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The Reattachment Process in Laminar Hypersonic Flow

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Abstract

The reattachment process in shock wave/boundary layer interaction situations is either viscous or inviscid dominated depending on the perturbation strength of $O(\mathrm{Re}^{-1/4})$. While the reattachment process is viscous dominated for small values of perturbation strength, for large scale separations, the process is inviscid and the separation region is dominated by significant transverse pressure gradients.

1. Introduction

In the wee known Chapman's isentropic re-compression theory,[3] the reattachment process is assumed essentially to be inviscid. While this assumption largely holds at high Reynolds number and moderate Mach number supersonic flows, its validity in high Mach number, low to moderate Reynolds number hypersonic flows is not immediately evident. Typically, in hypersonic flows, the reattachment process is characterised by a pressure over shoot, which results from the so called 'necking' a consequence of coalescence of separation and reattachment shock waves and some times a leading edge shock, forming a triple point close to the surface.

Daniels[4] developed a reattachment theory based on the triple-deck approach of Stewartson[16] and Neiland.[10] He shows that, while largely inviscid across a streamwise distance of $O(\mathrm{Re}^{-1/2})$, the reattachment is still viscous dominated in a sublayer of thickness $O(\mathrm{Re}^{-5/8})$, which can be described by boundary layer equations. The whole extent of the reattachment process is spread across a region of $O(\mathrm{Re}^{-3/8})$ with pressure rise of $O(\mathrm{Re}^{-1/4})$. Daniels' theory essentially assumes small separation, characterised without a pressure plateau, wherein the pressure rise is induced, for example, by a compression corner of angle $\alpha^* = O(\mathrm{Re}^{-1/4})$. For large separations, when $\alpha^* = O(1)$ and with a well developed plateau, Burggraf[2] has described a reattachment process based on asymptotic theory.

In the present paper, we will discuss the reattachment process in hypersonic large scale separated flow. In particular, it is shown that the evolution of the dividing streamline velocity is dependent on the Reynolds number as well as the way separation is initiated. It is also shown that the maximum dividing streamline velocity is less than the Chapman isentropic value and that the flow decelerates rapidly from maximum to zero at reattachment.

2. Analysis

2.1 Flow near reattachment

The reattachment process is characterised by the mass flow scavenged from the plateau pressure (constant pressure) region by the shear layer that is then turned back into the recirculation region as a result of the pressure rise at reattachment (Figure 1). Burggraf[2] has analysed this process by considering the salient terms of the Navier-Stokes equations pertaining to the shear layer and estimating their order of magnitude. He considered three cases depending on the ramp angle α^*.

The first one is the case $\alpha^* \leq O(\mathrm{Re}^{-1/4})$, wherein the separation region is small and there is no distinct plateau. Under these circumstances the reattachment process is entirely viscous dominated and the transverse pressure gradient $\partial p/\partial y \approx 0$ as in standard thin shear layer approximations.

In the second case, where $O(\mathrm{Re}^{-1/4}) \leq \alpha^* << 1$, the reattachment is largely inviscid but $\partial p/\partial y$ variation can still be considered insignificant. In such circumstances, there will be a short plateau and both separation and reattachment are interdependent and the velocities in the recirculation region are significantly low and the flow incompressible.

