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Fault-tolerant trajectory reconfiguration for sub-orbital spacecraft reentry guidance

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Abstract
Sub-orbital spacecraft is a multi-purpose and reusable launch vehicle. Due to its short period and high frequency of launch, it provides challenge problems to the robustness and intelligence of the guidance system. Establishing a spacecraft online reconfiguration capability is one of the key technologies for advanced spacecraft. Based on the mesh refinement and the dynamic inversion method, an trajectory reconfiguration algorithm is proposed. By selecting the off-line trajectory as initial values and selecting the appropriate nodes, we can reduce the number of iterations to improve the speed of trajectory reconfiguration to satisfy the real-time requirement.

1. Introduction

Sub-orbital spacecraft is a hypersonic vehicle that can reach the top of near space but is not fast enough to run around the earth. The speed of sub-orbital spacecraft is generally between Mach 5 and 15. After the mission is completed, it can return to Earth and is reusable. Due to the ability to perform multi-missions and the extremely fast flight speed, sub-orbital spacecraft have received extensive attention [1]. For the new generation of reusable spacecraft, reentry guidance with online trajectory reconfiguration is more reliable and effective [2].

In general, guidance method of reentry guidance includes reference-trajectory tracking guidance and predictor-corrector guidance[3]. The basic idea of the reference-trajectory tracking guidance is to track the pre-designed reference trajectory by using the linear or nonlinear control theory to design the guidance law to meet different guidance requirements. The predictor-corrector guidance approach including analytic and numerical method is to eliminate the deviation between the actual predicted target point and the expected target point. The reference trajectory guidance adopted by Gemini and Apollo missions were the early successful mission with the guided entry. In order to reduce the dependence on aerodynamic and atmospheric models, Apollo entry guidance used a reference trajectory algorithm based on drag acceleration. During the past several decades, a large numbers of drag tracking control methods have been applied to spacecraft guidance problems[4].

Based on the mesh refinement algorithm and the nonlinear dynamic inversion control method, online trajectory reconfiguration for spacecraft under unexpected conditions in reentry phase is studied in this paper. In the nominal case, the mesh refinement algorithm is designed to calculate the off-line optimal trajectory that satisfies the reentry process and terminal constraints, and this off-line trajectory will provide the initial value for the on-line trajectory reconfiguration. In case of uncertainty conditions, the nominal trajectory will no longer meet the guidance requirements. Therefore, a feasible reentry trajectory is generated on-line by the mesh refinement algorithm, and the guidance command is updated on-line. By adopting the nonlinear dynamic inversion control law with an active disturbance rejection method, the reentry trajectory generated can be tracked. By selecting the off-line trajectory as initial values and selecting the appropriate nodes, we can reduce the number of iterations to improve the speed of trajectory reconfiguration to satisfy the real-time requirement. The simulation results of a typical spacecraft show that the trajectory generated in case of actuator failure meets the reentry flight constraints and real-time requirements, so that the spacecraft can safely fly to the specified landing point, improving the safety and reliability of the spacecraft.

2. Model of reentry guidance

The nonlinear equations for the reentry guidance is expressed as

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$$\begin{aligned}
&\dot{r}=V\sin\gamma\\
&\dot{V}=-D-g\sin\gamma\\
&\dot{\theta}=\frac{V\cos\gamma\cos\psi}{r\cos\varphi}\\
&\dot{\varphi}=\frac{V\cos\gamma\sin\psi}{r}\\
&\dot{\gamma}=\frac{1}{V}\left[L\cos\sigma-\left(g-\frac{V^2}{r}\right)\cos\gamma\right]\\
&\dot{\psi}=-\frac{1}{V\cos\gamma}\left[L\sin\sigma+\frac{V^2}{r}\cos^2\gamma\cos\psi\tan\varphi\right]
\end{aligned} \tag{1}$$

where θ and ϕ are the longitude and latitude, respectively; r is the radial distance; V is the velocity; γ and ψ are the velocity elevation angle and the velocity heading angle, respectively; σ is the bank angle; $g=\mu/r^2$ is the gravitational acceleration, where μ is the gravitational parameter. The lift and drag accelerations are expressed as

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(2)

$$L=\frac{\rho V^2 S C_L}{2m} \tag{3}$$

where C_L,C_D is the aerodynamic coefficients, S stands for the vehicle reference surface area and m is the mass of the vehicle.

