Planetary Rover Navigation via Visual Odometry: Performance Improvement Using Additional Image Processing and Multi-sensor Integration

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Abstract

Visual Odometry is very important for a mobile robot, above all in a planetary scenario, in such a way to accurately estimate the rover occurred motion. The present work deals with the possibility to improve a previously developed Visual Odometry technique by means of additional image processing, together with suitable mechanisms such as the classical Extended/Iterated Kalman Filtering and also Sequence Estimators. The possible employment of both techniques is then addressed and consequently a better behaving integration scheme is proposed. Moreover, the eventuality of exploiting other localization sensors is also investigated, leading to a final multi-sensor scheme.

1. Introduction

This work preliminarily quotes a Visual Odometry technique, previously developed at GRAAL Lab, for the estimation of the relative poses assumed by a moving vehicle at discrete instants of time (for details about the developed VO algorithm refer to [1], [2], [3] and [4]). Then it approaches the problem of improving the VO-provided estimation sequences: the suggested method reasonably plans to add to two successive acquisitions a further one linking the two; moreover suitable mechanisms allowing a sensible reduction of the drifting errors, generally occurring when the absolute poses are deduced via summation of the relative ones, can be devised. More specifically, to the aim of such drifting reduction, state estimation techniques (Extended Kalman Filtering and/or Iterated Kalman Filtering) are formerly suggested. Then, as an alternative to state estimation, the better technique of sequence estimation, even if computationally a little more cumbersome, is also suggested and tailored to the problem. Moreover the possibility of suitably integrating the two techniques (state estimation and sequence estimation) into an overall scheme, better behaving than when using the same technique separately, is analyzed and then suggested to be adopted. Finally the possible integration of other sensors (namely Inertial management Units, or IMU, and Wheel Odometry, or WO, systems) is also briefly investigated within the definition of an overall resulting, further improved, multi-sensor integration scheme.

2. Visual Odometry based Additional Image Processing

The Visual Odometry technique, once per-se considered, can be seen as nothing more than a sensing system capable of progressively constructing, generally in the form of *assumed independent* estimates, the chained set of frames of Fig. 1; each frame is associated to a point located on a realization of a path traveled by a vehicle, without actually keeping into account anything about its specific motion characteristics. In Fig. 1, each couple ${}^{i-1}r_i, {}^{i-1}\theta_i$ (relative position and orientation) represents the true realized motion, completely defining the relative position of frame $< i >$ with respect to its antecedent $< i-1 >$; concerning this, the VO technique progressively provides their estimates, that is the couple of quantities ${}^{i-1}\hat{r}_i, {}^{i-1}\hat{\theta}_i$, characterized by the associated variances ${}^{i-1}\Sigma_i, {}^{i-1}\Theta_i$ respectively, also provided by the VO technique itself. Note that just for sake of simplicity we have assumed the two estimates ${}^{i-1}\hat{r}_i, {}^{i-1}\hat{\theta}_i$ as uncorrelated among them (no cross-variance terms have been actually evidenced). As we shall see in the sequel, this assumption will however generally reveal without loss of generality. Moreover also note that, due to the specific way the Visual Odometry actually works, each couple of estimates ${}^{i-1}\hat{r}_i, {}^{i-1}\hat{\theta}_i$ can be considered as independent from all the others;

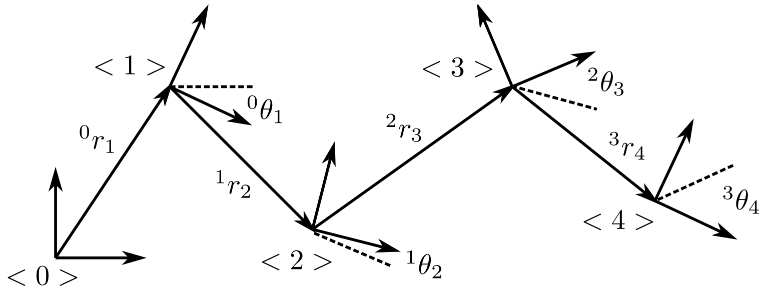

Figure 1: Sequence of positions-orientations attained by the vehicle

this fact allows to write the following relationships

$$^{i-1}r_i = \,^{i-1}\hat{r}_i + \,^{i-1}\varepsilon_i \; ; \; ^{i-1}\varepsilon_i = N\left(^{i-1}\varepsilon_i, 0, \,^{i-1}\Sigma_i\right) \tag{1a}$$

