Analytical models for Li-ion batteries developed at the IDR/UPM Institute

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Abstract

The present document summarizes the work carried out by the IDR/UPM Institute in relation to the development of Analytical models of Li-ion batteries. The proposed models are based on the energy discharge level (in relation to full capacity). This represents a new approach in relation to battery assessment with applications at the design phases of the power subsystem of satellites.

1. Introduction

Different battery performance models, mainly based on equivalent circuits, have been proposed in the recent past by several authors such as Newman *et al* [1] He *et al* [2] Mousavi *et al* [3] and Fotoui *et al* [4]. The model proposed in the present work for studying Li-ion batteries performance, is based on from a new and different approach. One of the problems that arise when developing analytical models for batteries is their dependency on the charge/discharge rate. The capacity of the battery is normally defined in relation to the charge/discharge rate. Furthermore, other variables used to measure the state of a battery such as the State of Charge (SoC) or the Deep of Discharge (DoD) are also defined as a function of the discharge rate. Once this problem was considered, our research group was interested in find a new variable that could define the state of a battery without any dependency on the aforementioned charge/discharge rate. The new variable that met this requirement was the energy discharge level [5]. This new variable was considered after a test was carried out to the UPMSat-2 battery in the thermal vacuum chamber of the IDR/UPM institute, where the efficiency of the battery was measured between 97.5% and 99%. As a result of the high efficiency, energy was then considered as the main candidate for the study of Li-ion batteries. Later on, and during the elaboration of the model, the variable finally considered was the total amount of energy that was discharge level (between 97.5%).

The present work is organized as follows. In section 2 the proposed model is described. The results are discussed in section 3. In section 4 the current work in shown, together with future lines of research. Section 5 closes the document with the conclusion extracted from this work.

2. Li-ion Battery models

2.1 Battery modeling.

The model presented in this work is based on the following assumptions:

- the efficiency of the battery is high enough to consider that there are no energy losses, and
- variations of the battery state due to temperature or the life cycle are not considered.

As previously said, the core of this work is based on the energy discharge level, ϕ . This energy discharge level is defined respectively for the discharging and charging processes as:

$$\phi = \phi_0 + \int_{t_0}^{t} E^d I dt = \phi_0 + \int_{t_0}^{t} (VI + R_d I^2) dt,$$
(1)

$$\phi = \phi_0 - \int_{t_0}^t E^c I dt = \phi_0 - \int_{t_0}^t (VI - R_c I^2) dt,$$
(2)

where E^d , E^c are the internal voltage of the battery for the discharging and the charging process respectively. R_d and R_c are the internal resistors of the battery within the corresponding process, and ϕ_0 is the initial value of the energy discharge level at the beginning of the process. The equivalent circuits that illustrate the mentioned battery processes are shown separately in Figure 1.



Figure 1. Equivalent circuit model for the charge (Left) and discharge (right) process of a Li-ion battery

Therefore, the output voltage can be obtained with the following expressions:

$$V(\phi, I) = E^d - R_d I, \tag{3}$$

$$V(\phi, I) = E^c + R_c I, \tag{4}$$

where E^d and E^c are mathematical functions that express the internal voltage of the battery, and therefore the state. The most important part of the work was carried out in this particular subject. Mathematically, E^d and E^c are functions of the energy discharge level ϕ and a series of parameter to be extracted from experimental data, obtained from testing the batteries:

$$E^{d} = f(\phi, E_{0}^{d}, E_{1}^{d}, E_{2}^{d} ...),$$
(5)

$$E^{c} = g(\phi, E_{0}^{c}, E_{1}^{c}, E_{2}^{c}...).$$
(6)

Both equivalent circuits were implemented in a single equivalent circuit as shown in Figure 2. This circuit only takes into account the static performance (this static performance is referred to charging or discharging processes at constant current, whereas dynamic performance is referred to those process where current does not remain constant). To consider the dynamic performance the model is completed using the a Thevenin circuit consisting in adding a pair of resistors and capacitators connected in parallel between them and in series to the rest of the battery as it is exposed in Figure 3. Obviously, with this modification, the following condition must be satisfied.

$$R_d = R_{\rm int} + R_1 + R_2. \tag{7}$$

The procedure to obtain the proposed model starts by testing the desire battery, the testing must consist on several charges and discharge cycles at different rates plus a dynamic discharge. From the static discharges and charges the parameters that define the E^d and E^c expressions are extracted. The following step is to redistribute the value of the internal resistor, obtained in the previous step, between the different resistors and simultaneously calculate the value of the capacitators, with the experimental data obtained from the dynamic test.







Figure 3. Proposed model for Li-ion battery for the dynamic and static performance

2.2 Modelling of the internal voltage during discharging processes.

The experimental data of the discharge of a Li-ion battery is shown in Figure 4. As shown in this figure, these discharging curves can be divided into two different parts. The first one could be approximated by a linear expression, whereas the second one is a curved part, corresponding with the end of the discharge process, which can be approximately modelled with an exponential approximation. Based on the previous statement different models for the internal voltage of the battery during a discharging process have been proposed.