The exponential atmospheric density model as following

$$\rho=\rho_0\exp(-\frac{r-r_0}{h_s}) \tag{4}$$

where ρ_0 is the density at the reference radius r_0 and h_s is the constant scale height.

3. Trajectory reconfiguration with mesh refinement technique

According to the different forms of performance indicators, the optimal control problems can be divided into Mayer problem, Lagrange problem and Bolza problem. The Mayer problem and the Lagrange problem can all be regarded as special forms of the Bolza problem. Therefore, without loss of generality, we consider the Bolza problem in the following. The Bolza problem can be described as: determining the control variable $\boldsymbol{u}(t)$ to minimize the following objective function

$$J=M\left(x(t_0),t_0,x(t_f),t_f\right)+\int_{t_0}^{t_f}L\left(\boldsymbol{x}(t),\boldsymbol{u}(t),t\right)\mathrm{d}t \tag{5}$$

where the system state $\boldsymbol{x}(t)\in\mathbb{R}^n$, the initial time t_0 and the final time t_f satisfy the state function

$$\dot{\boldsymbol{x}}=\boldsymbol{f}\left(\boldsymbol{x}(t),\boldsymbol{u}(t),t\right),\quad t\in[t_0,t_f] \tag{6}$$

and the boundary conditions

$$\boldsymbol{\Phi}\left(\boldsymbol{x}(t_0),t_0,\boldsymbol{x}(t_f),t_f\right)=\boldsymbol{0} \tag{7}$$

and the path constraint

$$\boldsymbol{C}\left(\boldsymbol{x}(t),\boldsymbol{u}(t),t\right)\le\boldsymbol{0},\quad t\in[t_0,t_f] \tag{8}$$

The parameters M, L, $\boldsymbol{f}$, $\boldsymbol{\Phi}$ and $\boldsymbol{C}$ are defined as

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$$\begin{aligned} M &: \mathbb{R}^n \times \mathbb{R} \times \mathbb{R}^n \times \mathbb{R} \to \mathbb{R} \\ L &: \mathbb{R}^n \times \mathbb{R}^m \times \mathbb{R} \to \mathbb{R} \\ \boldsymbol{f} &: \mathbb{R}^n \times \mathbb{R}^m \times \mathbb{R} \to \mathbb{R}^n \\ \boldsymbol{\Phi} &: \mathbb{R}^n \times \mathbb{R} \times \mathbb{R}^m \times \mathbb{R} \to \mathbb{R}^\phi \\ \boldsymbol{C} &: \mathbb{R}^n \times \mathbb{R}^m \times \mathbb{R} \to \mathbb{R}^c \end{aligned}$$

We use the local collocation method to discretize it into a nonlinear programming problem. Suppose the N+1 discrete points on the unit interval [0, 1] are

$$\begin{aligned} \boldsymbol{G} = \{\tau_i : \tau_i \in [0,1],\ i = 0,1,\ \cdots,\ N;\ \tau_0 = 0, \tau_N = \tau_f = 1; \\ \tau_i < \tau_{i+1},\ \ i = 0,1,\ \cdots,\ N-1\} \end{aligned} \tag{9}$$

Where τ_i is called a node or a grid point, and τ_i can be evenly distributed on [0, 1] or non-uniformly distributed. For ease of writing, a shorthand $\boldsymbol{x}_i = \boldsymbol{x}(\tau_i)$, $\boldsymbol{u}_i = \boldsymbol{u}(\tau_i)$.

