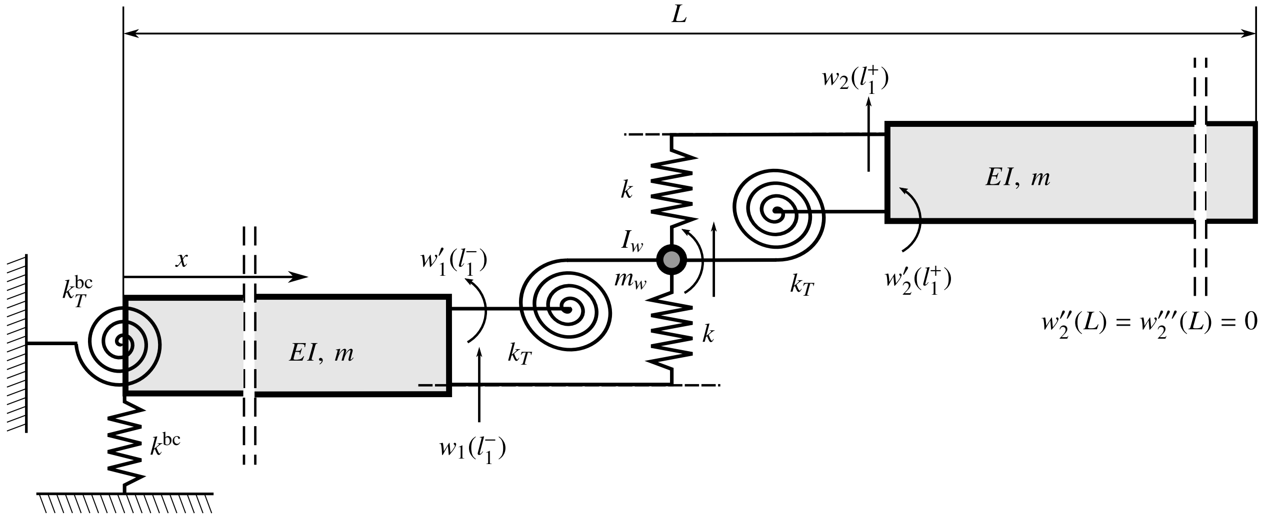

Figure 1: Welded beam model for the experimental setup detailed in Sec. 3.1. The welding is represented by linear springs and a lumped inertia to mimic the rigidity and the added mass of the welding connection. The clamp boundary condition is modeled by two linear springs opposing to translation and rotation of the fixed end. Here, $\langle\circ\rangle'$ stands for $\partial\langle\circ\rangle/\partial x$.

For the experimental setup described in Sec. 3, the attachment at $x = 0$ is modelled by two massless springs (see Figs. 1 and 2) that oppose to the beam displacement and rotation

$$EI\frac{\partial^2 w_1}{\partial x^2} + k_T^{\rm bc}\frac{\partial w_1}{\partial x} = 0, \quad x = 0; \tag{3a}$$

$$EI\frac{\partial^3 w_1}{\partial x^3} - k^{\rm bc}\, w_1 = 0, \quad x = 0; \tag{3b}$$

while zero-stress boundary condition is assumed at the free end $x = L$

$$\frac{\partial^2 w_2}{\partial x^2} = \frac{\partial^3 w_2}{\partial x^3} = 0, \quad x = L. \tag{4}$$

The dynamic response enclosed in Eqs. (1–4) is solved by means the Finite Element Method. We consider two classical FEM objects for the beam portions: the beam bending element for the stiffness matrix, and the lumped mass for the mass matrix.[19] The bond element, on the other hand, is represented by the following mass and stiffness matrices

$$\mathbf{M}_w = \begin{pmatrix} 0 & 0 & \dots & & & \\ & 0 & 0 & \dots & & \\ & & m_w & 0 & \dots & \\ & & & I_w & 0 & \dots \\ & \text{sym} & & & 0 & 0 \\ & & & & & 0 \end{pmatrix}, \quad \mathbf{K}_w = \begin{pmatrix} k & 0 & -k & 0 & \dots & \\ & k_T & 0 & -k_T & 0 & \dots \\ & & 2\,k & 0 & -k & 0 \\ & & & 2\,k_T & 0 & -k_T \\ & \text{sym} & & & k & 0 \\ & & & & & k_T \end{pmatrix}, \tag{5}$$

respectively. These matrices are assembled with FEM regular elements to obtain the global stiffness and mass matrices, denoted by $\mathbf{K}$ and $\mathbf{M}$.

