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Passively Safe Relative Orbit Configurations over Long Time Intervals for Heterogeneously Distributed Spacecraft Clusters

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Abstract
The problem of relative orbit configurations over a long time intervals incorporating realistic dynamics, platform differences and operational constraints for safety, station keeping and inter-spacecraft communications is addressed. Two different approaches are developed. In the first approach, relative mean orbital mean elements are found through minimizing deviations from reference mean orbit. In second one, relative configurations are found from a reference initial condition through minimizing probability of collision between the spacecraft in the cluster which are propagated numerically through full force models. Effectiveness of the approaches is demonstrated through simulations.

1. Introduction

Distributed space systems and spacecraft clusters have several advantages when compared to missions carried out using single spacecraft in terms of flexibility and robustness which would increase the net value or mission return [1]. One of the important factors for these distributed space systems is the design of relative orbit configurations for a cluster of spacecraft. For the case of cluster flying with a heterogeneously distributed system, it can be considered that a number of spacecraft, differing in platform characteristics, fly closely in formation with relatively loose geometry constraints **Hata! Başvuru kaynağı bulunamadı.**. In this manner, there are several aspects to consider for the realization of cluster flying in terms of safety, and inter-spacecraft communication availability [3] in addition to the regular station keeping and mission operations constraints.

So far, design and control of relative motion are widely studied in academia and industry with an emphasis on station keeping and/or reconfiguration objectives and constraints. Among these, [4] provides a basis for relative orbit design with safety considerations. [3] and [5] discusses the application of cluster flying for fractionated space systems. Also, the coordination, control, reconfiguration and optimization issues are widely analysed by [6], [7] and [8] providing an insight on the formulation of the relative dynamics, definition of constraints and linearization techniques. A methodological development of cluster control algorithms supporting the various use cases of the mission with an emphasis on the algorithm’s structure, information flow, and implementation is presented in [9]. This important literature guides the development of an infrastructure for cluster flying design; however, an approach combining these building blocks with realistic operational considerations is still to be developed and/or advanced.

In this paper, two different approaches are presented with the aim of incorporating realistic operational considerations and mission/platform parameters while designing relative orbits for spacecraft clusters. In the first one the objective function is defined such that the deviation from reference orbit for each spacecraft within the cluster is minimized, hence maximizing the station-keeping objective. In the second one, state uncertainties are also considered and the probability of collision between the spacecraft is minimized with the aim of maximizing the safety objective. For both approaches, minimum and maximum bounds on distances between the spacecraft and on the design variables (relative orbital elements) are defined by considering realistic mission and system specific parameters such as minimum and maximum ranges. In addition, different physical characteristics are introduced through ballistic coefficients and the radiation pressure parameters in order to simulate a heterogeneous system.

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Finally, these two approaches are formulated and solved providing passively safe long term operations which would not require any reconfiguration over a specific time interval therefore maximizing the mission return without any manoeuvre operation.

2. Cluster Flying Design, Evaluation and Optimization Infrastructure

2.1 Relative Orbit Definition and Propagation

Designing cluster flying configurations is basically equivalent to defining relative orbital elements for several spacecraft based on a reference orbit. Here, the reference orbit is specific for each type of mission derived from several needs and constraints depending on the application. However, once it is fixed, relative orbits can be defined by introducing small differences to reference orbital elements, such as $oe_l = oe_r + \Delta oe_l$ where r indicates the reference orbit, l indicates the spacecraft number within the cluster and relative orbital elements are defined as $\Delta oe = [\Delta a \quad \Delta e \quad \Delta i \quad \Delta\Omega \quad \Delta\omega \quad \Delta\upsilon]$. This will also provide orbital elements, or initial conditions, for each spacecraft which can be propagated through proper analytical models or numerical integrators with proper parameters and force models. Here, analytical models are obtained through a specific perturbation theory and they can be used for coarse mission analysis purposes without a significant computational demand. On the other hand, high precision numerical orbit propagators can provide accurate state information with much more computational effort. The models and propagators utilized in this study are summarized in Table 1:

