DOI: 10.13009/EUCASS2019-1052

8TH EUROPEAN CONFERENCE FOR AERONAUTICS AND AEROSPACE SCIENCES (EUCASS)

An Energy-Based Approach to Satellite Attitude Control in presence of disturbances

Mauricio Muñoz-Arias
Faculty of Science and Engineering
University of Groningen, Nijenborgh 4, 9747 AG,
Groningen, The Netherlands
m.munoz.arias@rug.nl

Abstract

The aim of this paper is to present a novel control strategy of the satellite attitude control problem on an energy-based setting. Controlling the orientation with respect to an inertial frame of reference becomes challenging in presence of nonlinear disturbances such as the gravity-gradient torque. We make use of the advantages of representing the system under study via the port-Hamiltonian framework due to its clear control design philosophy. The key strategies depend on position and velocity measurements, together with and integral action of the system's output. Finally, numerical simulation results are provided to demonstrate effectiveness.

1. Introduction

The rigid-body attitude control problem is inspired by aerospace systems such as atmospheric flight, spacecraft, underwater and ground vehicles, together with robotic systems, which includes attitude maneuvers and attitude stabilization, [4, 13]. Furthermore, Euler's equations of motion describe the dynamics of a rigid-body, and then attitude kinematic stabilization becomes a requirement. More specifically, attitude control of a satellite is done via attitude parametrizations due to the fact that the set of attitudes is not a Euclidean space (see [15], and [17] and the references therein). In addition to this, it is well-known that satellite systems are affected by external torques such as gravity gradient [14], solar pressures and norm-based disturbances [3], and the J_2 effect [2].

The port-Hamiltonian framework (PH) is an energy-based approach which describes (physical) systems in terms of power ports, energy variables, and their interconnection structure, [8, 18, 19]. The transfer of energy between the physical system and the environment is given via dissipation and energy elements, together with power preserving ports. The PH method has the additional advantage of preserving the PH structure for the closed-loop system.

Recently, a PH formulation of the rigid-body attitude problem that enhances the set of tools for its modeling and control is presented in [9], and [10]. More specifically, in [9] a novel approach on both dynamics and kinematics equations is provided such that a standard energy-balancing passivity-based controller (PBC) is used for set-point control. In addition to the PBC controller, a variation of the controller designed by [6] is given. Its mayor advantage is the achievement of the set-point without velocity measurements. Nevertheless, the controller proposed by [9] becomes ineffective when nonlinear disturbances are not neglected. The disturbances are considered as external forces affecting satellite attitude control.

Here, we have proposed a novel controller inspired by [5] and [9], by which a desired attitude kinematics and a attitude dynamics configuration of a satellite system is attained. Simulation results show how an integral action via an adapted momenta obtains asymptotic stability in presence of nonlinear disturbances, i.e. a type of gravity-gradient external torque.

The paper is structured as follows. Section 2 introduces the port-Hamiltonian framework, and later a class of standard mechanical systems for a satellite is introduced as in [9]. In Section 3, a novel control strategy based on the advantages of the port-Hamiltonian setting is given where the system's structure is preserved. Furthermore, in Section 4 simulation results are presented in order to demonstrate the effectiveness of our energy-based approach. Finally, Section 5 provides our main concluding remarks.

DOI: 10.13009/EUCASS2019-1052

1.1 Notation

The “cross” map $(\cdot)^{\times} : \mathbb{R}^3 \rightarrow so(3)$ is defined as

$$v^{\times} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}^{\times} := \begin{bmatrix} 0 & -v_3 & v_2 \\ v_3 & 0 & -v_1 \\ -v_2 & v_1 & 0 \end{bmatrix} \tag{1}$$

while the “vee” map $(\cdot)^{\vee} : so(3)$ denotes the inverse operation of “cross”, namely

$$(a^{\times})^{\vee} = a \tag{2}$$

Furthermore, the gradient of a scalar vector is given by

$$\nabla_x := \frac{\delta}{\delta x}. \tag{3}$$

All vectors are considered as column vectors, and $\mathrm{tr}(A)$ is the trace of the matrix $A \in \mathbb{R}^{n \times n}$.