Assuming harmonic motion $\propto e^{i\omega t}$, the characteristic equation for negligible damping is

$$\det\left(\mathbf{K} - \omega^2\mathbf{M}\right) = 0, \tag{6}$$

whose roots are the natural frequencies ω of the structure, depending on the following dimensionless parameters

$$\delta = \frac{k\,L^3}{EI}, \quad \delta_T = \frac{k_T\,L}{EI}, \quad \delta^{\rm bc} = \frac{k^{\rm bc}L^3}{EI}, \quad \delta_T^{\rm bc} = \frac{k_T^{\rm bc}L}{EI}, \quad \lambda = \frac{L}{l_1} - 1, \quad s = \frac{m_w}{mL}, \quad r_\theta = \frac{I_w}{mL^3}. \tag{7}$$

Remark that structural damping is neglected because, for typical metallic materials as the steel used in experiments (see Sec. 3.1), this value is between 1–2 % at most. Therefore, results will be barely affected at leading order.

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Previous analysis of Salgado Sánchez et al. (2016)[3] found that 10^2 elements were enough to reproduce the analytical solution of Eqs. (1–4) with less than a 0.2 % error. Therefore, 10^2 elements will be used for the results discussed throughout this paper.

In order to calculate welding structural properties, we use an inverse procedure for the solution of Eq. (6). From experiments, natural frequencies and the welding location λ are measured, while rough estimations are used for the inertia properties s and, in particular, r_θ. Following the work of Rafael de la Cruz et al. (2019),[4] we calculate them based on the measured added mass value and assuming an oval spatial distribution. Given this set of parameters, the values of $\delta = kL^3/(EI)$ and $\delta_T = k_T L/(EI)$ can be obtained afterwards by optimization, searching for the minimum error between calculated and measured natural frequencies. These obtained rigidities, therefore, will have certain degree of uncertainty, provided the unknown precise values of the welding inertia. We investigate here the system sensitivity to this inertia, as detailed below.

For further details of the FEM implementation, refer to the works of Salgado Sánchez et al. (2016)[3] and Rafael de la Cruz et al. (2019).[4]

2.2 Inverse eigenvalue problem formulation

The mass and rigidity matrices depend on various design parameters, an inverse engineering design deals with which values are the most adequate so that the designed physical system displays the specified or desired dynamic properties (natural frequencies and/or modal shapes).

Let us consider the (reduced) eigenvalue problem

$$\mathbf{K}\{\phi_i\} = \lambda_i\,\mathbf{M}\{\phi_i\}, \tag{8}$$

where the eigenvalues λ_i are the squared natural frequencies, and the eigenvectors $\{\phi_i\}$ are the normal modes, satisfying

$$\{\phi_i\}^T\,\mathbf{M}\{\phi_i\} = 1, \tag{9}$$

after their normalization.