Table 1: Utilized Orbit Propagators

	Numerical	**Analytical**
Method	6^{th} Order Symplectic Integrator	Simplified General Perturbations Theory (SGP4)
Gravitational Terms	Degree: 8, Order: 8	Degree: 4, Order: 0
Point Masses	Sun, Moon	-
Atmospheric Density Model	NRLMSISE00	Specific Model with a Drag Term, β^*
Solar Radiation Pressure Model	Spherical Body, Conic Shadow Model	-
Input	Initial State in Earth Centred Inertial (ECI) Cartesian Elements	Two Line Elements (TLE)
Output	ECI Cartesian States	ECI Cartesian States
Additional Information	Parameters such as mass, drag area and coefficient, solar radiation pressure area and coefficient, Earth Orientation Parameters and Space Weather Data are also input	Spacecraft specific differences (mass, area, etc.) can be defined indirectly by β^*

It should be noted that given a time interval, a TLE can be fitted using an orbital ephemeris data which can be generated from a high precision orbit propagator. Also, spacecraft specific parameters such as, mass, area, etc., can be input to both propagators which makes it possible to propagate different spacecraft within a heterogeneously distributed space system. Using the propagation of orbits, relative states in ECI can be obtained by subtracting the position states of a specific spacecraft from the ones obtained from the reference orbit as follows:

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$$\Delta \mathrm{X}_l = \begin{bmatrix} X_l - X_r \\ Y_l - Y_r \\ Z_l - Z_r \end{bmatrix} \tag{1}$$

This relative state is then transformed from ECI to Local Vertical Local Horizontal, or orbital, frame which is defined by the reference orbit. Here, we can further decompose these relative states into radial (R), along-track (T) and cross track (N) components for a circular orbit and the relative state vector for each spacecraft becomes $\Delta \mathrm{X}_{l,LVLH} = [\Delta R_l \quad \Delta T_l \quad \Delta N_l]$. Finally, the relative distance between any two spacecraft at a specific discrete time instant can be written as $\Delta \mathrm{X}_{l,m,j} = [\Delta R_{l,m,j} \quad \Delta T_{l,m,j} \quad \Delta N_{l,m,j}]$ where $l \neq m$ are spacecraft indices, and j indicates a discrete time instant. In this manner, $\Delta X_{l,m,j}$ describes the relative distance between the spacecraft l and m at the time instant j.

2.2 Uncertainty Propagation and Probability of Collision

In reality, an orbit determination result, i.e. orbital elements or a state vector, almost always comes with covariance information. Therefore, when assessing the collision or evaporation risks the state uncertainty should be also considered while propagating an orbit, or initial conditions. Here, unscented transform [10] is used to propagate the uncertainties by utilizing the nonlinear dynamics of the system. This is achieved by propagating 2n+1 particles (mean and the 2n distributed around the mean) derived from the state vector. Then a new mean at the terminal instant is synthesized using the propagated particles. The formulation of the unscented transform is provided in Equation 2:

$$\begin{aligned} y_i &= f(x_i, t) \\ \bar{y} &= \sum_{i=0}^{2N} W_i^{(m)} y_i \\ P_y &= \sum_{i=0}^{2N} W_i^{(c)} \{y_i - \bar{y}\}\{y_i - \bar{y}\}^T \end{aligned} \tag{2}$$

Here, f represents the nonlinear system dynamics which propagates the state x of the particle i at a given time t. $\bar{y}$ is the synthesized mean using the weights $W_i^{(m)}$ defined for each particle. Similarly, final covariance is synthesized using the weights $W_i^{(c)}$ for each particle, and the deviation of the states y_i from the mean $\bar{y}$.

Having the state uncertainty information for spacecraft, it is also possible to calculate the probability of collision. For this, standard methods developed in [11] and [12] are utilized. Here, the combined covariance of any 2 spacecraft is transformed into conjunction plane (also called B plane) which is perpendicular to the relative velocity vector. Then, the 2 dimensional integral of projected uncertainty (C_B) is calculated over the conjunction area (A_C) which is centred at the relative position vector with a radius (R_C) calculated as the sum of the radii of 2 spacecraft. This is summarized in the Figure 1 and the Equation 3.