2. port-Hamiltonian framework

In this section, we present the port-Hamiltonian (PH) formalism for a general class of physical systems, and later we present a formulation for the attitude control dynamics.

We apply the results of [6] in order to reinforced the proposal proposal of [9] in front of nonlinear disturbances.

The PH framework is based on the description of systems in terms of energy variables, their interconnection structure, and power port pairs.

PH systems include a large family of physical nonlinear systems which includes the dynamics of satellites. The transfer of energy between the physical system and the environment is given through energy elements, dissipation elements and power preserving ports [8, 18, 19].

A time-invariant PH system corresponds to the

$$\Sigma \begin{cases} \dot{x} = [\mathcal{J}(x) - \mathcal{R}(x)] \nabla_x H(x) + g(x) u, \\ y = g(x)^{\top} \nabla_x H(x), \end{cases} \tag{4}$$

where the state variable is given by $x \in \mathcal{R}^{\mathcal{N}}$, and the input-output port-pair representing flows and efforts are given by

$$u \in \mathcal{R}^{\mathcal{N}}, \tag{5}$$

$$y \in \mathcal{R}^{\mathcal{M}}, \tag{6}$$

respectively. Furthermore, the matrices input, interconnection and dissipation matrices of (4) are given by

$$g(x) \in \mathcal{R}^{\mathcal{N} \times \mathcal{M}}, \tag{7}$$

$$\mathcal{J}(x) = -\mathcal{J}(x)^{\top}, \; \mathcal{J}(x) \in \mathbb{R}^{\mathcal{N} \times \mathcal{N}}, \tag{8}$$

$$\mathcal{R}(x) = \mathcal{R}(x)^{\top} \geq 0, \; \mathcal{R}(x) \in \mathbb{R}^{\mathcal{N} \times \mathcal{N}}, \tag{9}$$

where $\mathcal{M} \leq \mathcal{N}$ being $\mathcal{M} = \mathcal{N}$ a fully actuated system, and $\mathcal{M} < \mathcal{N}$ an underactuated one. Furthermore, the energy function of system (4) is

$$H(x) \in \mathbb{R}. \tag{10}$$

Differentiating the Hamiltonian along the trajectories of $\dot{x}$, we recover the energy balance

$$\dot{H}(x) = -\nabla_x^{\top} H(x) \mathcal{R}(x) \nabla_x H(x) + y^{\top} u \leq y^{\top} u \tag{11}$$

where we clearly see how we consider the system (4) conservative.

DOI: 10.13009/EUCASS2019-1052

2.1 port-Hamiltonian formulation of satellite (rigid body)

Given a rigid-body in space (satellite), we define its inner energy (Hamiltonian) function as

$$H(q,p) := \frac{1}{2} p^\top \mathcal{I}^{-1} p \tag{12}$$

with $x = \mathrm{col}(q,p)$ being the state variable that depends on the (generalized) position $q \in \mathbb{R}^3$, and generalized momenta $p \in \mathbb{R}^3$. Furthermore, the matrix $\mathcal{I} := \mathrm{diag}\left(I_x, I_y, I_z\right)$ is the (principal) inertia matrix. Also, $p := \mathcal{I}\omega$ being $\omega \in \mathbb{R}^3$ the angular velocity vector. The dynamics of p is then given by

$$\dot{p} = p^\times \nabla_p H(p) + u \tag{13}$$

with $u \in \mathbb{R}^3$ being the applied control torques to the rigid body (satellite). Based on (12) and (13), we obtain the following PH formulation