If the solid rigid motion is not considered, rigidity and mass matrices are positive defined and symmetric. Furthermore, if they are smooth and with continuous derivative with respect to the design parameters, then the associated natural frequency and normal mode also have continuous derivative with respect to the design variables. Premultiplying equation (8) by $\{\phi_i\}^T$, we obtain

$$\{\phi_i\}^T\,\mathbf{K}\{\phi_i\} = \lambda_i\,\{\phi_i\}^T\,\mathbf{M}\{\phi_i\}. \tag{10}$$

Let $\mathbf{K}$ and $\mathbf{M}$, and consequently their eigenvalues and eigenvectors, depend on a certain design (physical or geometrical) parameter b_k. A small perturbation with respect to its nominal value $\widehat{b_k}$ can be expressed as

$$b_k = \widehat{b_k} + \epsilon\,\Delta b_k, \tag{11}$$

where ϵ is the perturbation parameter and Δb_k the finite (small) increment, so that the system response can be linearized. After combining Eqs. (11) and (10), the following leading order equation is derived

$$\frac{\partial \lambda_i}{\partial b_k} = \{\phi_i\}^T\left(\frac{\partial \mathbf{K}}{\partial b_k}\right)\{\phi_i\} - \lambda_i\,\{\phi_i\}^T\left(\frac{\partial \mathbf{M}}{\partial b_k}\right)\{\phi_i\}. \tag{12}$$

We call $\mathbf{S}_{ik} = \partial\lambda_i/\partial b_k$ to the resulting non-squared matrix, whose columns and rows are given by the number of natural frequencies to be optimized or placed at the desired values, and by the (number of) selected parameters of the design process, respectively. For the present work, three measured frequencies are used, while the design parameters considered are the inertia properties of the added mass m_w and I_w. Therefore, $\mathbf{S}_{ik}$ is a 2×3 matrix.

Let us define the difference between the ith measured and calculated eigenvalue as

$$\Delta\lambda_i = (\omega_{\exp,i})^2 - (\omega_i)^2, \tag{13}$$

where $\omega_{\exp,i}$ is the measured natural frequency and ω_i is the calculated natural frequency with the estimated values of the design parameters in a first guess. Then, the sensitivity of the system to design variables is obtained as

$$\Delta b_k = \mathbf{S}_{ik}^{-1}\,\Delta\lambda_i, \tag{14}$$

where the largest value of $\partial\lambda_i/\partial b_k$ will show the most sensitive element for the ith eigenvalue.

If it is assumed that the first time calculated eigenvalues and eigenvectors of the system are obtained from an initial FEM model, thus obtained by an initial guess of the unknown design parameters, the required change in the

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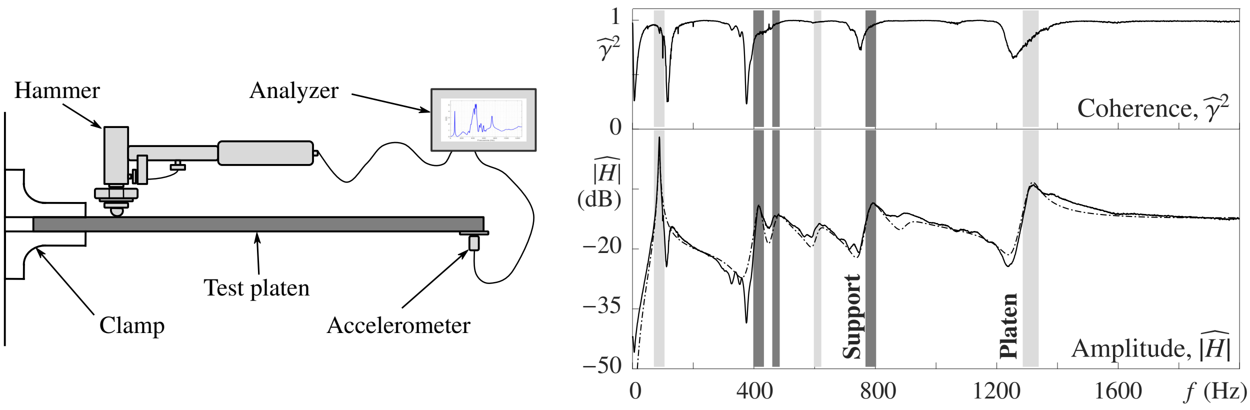

(a) - Experimental setup

(b) - Frequency Response Function (FRF)

Figure 2: (a) Sketch of the experimental setup for modal analysis. (b) Frequency Response Function (FRF) including the coherence $\widehat{\gamma}^2$ and amplitude $|\widehat{H}|$ functions. Platen and support resonances are highlighted by light and dark gray bands, respectively.