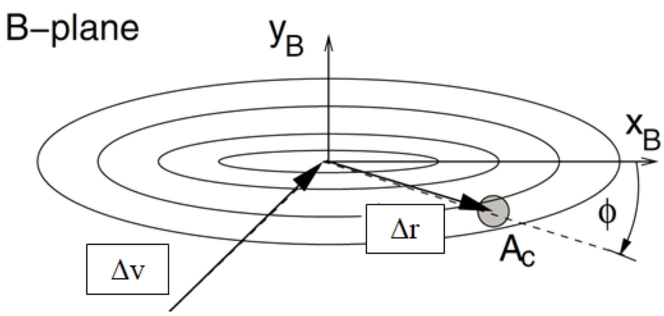

Figure 1: Representation of Conjunction Plane (B-plane), Combined Covariance (C_B) and Conjunction Area (A_c)

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Here, Δr and Δv represent the relative position and velocity, x_B and y_B represent the major and minor axes and ϕ represents the angular position of the conjunction area, A_c. Then the probability of collision can be calculated through the integral provided in Equation 3

$$P_c = \frac{1}{2\pi\sqrt{\|C_B\|}} \int_{-R_c}^{+R_c} \int_{-\sqrt{R_c^2 - x_B^2}}^{+\sqrt{R_c^2 - x_B^2}} \exp(\frac{1}{2}\Delta r_B{}^T C_B{}^{-1} \Delta r_B)\, dy_B dx_B \tag{3}$$

The implemented infrastructure for the cluster flying state and uncertainty propagation as well as probability of collision calculation is summarized in the flowchart given in Figure 2.

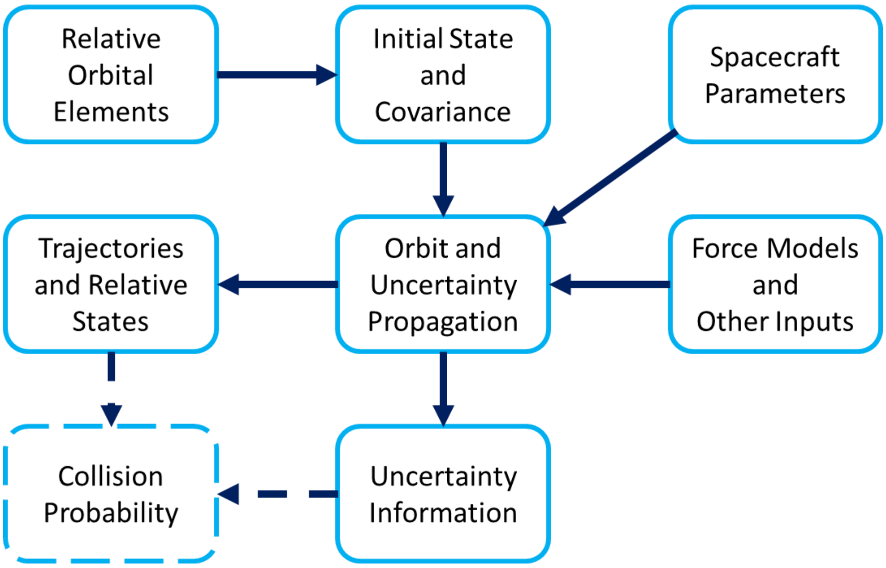

Figure 2: Flow Chart for the Calculation of Relative States, Uncertainties and Probability of Collision

This infrastructure is utilized when the numerical orbit propagator is used to evaluate the objective and constraint functions for the cluster flying design variables, i.e. relative orbital elements. For the case of analytical model (SGP4) utilization, this infrastructure is simplified into the propagation of relative orbital elements and therefore the evaluation of objective and constraints functions is based on relative states only.

2.3 Cluster Flying Constraints

Mission and platform characteristics, which may vary for different spacecraft, play an important role while realizing flight dynamics operations. In this manner, it is also important to consider realistic parameters, objectives and constraints stemming from several operational scenarios and limitations while solving cluster flying design problem. Here, operational constraints on station-keeping, safety and inter-spacecraft communications can be introduced by defining bounds on minimum and maximum distances between the spacecraft. For instance, a minimum distance constraint can be defined to ensure the safety through collision avoidance and a bound on maximum distance can be defined to ensure inter-spacecraft link availability through evaporation (or maximum range violation) avoidance for the cluster. Station-keeping can be also ensured if the minimum and maximum distance constraints are defined such that the deviations from the reference orbit are within the acceptable limits. The constraints derived from these considerations are summarized in Equation 4:

$$\begin{gathered} \Delta R_{l,m,j}^2 + \Delta N_{l,m,j}^2 \geq d_{\min} \\ \Delta R_{l,m,j}^2 + \Delta T_{l,m,j}^2 + \Delta N_{l,m,j}^2 \leq d_{\max} \\ u_{l,\min} \leq u_l \leq u_{l,\max} \end{gathered} \tag{4}$$