$$^{i-1}\theta_i = \,^{i-1}\hat{\theta}_i + \,^{i-1}\eta_i \; ; \; ^{i-1}\eta_i = N\left(^{i-1}\eta_i, 0, \,^{i-1}\Theta_i\right) \tag{1b}$$

with ε_i, η_i independent (among them as above assumed) white sequences; and further reasonably considered Gaussian with zero mean as a working assumption. Keeping into account the above considerations, we can immediately see how the VO technique can provide estimates of the *absolute* positions and orientations progressively attained by the vehicle, only in the form (i.e. the classical product of, in this case, planar transformation matrices of each frame with respect to its antecedent)

$$^0\hat{T}_i = \,^0\hat{T}_1 \,^1\hat{T}_2 \cdots \,^{i-1}\hat{T}_i \tag{2a}$$

with

$$^{i-1}\hat{T}_i \doteq \left[\begin{array}{c|c} ^{i-1}\hat{R}_i & ^{i-1}\hat{r}_i \\ \hline 0 & 1 \end{array}\right] \quad ; \quad ^{i-1}\hat{R}_i \doteq R\left(^{i-1}\hat{\theta}_i\right) \tag{2b}$$

which is clearly prone to an error progressively increasing with the number of stages. As a matter of fact such evidenced drawback simply arises from the fact that the VO algorithm, as above applied, leads per-se to a linear open chain of frames, with independent positioning of each one with respect to its antecedent, without any further constraint. Hence, in the following we shall primarily investigate about the possibility of improving this technique, by forcing it to provide further measurements, introducing some (probabilistic) constraints among the basic chained frames, in order to improve the vehicle absolute localization. To this aim, a feasible possibility is that of adding to the basic measurements the estimation of frame $< i >$ position also with respect to frame $< i-2 >$; in this way, it is possible to refer to the progressively growing graph of frames (now including closed loops connections) reported in Fig. 2, where only the additional estimate true values are indicated. As it can be easily seen, since the following relationships among the *true*

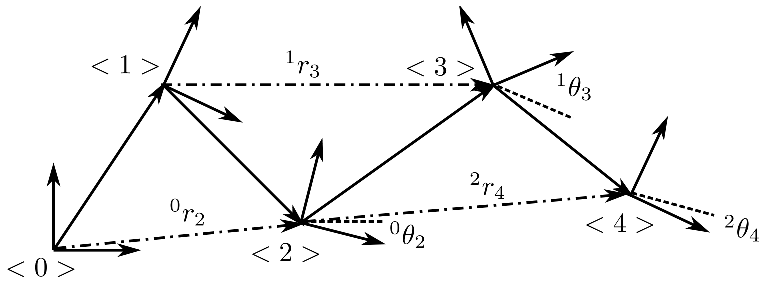

Figure 2: Adding extra image tracking to the basic ones

values always hold true (binding together two successive relative frame positions and attitudes)

$$^{i-2}r_i = \,^{i-2}r_{i-1} + R\left(^{i-2}\theta_{i-1}\right) \,^{i-1}r_i \tag{3a}$$

$$^{i-2}\theta_i = \,^{i-2}\theta_{i-1} + \,^{i-1}\theta_i \tag{3b}$$

it follows that the new measurements ${}^{i-2}\hat{r}_i, {}^{i-2}\hat{\theta}_i$ can be exploited for estimation improvement purposes. Note, however, that the assumption about the availability of the additional measurements requires that the stereo camera can recognize, within the current frame, not only a sufficient number of features belonging to the image just before this last, but also a sufficient number of those belonging to the two-step delayed one; this requirement, even if a little strict, seems reasonable to be implemented.