Define

$$\begin{aligned} &\boldsymbol{X} = \left\{\boldsymbol{x}_0, \boldsymbol{x}_1, \cdots, \boldsymbol{x}_N\right\};\quad \boldsymbol{U} = \left\{\boldsymbol{u}_0, \boldsymbol{u}_1, \cdots, \boldsymbol{u}_N\right\} \\ &\bar{\boldsymbol{G}} = \left\{\tau_{ij} \in \left[0,1\right] : \tau_{ij} \notin \boldsymbol{G},\ \ i = 0,1,\cdots,\ \ N-1,\ \ 1 < j < q\right\} \\ &\bar{\boldsymbol{X}} = \left\{\boldsymbol{x}_{ij} : \tau_{ij} \in \bar{\boldsymbol{G}}\right\};\quad \bar{\boldsymbol{U}} = \left\{\boldsymbol{u}_{ij} : \tau_{ij} \in \bar{\boldsymbol{G}}\right\} \end{aligned}$$

The nonlinear programming (NLP) problem can be obtained by discretizing the above continuous Bolza problem.

$$J = M\left(\boldsymbol{x}_0, t_0, \boldsymbol{x}_f, t_f\right) + \Delta t \sum_{i=0}^{N-1} h_i \sum_{j=1}^{q} \beta_j L_{ij} \tag{10}$$

where the constraints are satisfied

$$\boldsymbol{\xi}_i = \boldsymbol{x}_{i+1} - \boldsymbol{x}_i - \Delta t \cdot h_i \sum_{j=1}^{q} \beta_j \boldsymbol{f}_{ij} = \boldsymbol{0},\quad \left(i = 0,1,\ \cdots,\ N-1\right) \tag{11}$$

$$\boldsymbol{C}_i = \boldsymbol{C}\left(\boldsymbol{x}_i, \boldsymbol{u}_i, \tau_i; t_0, t_f\right) \le \boldsymbol{0},\quad \left(i = 0,1,\ \cdots,\ N\right) \tag{12}$$

$$\bar{\boldsymbol{C}}_{ij} = \boldsymbol{C}\left(\boldsymbol{x}_{ij}, \boldsymbol{u}_{ij}, \tau_{ij}; t_0, t_f\right) \le \boldsymbol{0},\quad \tau_{ij} \in \bar{\boldsymbol{G}} \tag{13}$$

$$\boldsymbol{\Phi}\left(\boldsymbol{x}_0, t_0, \boldsymbol{x}_f, t_f\right) = \boldsymbol{0} \tag{14}$$

and the parameter ξ_i is the discrete residual of the equation of state.

The NLP transformed by the optimal control problem is a large-scale sparse problem with many constraints, and there are equality constraints that are always active, and the calculation of constraints is large. Sequential Quadratic Programming (SQP) has fewer calculations for objective functions and constraints, and has strong processing power for constraint problems. Therefore, this project uses SQP to solve NLP. Currently, mature software packages such as SNOPT have been developed based on SQP. In order to improve the robustness and optimization efficiency of the established optimization method, it is also necessary to consider NLP normalization processing, NLP sparsity, numerical differentiation method, initial value selection strategy, discrete node distribution strategy.

4. Reentry guidance algorithm

This paper adopts drag acceleration tracking method based on the feedback linearization theory with the extended state observer to track the reference trajectory. Considering the trajectory range of the vehicle depends on the

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magnitude of the drag acceleration, and the drag acceleration is determined by the bank angle. Thus, the bank angle is the main control variable for tracking trajectory.

$$\dot{D} = -\frac{1}{h_s} DV \sin\gamma - \frac{2D}{V}\left(D + g\sin\gamma\right) \tag{15}$$

$$\ddot{D} = a + bu \tag{16}$$

where the terms a and b are nonlinear functions of the state variables.

$$\begin{aligned} a = {} & D\left[\frac{1}{h_s}\left(D + g\sin\gamma\right)\sin\gamma + \frac{1}{h_s}\cos^2\gamma\left(g - \frac{V^2}{r}\right)\right] \\ & + D\left[-\frac{2\left(D + g\sin\gamma\right)^2}{V^2} + \frac{2g^2\cos^2\gamma}{V^2} - \frac{2g\cos^2\gamma}{r} + \frac{4g\sin^2\gamma}{r}\right] + \dot{D}\left(-\frac{1}{h_s}V\sin\gamma - \frac{4D}{V} - \frac{2g\sin\gamma}{V}\right) \end{aligned} \tag{17}$$

$$b = -DL\cos\gamma\left(\frac{2g}{V^2} + \frac{1}{h_s}\right) \tag{18}$$

and $u = \cos\sigma$.