Table 1: Platen characteristics as function of the welding parameters: intensity (A) and speed, providing different weld qualities: good, medium and poor.

#	Intensity (A)	Speed	Quality
1	75	Slow	Good
2	75	Fast	Medium
3	50	Fast	Poor
4	75	—	Discontinuous

welding properties to achieve the structure measured frequencies can be obtained (see Djoud and Bahai (2002)[11]). This procedure allows to optimize welded structures that require small modifications (less than 10 %) in the computed and measured natural frequencies.

We note that sensitivity of eigenvectors with respect to welding parameters is not considered here, consistently with real applications where mode shapes are usually left out of the analysis. This, however, can be generalized by applying the method presented in Lee et al. (1999).[20]

3. Experiments

3.1 Experiment setup and modal analysis

The heart of the experiment is a test platen attached to a real wall in a cantilever configuration, as sketched in Fig. 2(a). The platen is vibrated by hitting it with a hammer, and the dynamic response is measured by an accelerometer. Both driving and recording devices datasets are processed by an analyzer that registers and treats their signals.

Six platens, made of steel[1] are tested. Each platen has an effective length of 200 mm and is 30 mm wide and 6 mm thick, displaying a cross section moment of inertia $I = 5.4 \cdot 10^{-10}$ m^4 and a mass per unit length $m = 1.476$ kg m^{-1}. Platens #1–4 are selected to perform a single welding at (nearly-)identical positions; while two platens are used for reference, with no welding performed.

Different welding qualities are applied to each welded platen. Weldings are executed manually by electric arc, permitting for controlling two main parameters: intensity and soldering velocity. Two intensities are regulated: 75 and 50 A. Based on the electrode characteristics (size and type) and the thickness of the platen, we consider an intensity of 75 A as suitable,[2] while 50 A is found to be less adequate. Welding speed is varied from low to fast, providing accurate welded elements and less precise solderings, respectively. For clarity, welding details in the different platens are summarized in Table 1, including a descriptive evaluation.

[1]Measured mean density $\rho = 8200$ kg m^{-3} and Young's Modulus $E = 231$ GPa.

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The test structural components are screwed to the clamping support and hit uniformly in 6 predefined locations. All measured data is processed with the digital analyzer, which is configured with a sampling frequency of 6000 Hz and a total register time of 1.37 s, complying with the standard procedure of modal analysis.[21] This yields the frequency response of the structure and measurements of the experimental data quality i.e., coherence. We illustrate in Fig. 2(b) the typical Frequency Response Function (FRF) of an experiment. This response features both the three first platen resonances of interest, and clamping modes.

For the experiments, geometry parameters are $L = 200$ mm and $\lambda = 0.428$, while inertia parameters, estimated as commented above, are fixed first to $m_w = 0.002$ kg and $I_w = 5 \cdot 10^{-6}$ kg m^2. Starting from this initial guess, welding rigidities can be calculated as explained hereafter.

3.2 Procedure to determine the welding stiffnesses

We start measuring resonances on reference and welded platens, referred to $\mathbf{f}_{\text{ref}}$ and $\mathbf{f}_{\text{exp}}$, with the associated geometrical λ, and inertia parameters s, r_θ of corresponding experiments.

First, reference platens are used to characterize the clamping support. We proceed iteratively in order to match numerical predictions $\mathbf{f}$ and experimental $\mathbf{f}_{\text{ref}}$ results by means of an *active-set* optimization technique, based on the gradient calculation. This yields the equivalent rigidities at the support δ^{bc}, δ_T^{bc}, which are assumed constant between experiments. Following an analogous approach, welding stiffnesses δ, δ_T are calculated to best reproduce the experimental resonances $\mathbf{f}_{\text{exp}}$.