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With these constraints, it is possible to ensure a minimum distance, d_{min}, in radial - cross track (RN) plane and a maximum range, d_{max}, in 3 dimensions. Here, the reason for introducing minimum distance constraint only on the RN plane is to ensure a safe distance between any spacecraft without relying on the distance in in-track direction for which the navigation uncertainties are usually very high when compared to other directions. Also, minimum and maximum bounds on the design variable vector *u*, can be introduced to form a design space based on the relative orbit elements.

2.4 Objective Functions and Design Variables

Depending on the use cases and requirements of the cluster flying, different objective functions can be defined based on relative distances or parameters such as probability of collision to maximize the station keeping and/or safety. For the station-keeping, an objective function which would minimize the relative distances to the reference orbit can be written as:

$$\sum_{l=1}^{L}\sum_{j=1}^{N}(\Delta R_{l,j}^2 + \Delta T_{l,j}^2 + \Delta N_{l,j}^2) \tag{5}$$

This objective function will imply that any deviation, or distance, of each spacecraft, *l*, for each discrete time instant, *j*, from the reference orbit would increase the cost. Therefore the total cost would become the sum of the deviations of all spacecraft, *l=1, 2,…, L*, over the simulated time interval which is discretized as *j=1, 2,…, N*. Similarly, the objective function which would maximize the safety can be written for the probability of collision as:

$$\sum_{l,m=1}^{L}\sum_{j=1}^{N} PoC_{l,m,j} \tag{6}$$

This objective function will imply that the probability of collision between any two spacecraft, *l* and *m* where $l \neq m$, for each time instant, *j*, would increase the cost. Therefore, the total cost would become the sum of the probabilities of collision between any two spacecraft, $l \neq m$*: 1, 2, …, L,* over the simulated time interval which is discretized as *j=1, 2, …, N*.

After specifying the objective functions, design variables which would be a selected set of relative orbital elements and their minimum and maximum bounds can be also defined. Initially, design variables can be specified as $u = [\Delta e \quad \Delta i \quad \Delta\Omega \quad \Delta\omega]$. With these variables relative orbit configurations can be formed in terms of relative eccentricity and relative inclination vectors which are defined by Δe, $\Delta\omega$ and Δi, $\Delta\Omega$ respectively. Using these, the maximum separations between the reference and relative orbits can be quantified in reduced form for the case of circular reference orbit as follows:

$$\begin{aligned} \Delta R_{\max} &\propto a\delta e \\ \Delta T_{\max} &\propto 2a\delta e \\ \Delta N_{\max} &\propto a\delta i \end{aligned} \tag{7}$$

where δe and δi are the magnitudes of the eccentricity and inclination vectors. Here, it should be noted that, the contribution of relative orbital elements to the maximum distances shall be in the same order. In this manner, the minimum and maximum bounds on the design vector are specified as follows:

$$\begin{aligned} -0.000525 \le \Delta e \le 0.000525 \\ -0.075° \le \Delta i \le 0.075° \\ -0.075° \le \Delta\Omega \le 0.075° \\ -0.003° \le \Delta\omega \le 0.003° \end{aligned} \tag{7}$$

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With this design space, the order of the maximum distances would be bounded by 500 km. when vector magnitudes are calculated for RTN directions. In order to check the relative effectiveness of the design variables, a random set which is composed of 1000 samples within the design space is formed and objective function values are calculated for 5 spacecraft each having 4 relative orbital elements around a circular orbit as design variables. The result of the sampled design space is shown in the Figure 3.