3. State Estimation

With the introduction of the assumed additional measurements, let us first observe how the sequence of acquisitions consequently evolves during time, with the aid of the scheme of Fig. 3. Then let us decompose the process into stages

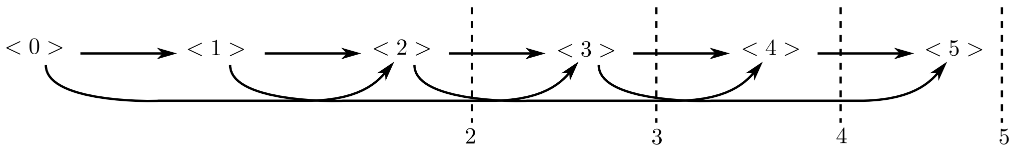

Figure 3: The considered sequence of stages

corresponding, each one, to when the interlaced additional acquisition ${}^{i-2}\hat{r}_i, {}^{i-2}\hat{\theta}_i$ has been completed, as indicated in Fig. 3; and then observe how the so resulting process can be modeled via the use of the following couple of separate, interacting, state-space models

- Basic Relative-Orientation dynamic

$$\begin{bmatrix} {}^{i-1}\theta_i \\ {}^{i-2}\theta_{i-1} \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} {}^{i-2}\theta_{i-1} \\ {}^{i-3}\theta_{i-2} \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} {}^{i-1}\hat{\theta}_i + \begin{bmatrix} 1 \\ 0 \end{bmatrix} {}^{i-1}\eta_i \qquad x_i^1 = A_1 x_{i-1}^1 + B_1 u_i^1 + B_1 \xi_i^1 \tag{4a}$$

$$\begin{bmatrix} {}^{i-2}\hat{\theta}_{i-1} \\ {}^{i-2}\hat{\theta}_i \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} {}^{i-2}\theta_{i-1} \\ {}^{i-1}\theta_i \end{bmatrix} - \begin{bmatrix} {}^{i-2}\eta_{i-1} \\ {}^{i-2}\eta_i \end{bmatrix} \qquad y_i^1 = C_1 x_i^1 + \zeta_i^1 \tag{4b}$$

- Basic Relative-Position dynamic

$$\begin{bmatrix} {}^{i-1}r_i \\ {}^{i-2}r_{i-1} \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ I & 0 \end{bmatrix} \begin{bmatrix} {}^{i-2}r_{i-1} \\ {}^{i-3}r_{i-2} \end{bmatrix} + \begin{bmatrix} I \\ 0 \end{bmatrix} {}^{i-1}\hat{r}_i + \begin{bmatrix} I \\ 0 \end{bmatrix} {}^{i-1}\varepsilon_i \qquad x_i^2 = A_2 x_{i-1}^2 + B_2 u_i^2 + B_2 \xi_i^2 \tag{5a}$$

$$\begin{bmatrix} {}^{i-2}\hat{r}_{i-1} \\ {}^{i-2}\hat{r}_i \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & R\left({}^{i-2}\theta_{i-1}\right) \end{bmatrix} \begin{bmatrix} {}^{i-2}r_{i-1} \\ {}^{i-1}r_i \end{bmatrix} - \begin{bmatrix} {}^{i-2}\varepsilon_{i-1} \\ {}^{i-2}\varepsilon_i \end{bmatrix} \qquad y_i^2 = C_2\left({}^{i-2}\theta_{i-1}\right) x_i^2 + \zeta_i^2 \tag{5b}$$

where also the compact standard notation has been reported (with obvious meaning of the introduced symbols). In particular note how the interaction between the two systems is solely represented by the influence that the component ${}^{i-2}\theta_{i-1}$ of the first system state exerts on rotation matrix $R\left({}^{i-2}\theta_{i-1}\right)$ which in turn appears within the second output of the second system. This is actually the sole cause of non-linearity of the resulting overall system that otherwise would have been fully linear and stationary. The signal flow scheme reported in Fig. 4 clearly shows the indicated interaction occurring between the two systems.