$$\Sigma_S \begin{cases} \begin{bmatrix} \dot{q} \\ \dot{p} \end{bmatrix} = \begin{bmatrix} 0_{9\times3} & r(q) \\ -r(q)^\top & p^\times \end{bmatrix} \begin{bmatrix} \nabla_q H(q,p) \\ \nabla_p H(q,p) \end{bmatrix} + \begin{bmatrix} 0_{9\times3} \\ G(q) \end{bmatrix} u \\ \\ y = G(q)^\top \nabla_p H(q,p) = G(q)^\top \omega \end{cases} \tag{14}$$

where the dissipation matrix is assume zero, i.e. $\mathcal{R}(q,p) = 0$, $G(q) \in \mathbb{R}^{3\times3}$ being the input matrix, and $r(q) : \mathbb{R}^9 \to \mathbb{R}^{9\times3}$ is computed as

$$r(q) := \begin{bmatrix} R_x^\times \\ R_y^\times \\ R_z^\times \end{bmatrix} \tag{15}$$

with

$$q := \mathrm{vec}\left\{R^\top\right\} = \begin{bmatrix} R_x & R_y & R_z \end{bmatrix}. \tag{16}$$

Notice from (14) that the dynamics of the generalized position q is given by

$$\dot{q} = r(q)\, p \tag{17}$$

with $r(q)$ as in (15). In [9], it is shown the full derivation of (14) with the matrix $r(q)$ as in (15), and the vector of position coordinates q as in (16).

In the follow up, we present the proposed control law to attain a desired attitude of a system in presence of nonlinear disturbances.

2.2 Definition of the desired attitude configuration

Inspired by [6] and [7], we make use of an *adapted momenta* strategy where information about the position is added to the momenta coordinate without loosing the system's structure, and at the same time attaining asymptotic stability for control purposes.

First, we define the desired attitude as

$$R_{\mathrm{ref}} := \begin{bmatrix} R_{\mathrm{ref},x} \\ R_{\mathrm{ref},y} \\ R_{\mathrm{ref},z} \end{bmatrix}, \tag{18}$$

where where $R_{\mathrm{ref},x}$, $R_{\mathrm{ref},y}$, and $R_{\mathrm{ref},z}$, denote the first, second and third, row of R_{ref}, respectively, then we make use of the energy candidate function

$$H_{\mathrm{ref}}(q) := \frac{1}{2} \mathrm{tr}\left[K_p \left(I - R_{\mathrm{ref}}^\top R(q)\right)\right] \tag{19}$$

with $K_p = \mathrm{diag}\left(k_{px}, k_{py}, k_{pz}\right) \succ 0$, and $I \in \mathbb{R}^{3\times3}$ an identity matrix which [1] has preliminary suggested. Then, the new Hamiltonian is proposed as

$$H_d(q,p) = H(q,p) + H_{\mathrm{ref}}(q), \tag{20}$$

and, inspired by [9], (an auxiliary) matrix is also proposed as

$$R_{\mathrm{aux}}(q) := K_p R_{\mathrm{ref}}^\top R(q) - R(q)^\top R_{\mathrm{ref}} K_p^\top \tag{21}$$

DOI: 10.13009/EUCASS2019-1052

such a the desired configuration $q_d \in \mathbb{R}^9$ is given by

$$q_d = r(q)^\top \nabla_q H_{\text{ref}}(q) = \frac{1}{2} R_{aux}(q)^\top = \begin{bmatrix} -R_{\text{aux},2,3}(q) \\ R_{\text{aux},1,3}(q) \\ -R_{\text{aux},1,2}(q) \end{bmatrix} \tag{22}$$

with $R_{aux}(q)$ as in (21). Notice that the new generalized coordinate $\bar{q} \in \mathbb{R}^n$ represents the error given by the reference matrix R_{ref} in (18) or, in other words, the desired attitude configuration.