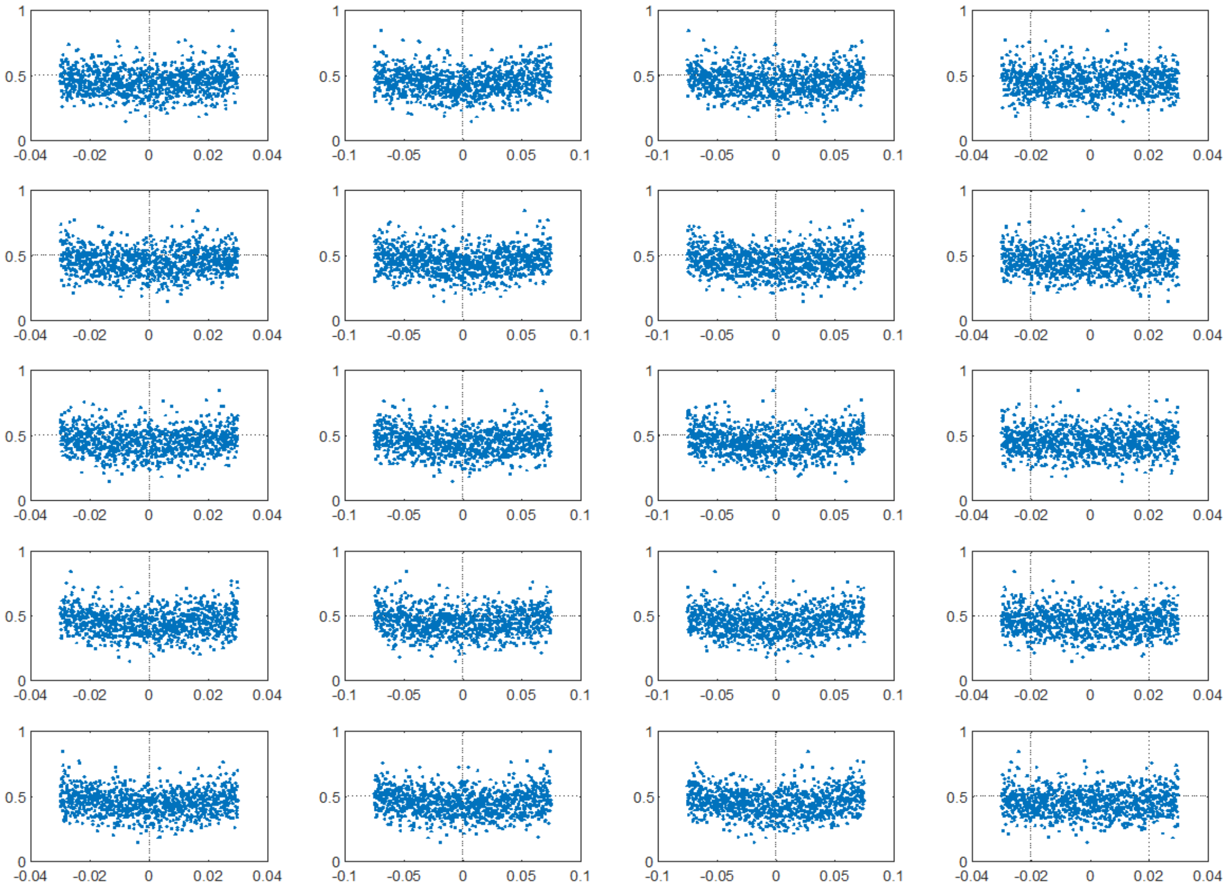

Figure 3: Relative Effectiveness of the Design Variables for 5 Spacecraft over a Design Space of 1000 Samples. Each row indicates the specific spacecraft, while the column indicates relative orbital element, Δe, $\Delta\omega$, Δi and $\Delta\Omega$ respectively.

In Figure 3, rows represent each spacecraft and columns represent the relative orbital elements for the specific spacecraft. For a specific row, i.e. spacecraft, each subplot shows the objective function values for a specific relative orbital element in the order of Δe, $\Delta\omega$, Δi and $\Delta\Omega$. From these figures, it can be seen that the change in each design variable, or the relative orbital element, contribute to the objective function.