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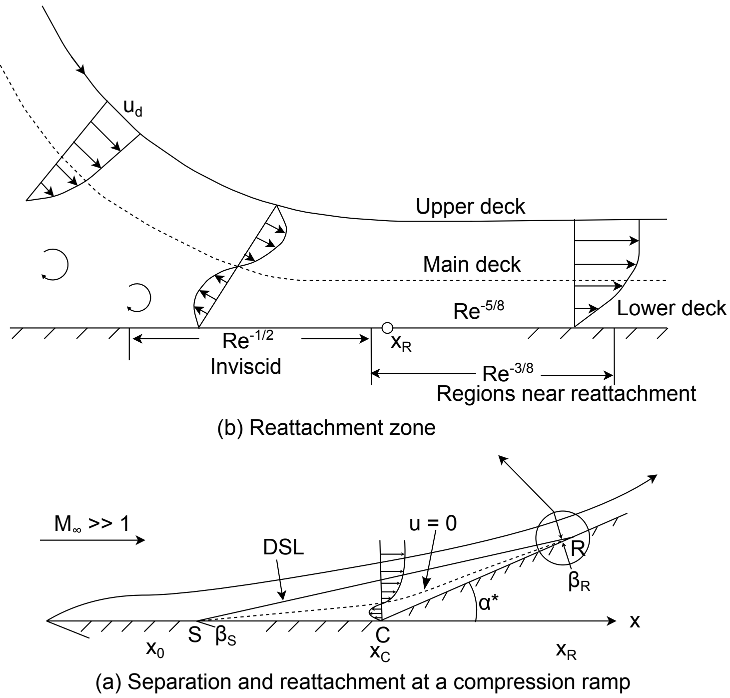

Figure 1: Separation and reattachment at a compression ramp

In the third case, we have $\alpha^* = O(1)$ and this implies a long and well developed plateau wherein separation and reattachment are independent of each other and the Neiland[10] and Burggraf[2] asymptotic theory applies. The transverse pressure gradient $\partial p/\partial y$ now becomes significant. The velocities in the recirculating region are high and the flow is compressible. The reattachment process is inviscid and Chapman's hypothesis becomes valid.

Following Burggraf,[2] the quantities in the shear layer entering the reattachment region scale as:

$$l_p \sim \alpha^{*^{3/2}} \; and \; l_R \sim (\alpha^* Re)^{-1/2} \tag{1}$$

Where l_p and l_R refer to lengths of plateau pressure and reattachment region respectively. The stream-wise and transverse pressure gradients are scaled as

$$\partial p/\partial x \sim \alpha^{*^{3/2}} Re^{1/2} \; and \; \partial p/\partial y \sim (\alpha^* Re)^{1/2} \tag{2}$$

We thus note that for small α (<<1), the transverse gradient vanishes and for $\alpha \sim O(1)$ or greater, it becomes significant. For large α with a long plateau, $l_p \sim O(1)$ and $\alpha^* \sim O(1)$, $l_R \sim O(Re^{-1/2})$.

The asymptotic calculation with the limit Reynolds number gives the long plateau length as (see Burggraf[2]):

$$l_p \sim 3.47(\alpha - 1.55)^{3/2} \tag{3}$$

This is valid for all $\alpha >> 1.55$, where α is the scaled angle as defined by Stewartson[16] and Rizzetta et al.[15] The validity of this relation is shown in Figure 2 with various experimental as well as numerical data. The notable feature of this relation is that it is independent of the wall temperature.

The reattachment region which is $O(\text{Re}^{-1/2})$, vanishes in the limit $\text{Re} \to \infty$. When this happens, the plateau length l_p and the separation bubble length l_B are the same. In fact, for high Reynolds number flows, the two lengths are used synonymously.

Burggraf[2] suggests that for a truly long plateau the boundary layer history before interaction becomes less relevant as separation and reattachment processes become independent of each other. A classic example of this is the leading edge separation that has been studied by our group for some time now (see Khraibut et al.;[7] Khraibut;[6] Prakash et al.;[13] Prakash et al.[14]). The configuration is shown in Figure 3. Here $l_R/l_p \sim 0.0245$, so that the Chapman's hypothesis that the reattachment is basically inviscid becomes valid.