3. A novel control strategy

Once we have define the error between the current and desired attitude configuration, i.e by (22), then we introduce the adapted momenta as

$$\bar{p} = p + K_p \bar{q} \tag{23}$$

where $\bar{p} \in \mathbb{R}^n$, with $n > 0$ constant, positive matrix $K_p > 0$, and desired configuration $\bar{q} = q - q_d$, with q as in (17), q_d to be defined later on. Then, the resulting output of the new PH systems is

$$\bar{y} = \bar{p}. \tag{24}$$

that contrapositioned with the original output y of (4), we see how the position becomes also relevant for control purposes.

Here, we also introduce an extended dynamics $z \in \mathbb{R}^n$ with an integral action on the output, i.e.

$$\dot{z} = -K_i \hat{y} \tag{25}$$

where a tuning matrix $K_i \in \mathbb{R}^{3\times 3}$, such that $K_i > 0$.

Based on the new output $\bar{y}$, and the extended dynamics z as in (25), we proposed the following control law.

Theorem 1 *Given a satellite dynamics represented by* (14) *with the generalized coordinate q and the generalized momenta p, we obtain asymptotic stability at the a desired configuration q_d as in* (22) *with the torque input vector u as*

$$u = -\frac{1}{2}\bar{q} - K_p \bar{y} + z \tag{26}$$

with a positive constant matrix $K_p > 0$.

Proof: Clearly, from (26), the adapted momenta $\bar{p}$ as in (23), the new output $\bar{y}$ as in (24) that depends on the position q and speed $\dot{q}$, together with an integral action on the dynamics of z as in (25), we obtain the closed loop system

$$\Sigma_{CL} \left\{ \begin{bmatrix} \dot{\bar{q}} \\ \dot{\bar{p}} \\ \dot{z} \end{bmatrix} = \begin{bmatrix} -\mathcal{I}^{-1} & r(\bar{q}) & 0_{9\times 3} \\ -r(\bar{q})^\top & -K_d & K_i \\ 0_{3\times 9} & -K_i & 0_{3\times 3} \end{bmatrix} \begin{bmatrix} \nabla_{\bar{q}} \bar{H}(\bar{q},\bar{p},z) \\ \nabla_{\bar{p}} \bar{H}(\bar{q},\bar{p},z) \\ \nabla_z \bar{H}(\bar{q},\bar{p},z) \end{bmatrix} \right. \tag{27}$$

with a Lyapunov Candidate function $\bar{H}(\bar{q},\bar{p},z)$ given by

$$\bar{H}(\bar{q},\bar{p},z) = \frac{1}{2}\bar{p}^\top \bar{p} + \frac{1}{2}\text{tr}\left[K_p\left(I - R_{\text{ref}}^\top R(\bar{q})\right)\right] + \frac{1}{2} z^\top K_i^{-1} z \tag{28}$$

First, we see how $\bar{H}(\bar{q},\bar{p},z) \geq 0$, and if we evaluate $\dot{\bar{H}}$ along the trajectories of (27), we obtain that $\dot{\bar{H}}(\bar{q},\bar{p},z) \leq 0$. Finally, since in R_{ref}, $\bar{H}$ has a minimum, then via the Lyapunov Stability theory [16], we conclude that the system (27) has a equilibrium point in $(\bar{q},\bar{p},z) = (q_d, 0, 0)$. ■

Theorem 1 shows how we can attain asymptotic stability on a desired attitude configuration R_{ref} given by (18). Even though, it depends on position and velocity measurements, the PH framework ensures robustness in presence of parameters uncertainties and disturbances, as demonstrated in [7, 18, 19]. In the next section, we incorporate such nonlinear disturbances to the system (14) and demonstrate the relevance our approach via numerical simulations.