Finally, since the objective and constraints are highly nonlinear and the problem is non-convex, a design space exploration based technique is utilized to solve the cluster flying design problem. Here, a design space exploration is performed through generating a set of many samples (in the order of thousands) and then objective and constraint functions are evaluated for each sample. The samples which are satisfying the constraints, i.e. feasible solutions, are filtered and the one providing the lowest cost value is selected as the initial condition. Finally, this filtered and selected initial condition is further optimized. The steps of the process are summarized in the Figure 4 as:

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Figure 4: Optimization Algorithm Flowchart

For the last part of the process, the filtered initial condition can be optimized by using a gradient based nonlinear optimizer such as MATLAB fmincon function [13], or another design space exploration can be performed within a smaller design space.

3. Cluster Flying Design by Minimization of Deviation from Reference Orbit

As it is described in the previous section, one of the objectives can be defined for maximizing station-keeping by minimizing the Equation 5. In this approach, design variables are selected as relative mean orbital elements for each spacecraft. Here, the relative orbital elements which are derived from a reference TLE are propagated through SGP4 propagator with no uncertainty. The final problem and simulation parameters for a reference simulation can be summarized in Table 2 as follows:

Table 2: Formulization of Cluster Flying Design with Station-Keeping Objectives

Parameter	**Specification**	
Objective Function	Equation 5	
Constraint Function(s)	Equation 4	
Bounds on Design Variables	Equation 7	
Propagator	Analytical (SGP4) with no uncertainty propagation	
Inputs and Assumptions	Reference Orbit	Sun Synchronous, LTDN 10:30 at 680 km altitude
	Spacecraft Number	10
	d_{min} in RN Plane	0.1 km
	d_{max} in RTN Plane	20 km
	Physical Differences	Defined by $\Delta\beta^*_l$
	Simulation Duration	2 days
	Step Size	10 sec.

The resulting optimized design solution for 10 spacecraft is simulated for 2 days and the distances between any two spacecraft during each time instant for 45 combinations is shown in the Figure 5.

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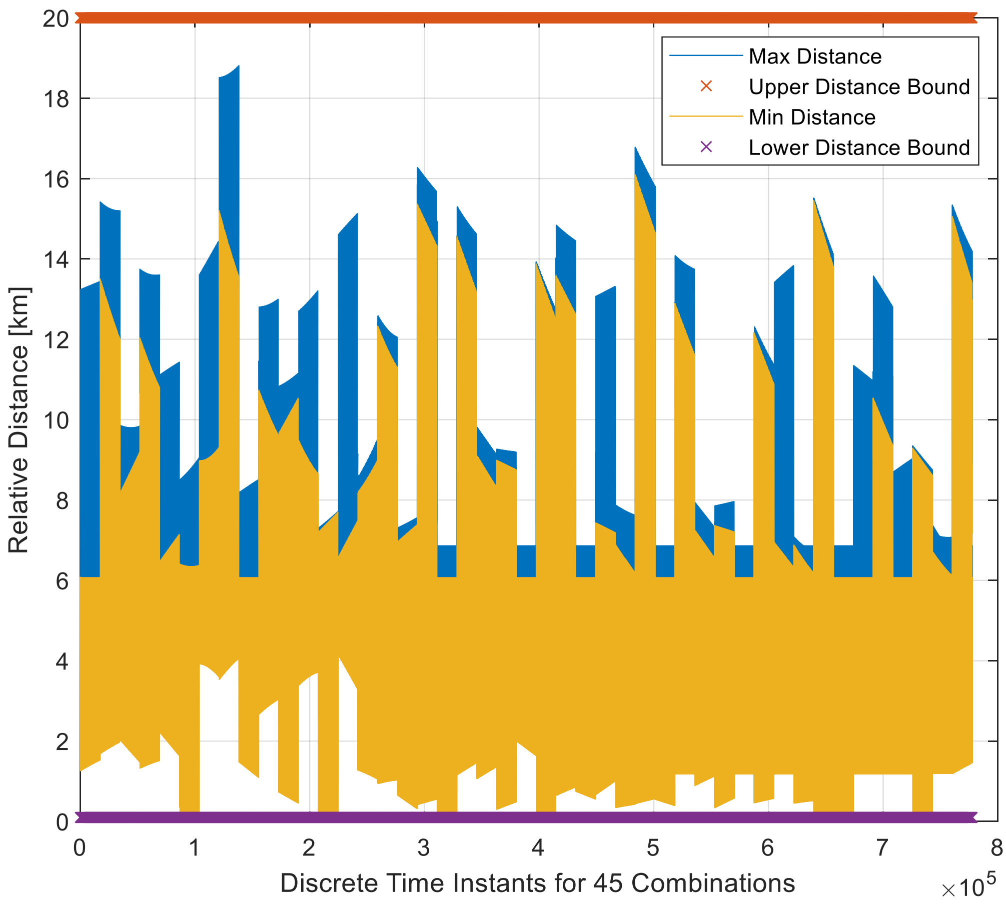