Let us define the relative errors ε_i between the ith frequency calculated $\mathbf{f}_i$ and measured $\mathbf{f}_{\text{exp},i}$ as

$$\varepsilon_i = \frac{\left|\mathbf{f}_i - \mathbf{f}_{\text{exp},i}\right|}{\left|\mathbf{f}_{\text{exp},i}\right|}, \tag{15}$$

to construct the following objective function $\mathcal{F}$ to be minimized

$$\mathcal{F} = \varepsilon_1^2\left(1 + \frac{10^{-2}}{|\varepsilon_2\varepsilon_3| + C}\right) + \varepsilon_2^2\left(1 + \frac{10^{-2}}{|\varepsilon_1\varepsilon_3| + C}\right) + \varepsilon_3^2\left(1 + \frac{10^{-2}}{|\varepsilon_1\varepsilon_2| + C}\right), \tag{16}$$

where $C = 10^{-10}$ is a small constant to avoid the potential division by zero. At convergence, the relative error at the nth iteration $\varepsilon_{(n)}$ is below

$$\varepsilon_{(n)} = \frac{\left|\mathcal{F}_{(n)} - \mathcal{F}_{(n-1)}\right|}{\left|\mathcal{F}_{(n-1)}\right|} < 10^{-6}, \tag{17}$$

and stiffnesses are determined. We note that the application for reference platens results from the straightforward change (exp) $\rightarrow$ (ref).

It is generally well-known the sensitivity of optimization techniques to the selected objective function, in particular, for multi-objective optimizations. The aim of this work is to reproduce the vibrational response of the test welded beams at their first three natural frequencies. We select Eq. (16) to obtain similar errors in all natural frequencies. Note that Eq. (16) has local theoretical minima along $\varepsilon_i = \varepsilon_j$, while it is penalized otherwise.

4. Results and discussion

4.1 Clamping support characterization

Based on the formulation of Sec. 2, two equivalent translational and torsional springs (see Fig. 1) model the clamping support, whose rigidities can be calculated as explained Sec. 3.2. The following dimensionless clamping stiffnesses

$$\delta^{bc} = 1089.1, \quad \delta_T^{bc} = 6.261 \tag{18}$$

are obtained. Table 2 summarizes the results for the clamped support characterization. As anticipated above, the numerical error in the calculated natural frequencies is $\varepsilon = 1.68$ % and almost identical in all of them.

The non-dimensional clamping stiffness have not a very high value. For an ideal cantilever beam, these values tend to infinity, far from the obtained ones, in the order of 10^3 for translation and of 10^1 for rotation. This fact, however, does not affect the objective of the present work, since the characterization of the clamping will be carried on throughout all the results presented. These rigidities, therefore, are fixed from now on.

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Table 2: Characterization of the non-ideal support: measured natural frequencies $\mathbf{f}_{\mathrm{ref}}$, numerical frequencies $\mathbf{f}$, relative errors (in absolute value) $|\varepsilon|$ and non-dimensional rigidities δ^{bc}, δ_T^{bc}.

$\mathbf{f}_{\mathrm{ref}}$ (Hz)	$\mathbf{f}$ (Hz)	$\lvert\varepsilon\rvert$ (%)	δ^{bc}	δ_T^{bc}
$\begin{pmatrix} 95.9 \\ 635.0 \\ 1709.0 \end{pmatrix}$	$\begin{pmatrix} 94.3 \\ 624.4 \\ 1737.6 \end{pmatrix}$	$\begin{pmatrix} 1.68 \\ 1.68 \\ 1.67 \end{pmatrix}$	1089.1	6.261