The first model is an approximation of the discharge curve using a lineal expression, see equation (8). This expression lacks accuracy at the end of the discharge, but it could be used for those applications where the battery works on the linear zone (that is, working with not high values of DoD):

$$E^{d}(\phi) = E_{0}^{d} + E_{1}^{d}\phi.$$
 (8)

The second model includes an exponential term that depends on the discharging current, *I*. This model represents an approximation for the whole curve of the discharge process, but it complexity and therefore the difficulty of the parameter extraction increases:

$$E^{d}(\phi, I) = E_{0}^{d} + E_{1}^{d}\phi + (E_{20}^{d} + E_{21}^{d}I + E_{22}^{d}I^{2})\exp[(E_{30}^{d} + E_{31}^{d}I)\phi].$$
(9)



Figure 4. Discharge curves of a Li-ion battery for different discharge rates

The third proposed model, which has been recently proposed [6], is a further complex expression that was created to obtain a better approximation at the end of the discharging process. This is important as at the end of the discharge process voltage drops quickly, at this high level of discharged energy the voltage of one the cells that compose the battery could drop under 2.7 V and damage permanently the cell (and therefore, the battery). This expression is derived from the second proposed model. In comparison, the resistor that was previously considered as a constant value it has a linear dependence in relation to the current now:

$$V(\phi, I) = E_0^d + \left(E_{10}^d + E_{11}^d I + E_{12}^d I^2\right)\phi + \left(E_{20}^d + E_{21}^d I + E_{22}^d I^2\right)\exp\left(\left(E_{30}^d + E_{31}^d I\right)\phi + E_{41}^d \exp\left(E_{42}^d\phi\right)\right) - \left(R_{d0} + R_{d1}I\right)I$$
(10)

2.3 Modelling of the internal voltage during the charge process.

Following the same steps as the discharging process, it can be observed that the charging curves (see Figure 5) show almost a linear behaviour. Only at the beginning of the charging processes a sharp increase of the voltage is produced. For the charging process the following models have been proposed. The first model is similar to the first one proposed for the discharging process, as it is based on a linear expression:

$$E^{c}(\phi) = E_{0}^{c} + E_{1}^{c}\phi .$$
(11)

The second model adds an exponential term that depends on the charge current.

$$E^{c}(\phi, I) = E_{0}^{c} - E_{1}^{c}\phi - E_{2}^{c}\exp[(E_{30}^{c} + E_{31}^{c}I)\phi].$$
(12)



Figure 5. Charge curves of a Li-ion battery at different charge rates.

3.Results

The results of the different models are presented in this section. Firstly, the results of the discharging process are shown, followed by the charging process and finally by the results of the dynamic performance of the battery. The battery which from the experimental results are extracted is a 17.2 A h Li-ion battery based on Samsung 18650-25R cells [7]. Several discharge and charge process were carried out, the discharging process between 0.5 A and 5 A current rate, and the charging process between 0.5 A and 3.5 A current rate. This asymmetry lies in the purpose of this work for the development of Li-ion battery models for small satellites, in which the discharge rates are larger than the charge rates. This is because the power consumption is periodically boosted due to communication with Ground Control. Besides, the limited area for solar arrays or limitations due temperature, might impose lower charging rates when compared to the discharging rates. The experimental data extracted from this battery have been already shown in Figures 4 and 5.

3.1 Discharging process

The results obtained for the first discharging model (equation (8)), are presented in Figure 6. As this model only consider the linear part of the curves it could be notice how this model fits the first part of the discharge process. In Table 1 the numerical values of the parameters are also shown.

The results of the second model (equation (9)), are included in Figure 7 (see also Table 1). In comparison with the previous model, now it can be noticed how the model fits the exponential behaviour of the discharge curve, but it lacks in accuracy at the very end of the discharge process.

The results of the third model which is been analysed at present (equation (10)), are shown in Figure 8. In comparison with the previous model, this new model improves the fitting of the curves at the very end of the discharge process. This is quite important as a deep discharge of the battery might cause permanent damage.



Figure 6. Comparison of model 1 (equation (8)) and the experimental data for the discharging process, expressed in relation to the discharge energy rate, ϕ .



Figure 7. Comparison of model 2 (equation (9)) and the experimental data for the discharging process, expressed in relation to the discharge energy rate, ϕ .



Figure 8. Comparison of model 3 (equation (10)) and the experimental data for the discharging process, expressed in relation to the discharge energy rate, ϕ .

Table 1. Coefficients of the fittings of model 1 and 2 for the discharging process (see Figures 6 and 7).

Coefficients	Model 1	Model 2
$R_{d0}(\Omega)$	0.13316	0.13433
E_0^d (V)	24.384	24.369
$E^d_{10} \; (\mathbf{V} \cdot \mathbf{W}^{-1} \cdot \mathbf{h}^{-1})$	$-1.271 \cdot 10^{-2}$	$-1.2276 \cdot 10^{-2}$
$E_{21}^{d}\left(V\right)$	-	$-1.0617 \cdot 10^{-9}$
$E^{d}_{21}\left(\Omega ight)$	-	$-4.468 \cdot 10^{-10}$
$E^d_{22} \; (\Omega \cdot A^{-1})$	-	$-4.783 \cdot 10^{-11}$
$E^{d}_{30} \; (W^{-1} \cdot h^{-1})$	-	$5.8151 \cdot 10^{-2}$
$E^{d}_{31} (\mathbb{W}^{-1} \cdot \mathbb{h}^{-1} \cdot A^{-1})$	-	$-3.3599 \cdot 10^{-3}$

3.2 Charging process

The result obtained for the first charging model (equation (11)) are presented in Figure 9. In Table 2 the numerical values of the parameters are also shown.