DOI: 10.13009/EUCASS2019-1052

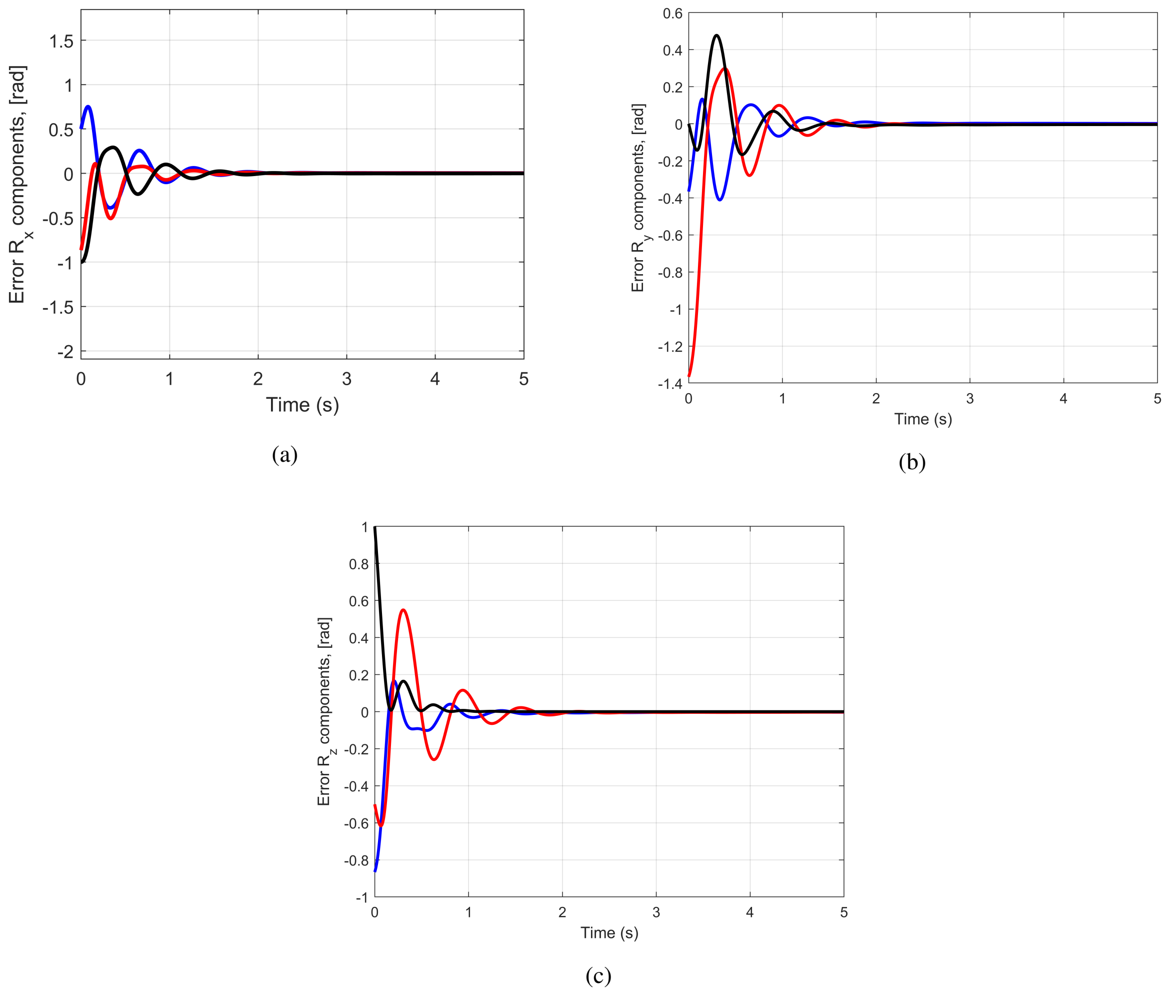

Figure 1: Simulation results of the attitude configuration for the system (14) with control law (26), and nonlinear disturbances (30). In a), b) and c), we have the attitude error for the rows $R_{\text{ref},x}$, $R_{\text{ref},y}$, and $R_{\text{ref},z}$, respectively. The solid lines in blue, red and and black represent the attitude error getting stabilized to zero, among the three elements of each vector row of R_{ref} as in (18) and the three elements of each row matrix of $R_{\text{ref},d}$ as in (29). Asymptotic stability is obtained after $t = 3$ s.