	(a) – **Welding rigidities characterization**					(b) – **Inertia parameters adjustment**			
Platen #	$\mathbf{f}_{\mathrm{exp}}$ (Hz)	δ (–)	δ_T (–)	$\mathbf{f}$ (Hz)	$\lvert\varepsilon\rvert$ (%)	$\widehat{\mathbf{f}} + \Delta\mathbf{f}$ (Hz)	$\widehat{m_w} + \Delta m_w$ (g)	$\widehat{I_w} + \Delta I_w$ (kg m^2)	$\lvert\varepsilon + \Delta\varepsilon\rvert$ (%)
1	$\begin{pmatrix} 96.2 \\ 603.5 \\ 1423.5 \end{pmatrix}$	$1.413 \cdot 10^5$	5.896	$\begin{pmatrix} 99.6 \\ 581.5 \\ 1473.0 \end{pmatrix}$	$\begin{pmatrix} 3.55 \\ 3.65 \\ 3.47 \end{pmatrix}$	$\begin{pmatrix} 99.4 \\ 584.5 \\ 1465.0 \end{pmatrix}$	3.4	$1.01 \cdot 10^{-6}$	$\begin{pmatrix} 3.29 \\ 3.15 \\ 2.92 \end{pmatrix}$
2	$\begin{pmatrix} 94.8 \\ 586.2 \\ 1331.3 \end{pmatrix}$	$5.346 \cdot 10^4$	4.170	$\begin{pmatrix} 99.3 \\ 554.6 \\ 1403.6 \end{pmatrix}$	$\begin{pmatrix} 4.80 \\ 5.42 \\ 5.43 \end{pmatrix}$	$\begin{pmatrix} 99.2 \\ 557.0 \\ 1397.3 \end{pmatrix}$	3.0	$1.00 \cdot 10^{-6}$	$\begin{pmatrix} 4.61 \\ 4.99 \\ 4.96 \end{pmatrix}$
3	$\begin{pmatrix} 92.3 \\ 562.8 \\ 1258.9 \end{pmatrix}$	$1.251 \cdot 10^5$	3.055	$\begin{pmatrix} 99.0 \\ 525.1 \\ 1344.1 \end{pmatrix}$	$\begin{pmatrix} 7.25 \\ 6.69 \\ 6.79 \end{pmatrix}$	$\begin{pmatrix} 98.7 \\ 527.4 \\ 1336.0 \end{pmatrix}$	3.5	$1.02 \cdot 10^{-6}$	$\begin{pmatrix} 6.97 \\ 6.29 \\ 6.10 \end{pmatrix}$
4	$\begin{pmatrix} 96.4 \\ 602.3 \\ 1333.6 \end{pmatrix}$	$6.467 \cdot 10^4$	4.604	$\begin{pmatrix} 99.4 \\ 562.9 \\ 1423.0 \end{pmatrix}$	$\begin{pmatrix} 3.15 \\ 6.53 \\ 6.72 \end{pmatrix}$	$\begin{pmatrix} 99.7 \\ 566.6 \\ 1427.5 \end{pmatrix}$	1.3	$1.0 \cdot 10^{-6}$	$\begin{pmatrix} 6.97 \\ 6.29 \\ 6.10 \end{pmatrix}$

Table 3: Results obtained by the optimization of Eq. (16). (a) Welding rigidities characterization: measured natural frequencies $\mathbf{f}_{\mathrm{exp}}$, design rigidities δ, δ_T, design numerical frequencies $\mathbf{f}$ and associated errors ε (in absolute value). (b) Inertia parameters adjustment: adjusted frequencies $\mathbf{f} + \Delta\mathbf{f}$, adjusted inertia parameters $\widehat{m_w} + \Delta m_w$, $\widehat{I_w} + \Delta I_w$, and errors $\varepsilon + \Delta\varepsilon$ (in absolute value) after the adjustment.

4.2 Welding rigidities characterization

As a first step, the measured natural frequencies of welded beams are used to determine the values of the non-dimensional rigidities δ and δ_T by means of the procedure detailed above. We remark that the initial guesses given in Sec. 3 of the welding mass $m_w = 0.002$ kg and moment of inertia $I_w = 5 \cdot 10^{-6}$ kg m^{-2} are used.