3.1 Extended Kalman Filtering (EKF)

The above introduced overall state space system (i.e. the aggregation of the two) results into a linear time-invariant one as regards its dynamic part, while it is partially non-linear in its overall output vector, as previously evidenced. As a consequence of this the overall system appears to be very suitable (i.e. certainly more suitable than systems exhibiting non-linearities also in their dynamic part) for having its state to be recursively estimated via the well known and celebrated *EKF* technique [5]. More precisely, by formally aggregating the two systems into the following overall one (again, with an obvious meaning for the introduced symbols)

$$x_i = A x_{i-1} + B u_i + B\xi_i \qquad Q_i \doteq cov(\xi_i) = Bdiag\left({}^{i-1}\Theta_i,\ {}^{i-1}\Sigma_i\right) \tag{6a}$$

$$y_i = C\left({}^{i-2}\Theta_{i-1}\right) x_i + \zeta_i \qquad S_i \doteq cov(\zeta_i) = Bdiag\left({}^{i-2}\Theta_{i-1},\ {}^{i-2}\Theta_i,\ {}^{i-2}\Sigma_{i-1},\ {}^{i-2}\Sigma_i\right) \tag{6b}$$

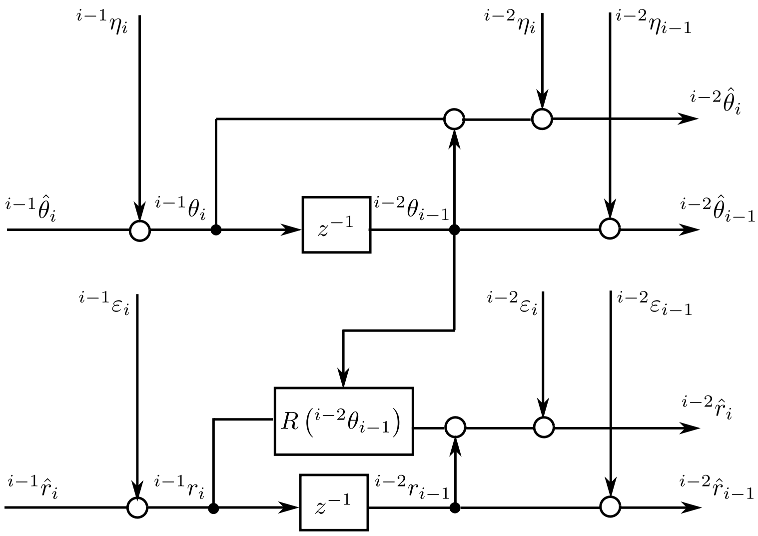

Figure 4: Block diagram of the considered state space model

Then, by keeping into account the resulting linearity of the overall dynamic part; as well as of the kind of structured state dependency exhibited by the output matrix $C\left({}^{i-2}\Theta_{i-1}\right)$; it is not difficult to verify (even if a little tedious) that the EKF associated to the above system actually takes on the form

$$\widehat{x}_{i/i-1} = A\ \widehat{x}_{i-1/i-1} + Bu_i \tag{7a}$$

$$\widehat{x}_{i/i} = \widehat{x}_{i/i-1} + K_i\left[y_i - C\left({}^{i-2}\ \widehat{\Theta}_{i-1/i-1}\right)\widehat{x}_{i/i-1}\right] \tag{7b}$$