DOI: 10.13009/EUCASS2019-1052

4. Simulation results

Given a satellite system as in (14), and a desired attitude configuration presented as

$$R_{\text{ref,d}} = \begin{bmatrix} R_{\text{ref},x,d} \\ R_{\text{ref},y,d} \\ R_{\text{ref},z,d} \end{bmatrix} = \begin{bmatrix} 0 & 0 & -1 \\ \cos\left(\frac{\pi}{3}\right) & -\sin\left(\frac{\pi}{3}\right) & 0 \\ -\sin\left(\frac{\pi}{3}\right) & -\cos\left(\frac{\pi}{3}\right) & 0 \end{bmatrix}, \tag{29}$$

where $R_{\text{ref}} R_{\text{ref}}^\top = I_{3\times3}$. Given also initial conditions for the generalized momenta, i.e. $p(0) = \text{col}(-5.0, 4.0, -3.0)$, and the inertial matrix of the system $\mathcal{I} = \text{diag}(1.0, 0.8, 1.0)$. It follows that the controller parameters are $K_i = \frac{1}{2} I_{3\times3}$, $K_p = 2I_{3\times3}$, and $K_d = I_{3\times3}$ which can always be fine tuned to achieve different performances depending on the desired transient response. We apply then the control law (26) to the system (14) in order to achieve the desired configuration (29). Finally, a nonlinear disturbance in the system's input is presented as a gravity-gradient external torque, similarly to [20], is given by

$$\tau_{\text{ref,d}} = \begin{bmatrix} d_x \sin t \cos t \\ d_y \sin t \cos t \\ d_z \sin t \cos t \end{bmatrix} \tag{30}$$

with constants $\text{col}\left(d_x, d_y, d_z\right) = \text{col}(0.1, 0.1, 0.1)\ Nm$. Figure 1 shows the simulation results. Clearly, we can see how there is a transient response of $t = 2\ s$, and finally the system is stabilized at $t \geq 3\ s$. Robustness is present in front of nonlinear disturbances which results from the model-based strategy together with the integral action proposed in (25).

5. Concluding remarks and future work

An novel energy-based control strategy is here presented in order to tackle the attitude control problem for a satellite system. The port-Hamiltonian framework allows the design and implementation of a controller based on the concept of adapted momenta and a simple integral action on the system's output. Robustness in presence of nonlinear disturbances is a key advantage of the presented strategy due to model-based control design. Shaping the energy (Hamiltonian) function via a Lyapunov Candidate function is a fundamental step towards the regulation problem, i.e attaining a desired and constant attitude configuration of the spacecraft. The paper presents only disturbance attenuation in presence of the gravity-gradient external torque. Further types of external torques such as magnetic, aerodynamic, solar radiation pressure, and mass-expulsion will be considered as a potential extensions to the current study. In addition to this, a comparison with classical and novel control strategies is another topic of ongoing work. Also, the design of a trajectory tracking control instead of the regulation problem is certainly a path we would like to follow. Finally, an experimental setup in space would be the ideal case to compare our simulation results with a real orbiting system. Low cost cub-sat missions such as [11] and [12] are of interest due to their accessibility.

6. Acknowledgments

We would like to thank the Design Group of the Faculty of Science and Engineering, specially to dr. Gerald Jonker and prof. dr. R.G.E. (Rob) Timmermans for the support to participate in the EUCASS 2019 conference and present the current results.

References

[1] Francesco Bullo and Andrew D Lewis. *Geometric control of mechanical systems: modeling, analysis, and design for simple mechanical control systems*, volume 49. Springer Science & Business Media, 2004.

[2] Lu Cao and Arun K Misra. Linearized j_2 and atmospheric drag model for satellite relative motion with small eccentricity. *Proceedings of the Institution of Mechanical Engineers, Part G: Journal of Aerospace Engineering*, 229(14):2718–2736, 2015.