Note that these parameters are estimated, provided the controlled environment of the laboratory, by weighting the platens in the case of the added mass, and assuming that it takes an oval shape distribution in the added moment of inertia. For any real application, however, measuring such parameters will be certainly difficult or even not possible and thus, they have to be roughly estimated. Therefore, the main objective of this paper is to ascertain how errors in this approximation may influence the assessment of the welding properties.

Table 3(a) compiles the obtained results for the four platens, using the first guess of the parameters. Both measured $\mathbf{f}_{\mathrm{exp}}$ and computed $\mathbf{f}$ natural frequencies, their relative error $|\varepsilon|$ (in absolute value) and the calculated non-dimensional rigidities δ and δ_T that minimize $\mathcal{F}$ are detailed.

From these results, it can be noticed that the all frequencies are similarly adjusted, providing relative errors ranging between 3 and 7 % for different platens. In particular, platen #3 presents the higher errors of 7.25 % at the first natural frequency. Although these results are somewhat affected by the guessed inertia properties, this initial approach provides good accuracy with differences below 10 %, acceptable for any typical combination of experimental and numerical work of this nature.

The obtained rigidities, on the other hand, capture fairly well the degradation of the welded section. In particular, for platens #1 and #3, decreasing rigidities are obtained as expected. For platen #4, the discontinuous welding

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procedure affects somewhat more the translational rigidity, while the associated torsional one displays a higher value compared to the incorrectly welded platen #3. Results from platen #2, on the other hand, suggest that the given bond was not welded with a medium quality as expected, providing the major reduction in δ.

One may compare these results to those that would be obtained if the FEM model is fitted to reproduce the resonances of reference platens; a reduction of two orders of magnitude in both δ and δ_T once welded is found. We remark that rigidity values in this ideal scenario are in the order of $\delta_0 = 10^7$ and of $\delta_{T0} = 500$ for translational and torsional stiffnesses, respectively. Overall, the method is able to assess the degradation of the different structural elements, captured in the change of δ and δ_T caused by the welding process.

Finally, compared to the recent work of Rafael de la Cruz (2019),[4] where an analogous system was adjusted by searching the minimum of $|\mathbf{f} - \mathbf{f}_{\rm exp}|$ with genetic algorithms, the same order of magnitude (but different in values) is obtained here in the rigidities, which suggest the reliability of the procedure. This supports, however, the aforementioned sensitivity of the optimization process to the selected objective function.

4.3 Natural frequencies sensitivity to inertia parameters

Now, the objective is to verify the sensitivity of the results to the guessed parameters used in the model: the added mass due to soldering and its moment of inertia. In a first evaluation, the system is linearized in the neighborhood of the previous calculated design point, in particular, for platen #1.

The modified sensitivity matrix $\widehat{\mathbf{S}}$, similar to the one presented in Sec. 2,

$$\widehat{\mathbf{S}} = \begin{bmatrix} \frac{\partial f_1}{\partial m_w} & \frac{\partial f_2}{\partial m_w} & \frac{\partial f_3}{\partial m_w} \\ \frac{\partial f_1}{\partial I_w} & \frac{\partial f_2}{\partial I_w} & \frac{\partial f_3}{\partial I_w} \end{bmatrix} = \begin{bmatrix} -0.23\,[\mathrm{g}^{-1}] & -0.52\,[\mathrm{g}^{-1}] & -0.62\,[\mathrm{g}^{-1}] \\ -0.24\cdot 10^5\,[\mathrm{kg}^{-1}\,\mathrm{m}^{-2}] & -10.01\cdot 10^5\,[\mathrm{kg}^{-1}\,\mathrm{m}^{-2}] & -1.9\cdot 10^5\,[\mathrm{kg}^{-1}\,\mathrm{m}^{-2}] \end{bmatrix} \mathrm{Hz} \tag{19}$$

is obtained and evaluated at the design point, permitting the straightforward identification of the most sensitive modes. In particular, the largest values of each row would be the most adequate frequency to be adjusted by the given parameter, resulting in the smallest modification.