The control system can be written as

$$\begin{bmatrix}\dot{x}_1 \\ \dot{x}_2\end{bmatrix} = \begin{bmatrix} x_2 \\ a(x_1, x_2) + b(x_1, x_2)u + w(t)\end{bmatrix} \tag{19}$$

where $x_1 = D$ and $x_2 = \dot{D}$. Let the control input

$$u = \frac{1}{b(D,\dot{D})}[P - a(D,\dot{D}) - w(t)] \tag{20}$$

The feedback linearization control law is designed as,

$$u = \frac{[-k_0(D - D_r) - k_1(\dot{D} - \dot{D}_r) + \ddot{D}_r - a(D,\dot{D}) - w(t)]}{b(D,\dot{D})} \tag{21}$$

The control law design parameters are the gains k_0 and k_1. By using the standard second order linear dynamics parameters, we obtain the tracking guidance law parameters $k_1 = 2\varsigma\omega$ and $k_0 = \omega^2$ with the damping ratio $0 \le \varsigma < 1$ and the natural frequency $\omega > 0$.

To get the unknown disturbance $w(t)$ in the control law, an extended state observer is designed to estimate its value. By considering $w(t)$ as an unknown extended state variable x_3 and setting its derivative as $h(t)$, we rewrite the guidance system into a state equation of the following form

$$\begin{cases}\dot{x}_1 = x_2 \\ \dot{x}_2 = a(x_1, x_2) + b(x_1, x_2)u + x_3 \\ \dot{x}_3 = h(t)\end{cases} \tag{22}$$

Adopt the following extended state observer,

$$\begin{cases} e_1 = z_1 - x_1 \\ \dot{z}_1 = z_2 - \beta_1 e_1 \\ \dot{z}_2 = z_3 - \beta_2\, fal(e_1, \alpha_1, \delta) + a(x_1, x_2) + b(x_1, x_2)u \\ \dot{z}_3 = -\beta_3\, fal(e_1, \alpha_2, \delta)\end{cases} \tag{23}$$

$\beta_1, \beta_2, \beta_3$ is the gains of the observer, z_1, z_2 is the estimate of the state x_1, x_2, and z_3 is the estimate of the unknown disturbance. $fal(\cdot)$ is a nonlinear function as follows

$$fal(e, \alpha, \delta) = \begin{cases} |e|^\alpha\, sign(e), & |e| > \delta \\ \dfrac{e}{\delta^{1-\alpha}}, & |e| \le \delta \end{cases} \tag{24}$$

where $0 < \alpha < 1$, and α_1, α_2 is usually chosen as 0.5 and 0.25 in the third-order extended state observer. By selecting the appropriate gain parameters β_i and $\delta > 0$. By substituting the unknown disturbance estimated by the extended state observer into (21), we obtain the final guidance law,

$$u = \frac{[-k_0(D - D_r) - k_1(\dot{D} - \dot{D}_r) + \ddot{D}_r - a(D,\dot{D}) - z_3]}{b(D,\dot{D})} \tag{25}$$

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4. Simulation results

The simulation results are shown in figure 1-2.