[3] Songyin Cao and Yunfeng Zhao. Anti-disturbance fault-tolerant attitude control for satellites subject to multiple disturbances and actuator saturation. *Nonlinear Dynamics*, 89(4):2657–2667, 2017.

[4] Nalin A Chaturvedi, Amit K Sanyal, and N Harris McClamroch. Rigid-body attitude control. *IEEE control systems magazine*, 31(3):30–51, 2011.

DOI: 10.13009/EUCASS2019-1052

[5] Daniel Dirksz and Jacquelien MA Scherpen. Port-Hamiltonian and power-based integral type control of a manipulator system. *IFAC Proceedings Volumes*, 44(1):13450–13455, 2011.

[6] Daniel Dirksz, Jacquelien MA Scherpen, and Romeo Ortega. Interconnection and damping assignment passivity-based control for port-Hamiltonian mechanical systems with only position measurements. In *2008 47th IEEE Conference on Decision and Control*, pages 4957–4962. IEEE, 2008.

[7] Alejandro Donaire and Sergio Junco. On the addition of integral action to port-controlled hamiltonian systems. *Automatica*, 45(8):1910–1916, 2009.

[8] Vincent Duindam, Alessandro Macchelli, Stefano Stramigioli, and Herman Bruyninckx. *Modeling and Control of Complex Physical Systems: The Port-Hamiltonian Approach*. Springer, Berlin, Germany, 2009.

[9] Paolo Forni, Dimitri Jeltsema, and Gabriel AD Lopes. port-Hamiltonian formulation of rigid-body attitude control. *IFAC-PapersOnLine*, 48(13):164–169, 2015.

[10] Kenji Fujimoto, Tomoya Takeuchi, and Yuki Matsumoto. On port-Hamiltonian modeling and control of quaternion systems. *IFAC-PapersOnLine*, 48(13):39–44, 2015.

[11] Marco Gómez-Jenkins, Julio Calvo-Alvarado, Ana Julieta-Calvo, Adolfo Chaves-Jiménez, Johan Carvajal-Godínez, Alfredo Valverde-Salazar, Julio Ramirez-Molina, Carlos Alvarado-Briceño, and Arys Carrasquilla-Batista. Irazú: Cubesat mission architecture and development. 2016.

[12] Jian Guo, Jasper Bouwmeester, and Eberhard Gill. From single to formation flying cubesats: An update from the delft programme. 27th Annual AAIA-USU Conference on Small Satellites, 2013.

[13] Taeyoung Lee. Exponential stability of an attitude tracking control system on SO (3) for large-angle rotational maneuvers. *Systems & Control Letters*, 61(1):231–237, 2012.

[14] Marco Lovera and Alessandro Astolfi. Global magnetic attitude control of spacecraft in the presence of gravity gradient. *IEEE transactions on aerospace and electronic systems*, 42(3):796–805, 2006.

[15] F Landis Markley and John L Crassidis. *Fundamentals of spacecraft attitude determination and control*, volume 33. Springer, 2014.

[16] Romeo Ortega, Julio Antonio Loría Perez, Per Johan Nicklasson, and Hebertt J Sira-Ramirez. *Passivity-based control of Euler-Lagrange systems: mechanical, electrical and electromechanical applications*. Springer Science & Business Media, 2013.

[17] Malcolm D Shuster. A survey of attitude representations. *Navigation*, 8(9):439–517, 1993.

[18] Arjan van der Schaft. *L_2-Gain and Passivity Techniques in Nonlinear Control*. Springer, London, UK, 2000.

[19] Arjan van der Schaft and Dimitri Jeltsema. Port-hamiltonian systems theory: An introductory overview. *Foundations and Trends® in Systems and Control*, 1(2-3):173–378, 2014.

[20] Bing Xiao, Shen Yin, and Ligang Wu. A structure simple controller for satellite attitude tracking maneuver. *IEEE Transactions on Industrial Electronics*, 64(2):1436–1446, 2016.