We adjust, for example, the third natural frequency by modifying m_w to obtain

$$\Delta m_w = \frac{f_{3,\rm exp} - f_3}{\partial f_3/\partial m_w} \approx 10\,\mathrm{g}. \tag{20}$$

Setting-up this value in the model, the calculated set of natural frequencies is $\mathbf{f} = (96.62, 523.41, 1422.5)$ Hz, providing an error in the third frequency of 0.007 %. This further improves the prediction of the first frequency reducing its error to 0.4 %, while substantially degrading the second one.

This suggest that the optimization of all three frequencies together cannot be easily done by hand, since the second frequency, in contrast to the other two, is always underestimated by the model. For this purpose, and provided $\widehat{\mathbf{S}}$ that reveals the system sensitivity, we modify the optimization method used in previous sections to adjust m_w and I_w as described hereafter.

4.4 Determination of inertia parameters

The objective now is to reduce as much as possible the value of $\mathcal{F}$ and, therefore, the error of numerical predictions. Remark that the rigidities of the welding (so-called design rigidities), δ and δ_T are kept constant and equal to the values computed in Table 3(a).

Table 3(b) summarizes the obtained results for the four platens: updated natural frequencies $\widehat{\mathbf{f}} + \Delta\mathbf{f}$, their relative errors $|\varepsilon + \Delta\varepsilon|$ (in absolute value) and adjusted inertia parameters $\widehat{m_w} + \Delta m_w$ and $\widehat{I_w} + \Delta I_w$ that minimize $\mathcal{F}$. It can be noticed that the error can just be reduced slightly, meaning that wrong evaluations of these parameters do not practically affect the model resolution.

Furthermore, for the particular case of the inertia mass m_w, the obtained values are consistent with the welding quality. In platens #1–3, the operator executed the welding differently but the amount of additional agent was similar. For platen #4, in contrast, the welding was executed by adding three discrete bond points, reducing the quantity of additional mass. This effect is consistently captured by the present optimization.

From these results, it can be concluded that the proposed model is a reliable and robust method to evaluate the quality of a welding. Errors of the order of 75 % in the added welding mass and of one order of magnitude in the moment of inertia barely affect the differences between the measured and calculated frequencies, once suitable values of the welding rigidities are determined. The model, however, cannot reduce the differences to even smaller values than 2.9 %, fact that, despite of being admissible for most of engineering process, suggests itself for further investigation.

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5. Conclusions

The sensitivity of a vibration-based method to evaluate mechanical properties on welded structures and assess their integrity is examined.

The model is successfully used for characterizing welded cantilever platens, despite departing from rough estimations of the uncertain parameters of the welding, particularly, the added mass and the moment of inertia. The different reduction of stiffness at the welded section, caused deliberately by the welding execution, is determined by the use of a new objective function that searches the global error minimization by reducing equally partial errors in all the natural frequencies considered in the analysis. Despite the novel approach, obtained rigidities are consistent with previous analyses, suggesting the robustness of the model besides the underlying process used to optimize it.

System sensitivity to welding inertia is analyzed subsequently in the neighborhood of the design points, adjusting their values to improve the model accuracy. The obtained modifications are in the order of 75 % in the added welding mass and of one order of magnitude in the rotational inertia, while the achieved reduction in the error is just 0.5 % at most. Furthermore, the obtained masses are able to predict consistently the added agent quantity during the welding.

From these results, it can be concluded that the proposed stiffness and lumped mass model, combined with its optimization, is a reliable and robust approach to evaluate the quality of a welding. It is shown how rough estimations of uncertain inertia parameters barely affect the differences between measured and predicted resonances and thus, the calculated welding stiffnesses. The model, however, is not capable to reproduce experiments with less than a 2.9 % error. This fact, despite of being admissible for most of engineering and industrial processes, suggests itself for further investigation.

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