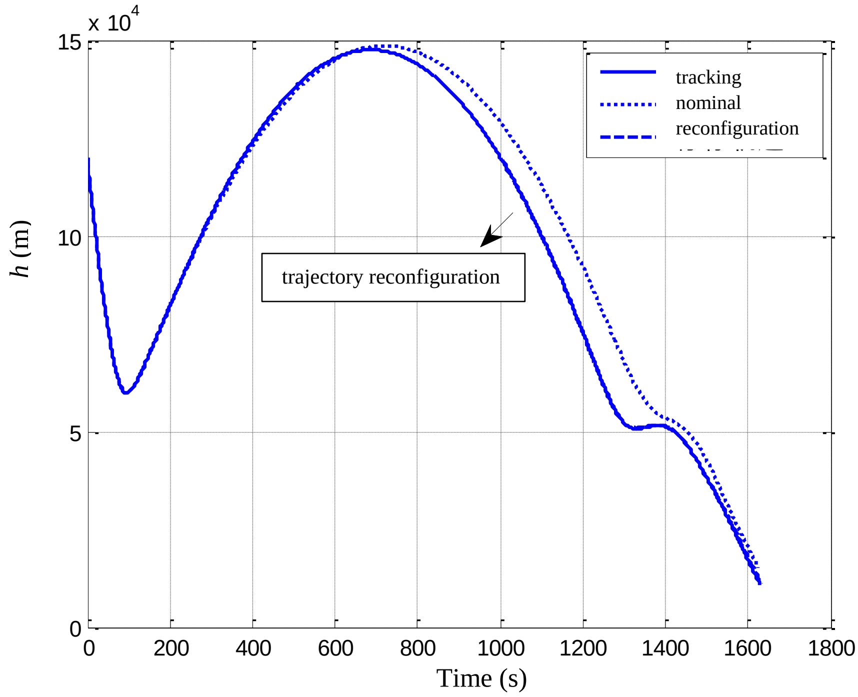

Figure 1: Tracking performance

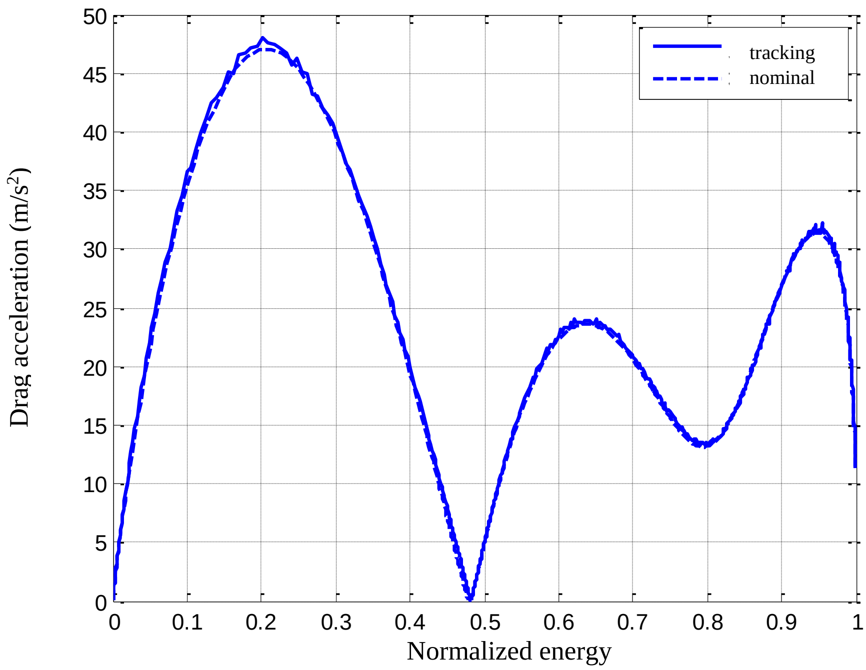

Figure 2: Drag tracking

It can be seen from the above simulation results that the feedback linearization guidance law with the trajectory reconfiguration method can greatly reduce the tracking error caused by the existence of random aerodynamic coefficient deviation during the entire reentry flight. The position error of the terminal is kept within the range that meets the design requirements.

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5. Conclusions

Based on the mesh refinement algorithm and the nonlinear dynamic inversion control method, online trajectory reconfiguration for spacecraft under unexpected conditions in reentry phase is proposed in this paper. The simulation results of a typical spacecraft show that the trajectory generated in case of actuator failure meets the reentry flight constraints and real-time requirements, so that the spacecraft can safely fly to the specified landing point, improving the safety and reliability of the spacecraft.

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