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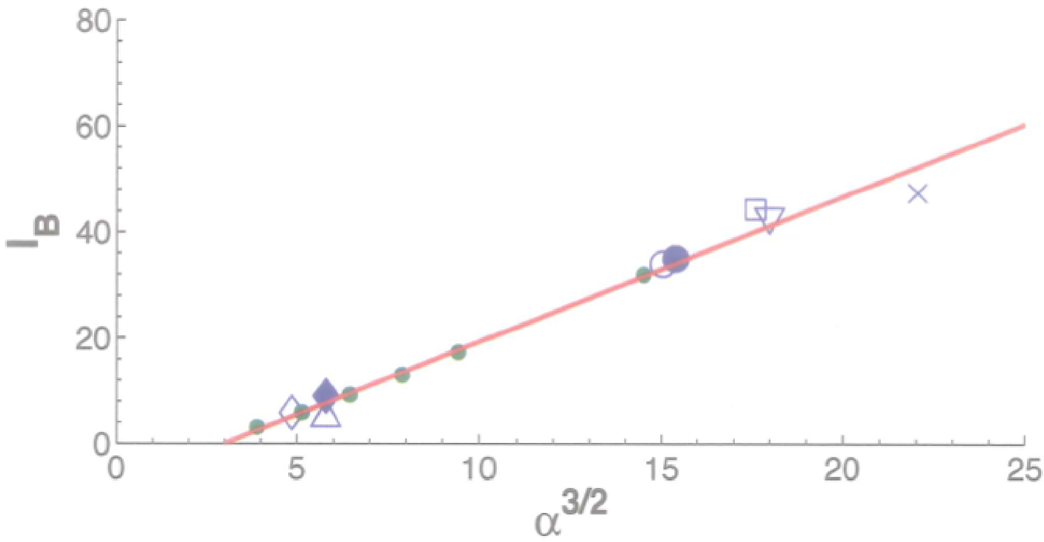

Figure 2: Variation of length of separation l_B with scaled angle α. ● : Compression corner data for angles 10° to 24° (T_w/T_0 = 0.125); ◇ : Katzer, 1989 (adiabatic); △ : Degrez et al., 1987 (adiabatic); ◆ : Rizzetta et al., 1978 (T_w/T_0 = 0.125); ○ & ● : Benay et al., 2006 (T_w/T_0 = 0.6 and 0.1, respectively); □ : Khraibut et al., 2017 (T_w/T_0 = 0.1); ▽ Chapman et al., 1958 (adiabatic); × : Swantek & Austin, 2012 ($T_w/T_0 \approx 0.04$)

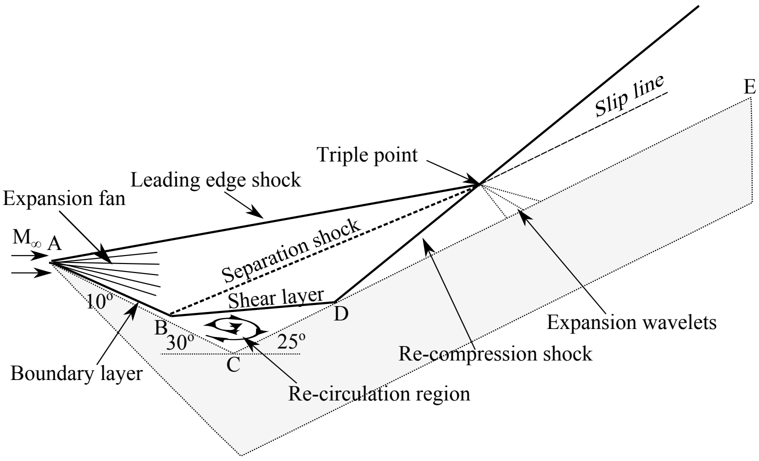

Figure 3: Flow schematic of a ‘tick’ configuration

2.2 Physical features of reattachment

From Oswatitsch’s relation,[11] the reattachment angle β_R is expressed as

$$tan\,\beta_R = 3\frac{d\tau_w/ds}{dp_w/ds} \tag{4}$$

where β_R is the angle that the dividing streamline makes with the surface, τ_w and p_w are the wall shear stress and pressure evaluated at reattachment. Figure 4 shows the streamline angle at reattachment with respect to the compression surface. It is seen that the reattachment angle β_R is shallower than the separation angle β_S. Figure 5 shows the pressure and skin friction coefficient at reattachment. The reattachment angle calculated using Oswatitsch relation gave 5.5° based on CFD calculations (Khraibut[6]) and 8.6° from DSMC calculations (Prakash et al.[13]).