The results of the second model (equation (12)) are included in Figure 10 (see also Table 2). Since the charge curve shows almost a completely linear behaviour the values of the exponential terms tend to be almost irrelevant for some of the curves.



Figure 9. Comparison of model 1 (equation (11)) and the experimental data for the charging process, expressed in relation to the discharge energy rate, ϕ .



Figure 10. Comparison of model 2 (equation (12)) and the experimental data for the charging process, expressed in relation to the discharge energy rate ϕ .

Table 2.	Coefficients	of the fitting	s of model	1 and 2 for the	e charging proce	ss (see Figures	9 and 10).
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Coefficients	Model 1	Model 2
$R_{c0}(\Omega)$	0.17547	0.15724
E_0^c (V)	24.446	24.421
$E_1^c (\mathbb{V} \cdot \mathbb{W}^{-1} \cdot \mathbf{h}^{-1})$	$-1.231 \cdot 10^{-2}$	$-1.2171 \cdot 10^{-3}$
E_2^c (V)	-	$-2.6493 \cdot 10^{-14}$
$E^c_{30} \; ({\rm W}^{-1} \cdot {\rm h}^{-1})$	-	$8.4402 \cdot 10^{-2}$
$E_{31}^c (\mathbb{W}^{-1} \cdot \mathbb{h}^{-1} \cdot A^{-1})$	-	$-3.8938 \cdot 10^{-3}$

3.3 Dynamic performance

A discharging process at 2 A rate, combined with 20-step dynamic discharging/charging periods of 360 s each hour was programmed to obtain data to adjust the complete model from Figure 3. In Figure 14, the discharging current during these dynamic periods is plotted.



Figure 11: 20-step dynamic discharging/charging period of 360 s programmed to measure the dynamic battery performance. This cycle is similar to the one included in the USABC battery test procedures manual [8].

The results obtained for the dynamic performance, correspond at using the second model of the discharging and charging processes (equations (9) and (12)). The simulation was programmed and run in Matlab/Simulink the results of the simulation are included in Figure 12. The values of the different components of the equivalent circuit, (resistors and capacitator) are the following. $R_{int} = 0.07 \ 707 \ \Omega$; $R_1 = 0.03 \ 777 \ \Omega$; $R_2 = 0.01 \ 949 \ \Omega$; $C_1 = 1100 \ F$; $C_2 = 950 \ F$.

In Figure 12 the results are expressed graphically. The comparison between the model and the battery performance during the second 20-step dynamic discharging/charging period is also plotted in the bottom graph of the figure, in order to compare the behaviour during sudden changes of battery discharge/ charge rate. In this graph, one can observe the good performance of the model, with deviations of $\Delta V < 0.1 V$ in relation with the measured performance of the battery.

4. Improvement of the model and future works

At present, the authors of this work are still working in the improvement of the analytical model. An improvement of the discharging process has already been proposed (equation (10)), meanwhile another model for the charging process is still under study. This new model aims to include the effect of the current rate on the resistor as it was stated in equation (10) for the discharging process.

Once the improved model is ready, it is planned to fit it to the experimental data from the 4-year monitoring and maintenance on the UPMSat-2 mission carried out by professors and students from IDR/UPM Institute. The prior concern is to stablish some pattern in relation to the evolution of the voltage disequilibrium within each series of the UPMSat-2 battery. This is a 18 A·h battery, composed of 4 series of Saft VES16 Li-ion cells, each one formed by 6 cells.

Other lines of research will lead us to analyse the aging process of the batteries through discharge/charge cycles and how the total amount of energy that the battery can held (that is, the capacity) is affected through the pass of cycles. This particular point is interesting for space missions, as the power subsystem is usually design in the end of life conditions, so predicting accurately the state of the battery at the end of life of a mission is of high interest through the design phases.



Figure 12. (Top) discharging process of the tested Li-ion at 2 A rate, combined with 20-step dynamic discharging/charging periods of 360 s each hour. Results of the second proposed model have also been included (dashed line). Results from the second 20-step dynamic discharging/charging period have been plotted in detail, in the bottom graph.

5. Conclusions

A model for a Li-ion battery performance based on the energy discharge level has been proposed. This model consists on an equivalent circuit that can predict the static and dynamic performance of these batteries. The core of the model is based on the correct modelling of the internal voltage of the battery for the charging and discharge processes. For this purpose, the authors have proposed different expressions that could predict the internal voltage of the battery in relation to the energy discharge level. Each model has its own application and complexity associated with it. Based on the results obtained it can be said that the model is able to reproduce quite well the performance of Li-ion batteries.

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