Figure 5: Cluster Flying Design Optimization 2 Day-Result for 10 Spacecraft with Station-Keeping Objective

From the Figure 5, it can be concluded that the 10 spacecraft configuration can be designed such that the minimum and maximum distances of 0.1 km and 20 km respectively are not violated over 2 days. Also, since the relative distances to the reference orbit are minimized, the station keeping objective can be considered as maximized.

4. Cluster Flying Design by Minimization of Collision Probability

When the objective is to maximize safety, the problem can be formulized as minimizing the Equation 6. However, since the probability of collision is associated with uncertainties, the process described in Figure 2 shall be utilized. The final problem and simulation parameters for a reference simulation for this approach can be summarized in Table 3 as follows:

Table 3: Formulization of Cluster Flying Design with Safety Objective

Parameter	**Specification**
Objective Function	Equation 6
Constraint Function(s)	Equation 4
Bounds on Design Variables	Equation 7
Propagator	Numerical with uncertainty propagation

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Inputs and Assumptions	Reference Orbit	Sun Synchronous, LTDN 10:30 at 680 km altitude
	Spacecraft Number	3
	d_{min} in RN Plane	0.1 km
	d_{max} in RTN Plane	100 km
	Physical Differences	Defined by differences in terms of mass, drag area and coefficient, solar radiation pressure area and coefficient
	Simulation Duration	2 days
	Step Size	10 sec.

The resulting optimized design solution for 3 spacecraft is simulated for 2 days and the distances between any two spacecraft during each time instant for 3 combinations is shown in the Figure 6.

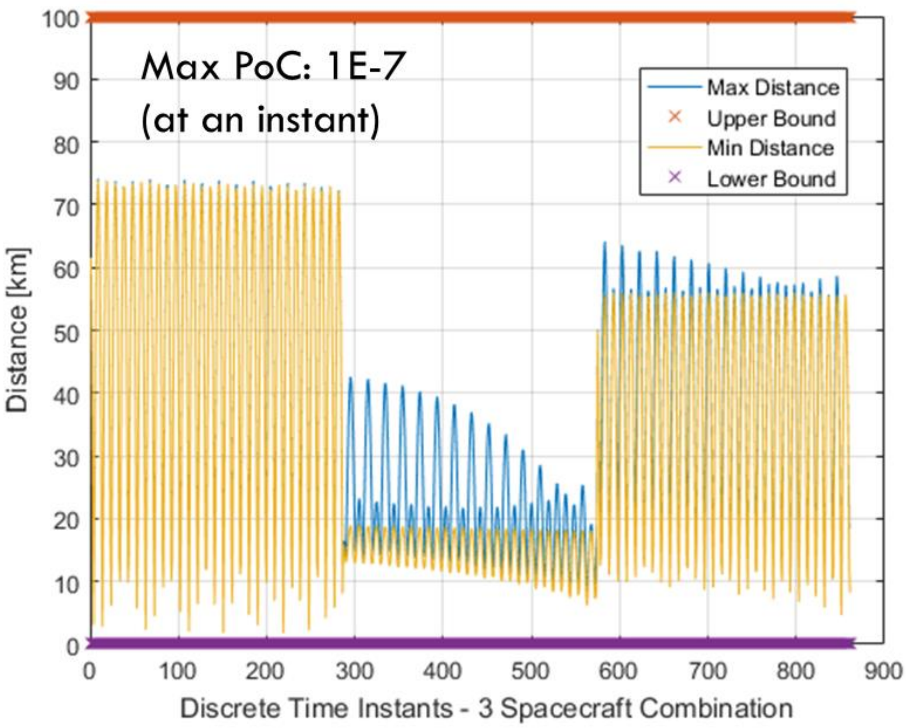

Figure 6: Cluster Flying Design Optimization 2 Day-Result for 3 Spacecraft with Safety Objective

From the Figure 6, it can be concluded that the 3 spacecraft configuration can be designed such that the minimum and maximum distances of 0.1 km and 100 km respectively are not violated over 2 days. Also, the maximum probability of collision between any 2 spacecraft over all time instants is calculated in the order of 10^{-7}. Since the probability of collision (PoC) values are minimized and the maximum encountered value is quite small, it can be considered that the safety is ensured and maximized for a spacecraft cluster of 3 for a long time interval which is in the orders of days.