It is seen that for large angle at separation, $\alpha^* \sim \beta_S$. Further, from Oswatitsch relation,

$$d\tau/ds >> dp/ds \tag{5}$$

This implies that for large separation angles, the slow rise in dp/ds is accompanied by an abrupt decrease in

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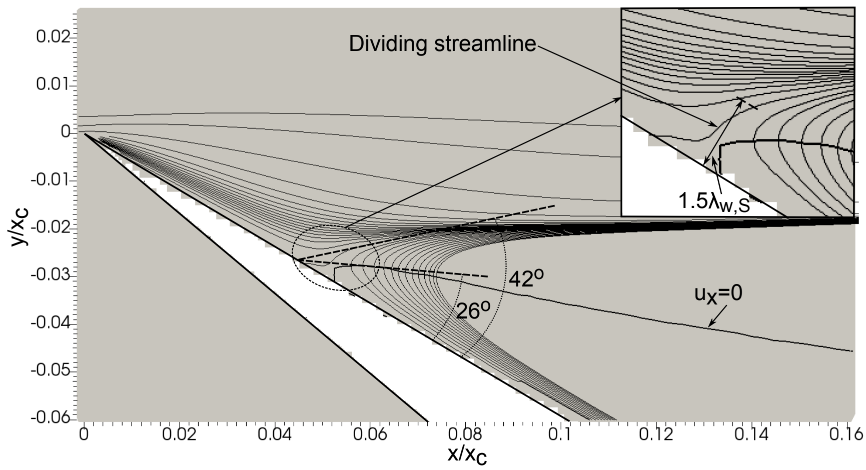
(a)



(b)

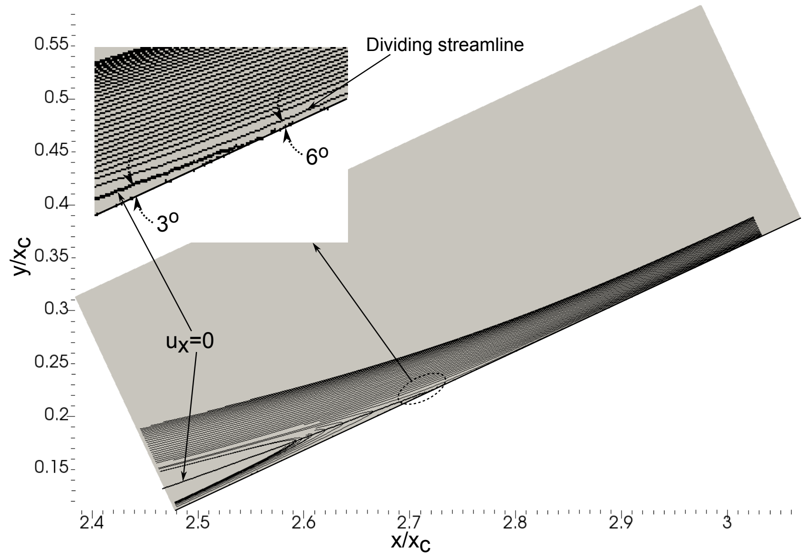


Figure 4: Dividing streamline angles: (a) Separation, (b) Reattachment.

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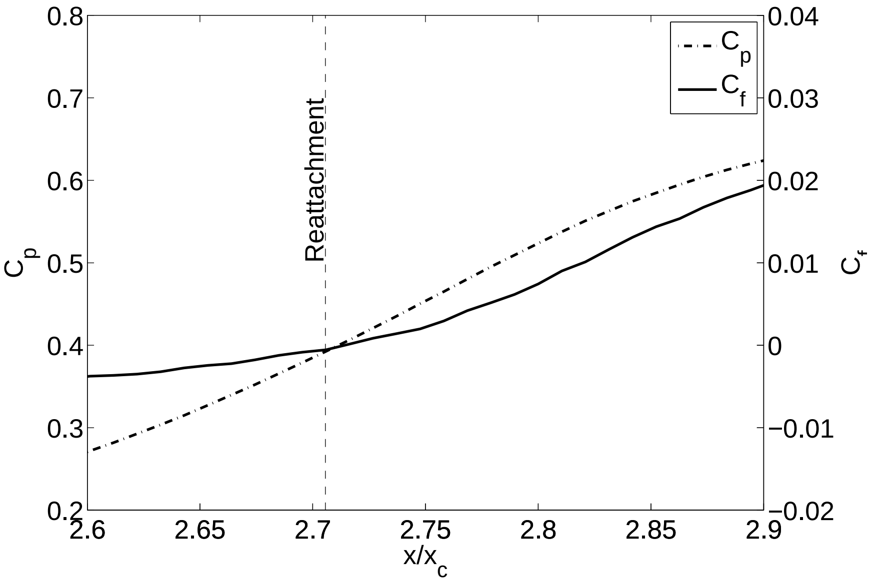

Figure 5: Pressure and skin friction coefficients around reattachment

$d\tau/ds$. On the other hand, at reattachment, there is an abrupt increase in pressure and the process is largely inviscid so that the reattachment angle is small.

2.3 The velocity on the dividing streamline

Another important aspect of reattachment process is the velocity variation along the dividing streamline (DSL) from separation to reattachment. Chapman's isentropic recompression theory assumes that the normalised dividing streamline velocity u_d^* is constant and equal to 0.587 and is independent of the Reynolds number. Subsequently, Baum and Denison[1] extended the Chapman model to include the existence of a Blasius boundary layer prior to separation and applied it to base flows. They showed that the velocity evolved gradually reaching the Chapman value asymptotically. On the other hand, base flow investigations of cylinder and blunt cone by Park[12] have shown that the dividing streamline velocity goes to zero at reattachment quite abruptly after reaching a peak value, which is considerably less than the Chapman's isentropic limit. Computations, both CFD and DSMC of leading edge separation configuration in hypersonic flow also show non-isentropic variation in dividing streamline velocity. Figure 6 shows the computed (DSMC) variation of the dividing streamline velocity. In this figure, S^* is a scaled distance along the dividing streamline normalised by the scaled distance along the surface from the leading edge to the point of separation as defined in Baum and Denison.[1] Here $S^* = 0$ indicates separation and $S^* \to \infty$ implies fully developed self-similar shear layer. Some interesting features may be noted. After initial slow rise, the velocity reaches a peak value of about 0.55 before rapidly falling to zero at $S^* \to 10^2$ which indicates reattachment. In comparison, the base flows data of Park[12] show smaller velocity peaks (~ 0.37) and smaller separation regions.

Reeves and Lees[8] in their study of separation and reattachment behind blunt bodies in hypersonic flow, point out that the presence of a surface and its radius of curvature in the vicinity of separation has a very significant influence on the evolution of the dividing streamline velocity u_d^*. This would explain the differences in u_d^* variation for the three configurations shown in Figure 6. While the axisymmetric blunt cone and circular cylinder are almost identical and both show strong influence of the pre-existing boundary layer, the leading edge separation geometry, which is a flat surface with very little fetch for boundary layer growth, shows a higher peak and a rather slower rise of u_d^* in the initial stages. The Reynolds number in all the three cases is of the same order (~ $3\text{x}10^4$) based on the characteristic length.