5. Conclusions

In this paper, two approaches for mission design and/or operational analysis purposes are developed and summarized for passively safe relative orbit configurations of heterogeneously distributed spacecraft clusters over long time intervals incorporating realistic operational constraints. Firstly, relative orbit definition and propagation are explained. Secondly, uncertainty propagation and calculation of the probability of collision are presented. Then, constraints, objectives and design variables specific to the presented cluster flying problem are defined. Finally, design optimization framework is summarized and simulation results for station-keeping and safety objectives are shown. With the presented methodology, a number of spacecraft can be distributed into relative orbits with specific distance bounds based on specific interests which can be station-keeping and/or safety. Using the provided design optimization framework, several configurations can be achieved by exploring the limits of cluster size in terms of spacecraft number, maximum distance bounds and long time intervals.

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References

[1] Brown, O. and Eremenko, P. 2006. The Value Proposition for Fractionated Space Architectures. AIAA Space 2006. San Jose, CA: American Institute of Aeronautics & Astronautics. Paper No. AIAA–2006–7506.
[2] Yaglioglu, B. 2011. A Fractionated Spacecraft Architecture for Earth Observation Mission. Master Thesis, Lulea University of Technology, Department of Computer Science, Electrical and Space Engineering.
[3] Yaglioglu B., Wang J., 2011. Cluster Flying Configuration Evaluation in the Case of Fractionated Spacecraft Architecture. *6th Ankara International Aerospace Conference*.
[4] Simone D'Amico and Oliver Montenbruck. 2006. Proximity operations of formation-flying spacecraft using an eccentricity/inclination vector separation. *Journal of Guidance,Control, and Dynamics, 29(3):554–563*
[5] Hur-Diaz, S. and O'Connor, B. 2014. Cluster Flight Application on System F6. *24th International Symposium on Space Flight Dynamics*.
[6] Michael Tillerson, Gokhan Inalhan, and Jonathan P How. 2002. Coordination and control of distributed spacecraft systems using convex optimization techniques. *International Journal of robust and nonlinear control, 12(2-3):207–242*.
[7] Joseph B. Mueller, Paul R. Griesemer, and Stephanie J. Thomas. 2013. Avoidance Maneuver Planning Incorporating Station-Keeping Constraints and Automatic Relaxation. *Journal of Aerospace Information Systems, Vol. 10, No. 6 (2013), pp. 306-322.*
[8] Morgan D., Chung S., and Hadaegh F. Y. 2014. Model Predictive Control of Swarms of Spacecraft Using Sequential Convex Programming. *Journal of Guidance, Control, and Dynamics*. Vol. 37, No. 6, pp. 1725-1740.
[9] Edlerman, E. and Gurfil, P. 2019. Cluster-Keeping Algorithms for the Satellite Swarm Sensor Network Project. *Journal of Spacecraft and Rockets*. Vol. 56 pp. 649–663.
[10] Julier, S., Uhlmann., J., Whyte, H. 2000. A new method for the nonlinear transformation of means and covariances in filters and estimators. *IEEE Transactions on Automatic Control 45, pg 477-482*.
[11] Alfriend, K. T., Akella, M. R., Frisbee, J., Foster, J. L., Lee, D.-J., and Wilkins, M. 1999. Probability of Collision Error Analysis. *Space Debris, Vol. 1, No. 1, pp. 21–35, ISSN: 1388-3828*.
[12] Chan, K. F., 2001. Analytical Expressions for Computing Spacecraft Collision Probabilities. *11th AAS/AIAA Space Flight Mechanics Meeting, No. 1, American Astronamical Society, pp.305–320*.
[13] The Mathworks, Inc. Find minimum of constrained nonlinear multivariable function (fmincon). June 2019, web: https://www.mathworks.com/help/optim/ug/fmincon.html