An estimate of the maximum value of the dividing streamline velocity u_d^* can be made following the analysis of Messiter et al.[9] of a separating shear layer behind a step in supersonic flow. They show that at the end of the shear layer prior to reattachment, the velocity on the dividing streamline is given by

$$(u_d^*)^2 \sim 2a_1^{4/3}k_o^{5/3}(x_f/L)^{2/3} \tag{6}$$

where a_1 is a shear constant (0.332 for a Blasius boundary layer), k_o is a positive constant, x_f is the length of the shear layer, and L is a characteristic length. In the case of the leading edge separation geometry, the characteristic

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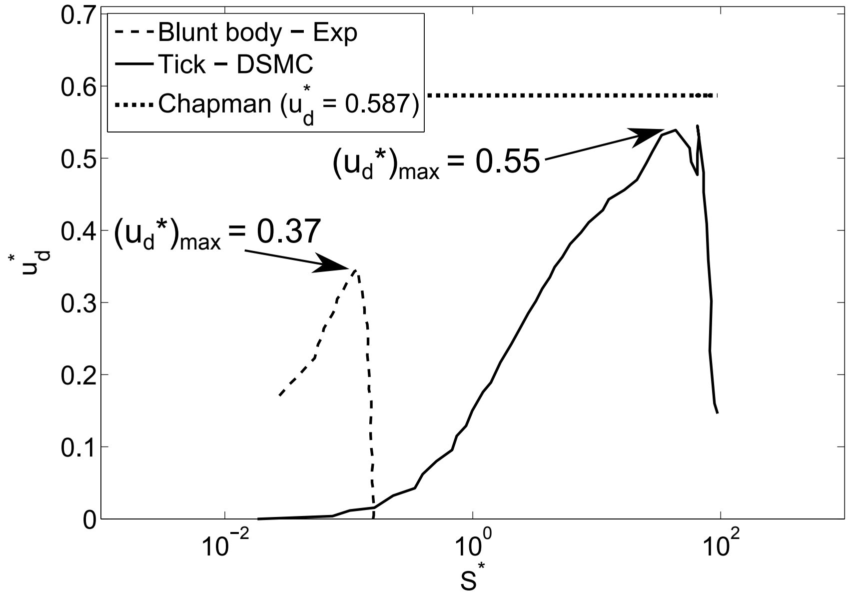

Figure 6: Variation of DSL velocity ratio

length is the expansion surface length, while for circular cylinder it is the cylinder diameter and for the blunt cone, the base diameter. Messiter et al.[9] give $0.6 \leq k_o \leq 0.73$ based on experimental and numerical data. For hypersonic flows, Gai[5] shows that $k_o \leq 0.655$. Using the appropriate values for the shear constant, the peak values of u_d^* for the leading edge separation is then estimated to be 0.53 which compares quite well with the computed value of 0.55. For the cylinder and blunt cone the corresponding estimate is 0.37, which again is in good agreement as seen from Figure 6.

3. Conclusions

Consideration is given to large-scale separation, wherein the plateau region is long and separation and reattachment can be treated independently. Using Burggraf's asymptotic analysis, the reattachment process is dependent on the perturbation strength α^*. When $\alpha^* = O(\mathrm{Re}^{-1/4})$, the reattachment process is viscous dominated and the transverse pressure gradient $\partial p/\partial y$ is negligible. With $O(\mathrm{Re}^{-1/4}) < \alpha^* < 1$, the reattachment process is largely inviscid and $\partial p/\partial y$ can still be assumed small. When $\alpha^* = O(1)$, however, the reattachment is an inviscid process and $\partial p/\partial y \neq 0$ and has to be accounted for. In the asymptotic limit of $\mathrm{Re} \to \infty$, the plateau length and separation length (the distance between separation point and the reattachment point) become of the same order and scale as $\alpha^{3/2}$. Available data, both experimental and numerical, verify this relation. Further, it is seen that this relation is independent of the wall temperature.

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