$$P_{i/i-1} = AP_{i-1/i-1}A^T + BQ_iB^T \tag{7c}$$

$$P_{i/i} = (I - K_iH_i)\,P_{i/i-1} \tag{7d}$$

with

$$K_i = P_{i/i-1}H_i^T\left(H_iP_{i/i-1}H_i^T + S_i\right)^{-1} \tag{7e}$$

$$H_i \doteq C\left({}^{i-2}\ \widehat{\Theta}_{i-1/i-1}\right) + C'\left({}^{i-2}\ \widehat{\Theta}_{i-1/i-1},\ {}^{i-1}\ \widehat{r}_{i/i-1}\right) \tag{7f}$$

and where $C'\left({}^{i-2}\ \widehat{\Theta}_{i-1/i-1},\ {}^{i-1}\ \widehat{r}_{i/i-1}\right)$ simply takes on the “almost empty” form

$$C'\left({}^{i-2}\ \widehat{\Theta}_{i-1/i-1},\ {}^{i-1}\ \widehat{r}_{i/i-1}\right) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0_{2\times1} & 0_{2\times1} & 0_{2\times1} & 0_{2\times1} \\ 0_{2\times1} & R'\left({}^{i-2}\ \widehat{\theta}_{i-1/i-1}\right)\ {}^{i-1}\ \widehat{r}_{i/i-1} & 0_{2\times1} & 0_{2\times1} \end{bmatrix} \tag{7g}$$

that is a 6×6 square matrix only having the last two elements of its second column as non-zero elements. Moreover, within the indicated expression for such non zero terms we have

$$R'\left({}^{i-2}\ \widehat{\theta}_{i-1/i-1}\right) \doteq \begin{bmatrix} -sin\ {}^{i-2}\ \widehat{\theta}_{i-1/i-1} & -cos\ {}^{i-2}\ \widehat{\theta}_{i-1/i-1} \\ cos\ {}^{i-2}\ \widehat{\theta}_{i-1/i-1} & -sin\ {}^{i-2}\ \widehat{\theta}_{i-1/i-1} \end{bmatrix} \tag{7h}$$

simply corresponding to the derivative w.r.t θ of rotation matrix $R(\theta)$ evaluated for ${}^{i-2}\ \widehat{\theta}_{i-1/i-1}$. We can now conclude this section by explicitly noting how the presence of the non-zero terms in Eq. (7g) is actually the sole responsible for having the filter of full-order 6. In fact, as it can be easily verified, in case the non-zero terms in Eq. (7g) were neglected, the overall filter would automatically reduce to two separate filters: the first one applied to system Eq. (3a) and independently estimating the orientations; the second one applied to system Eq. (3b) and estimating the position only; even if this is done by also acquiring the orientation estimates from the first filter, just for estimating (via direct insertion of such orientation estimates) the posture dependent output matrix appearing Eq. (3b) . Obviously enough, though neglecting such terms in Eq. (7g) certainly reduces the filter performances; the simplification gained in its implementation might however compensate for such performance reduction, provided it reveals acceptable. Certainly enough some simulation experiments should be performed for approving or not such conjecture.

3.2 Iterated Kalman Filtering (IKF)

The IKF [5], [6] is a quasi-optimal non-linear filter, which can actually be interpreted as an improvement introduced in the above seen EKF; in the sense that this last can in turn be seen as a simplification of *IKF*. We can introduce the filter by starting from the main part of the well known Chapman-Kolmogorov general relationship, stating that once a dynamic system having the generic form

$$x_i = f(x_{i-1}, u_i) + \xi_i \tag{8a}$$

$$y_i = g(x_i) + \zeta_i \tag{8b}$$

is made available, together with the progressive collection of its output and input measurements

$$I_i = \{(y_k, u_k)\,; k = 1, 2, \ldots, i\} \tag{9}$$

then, for the posterior probability density function (p.d.f.) $p(x_i/I_i)$ we have:

$$p(x_i/I_i) = \frac{1}{M_i} p(y_i/x_i)\, p(x_i/I_{i-1}, u_i) \tag{10}$$

In the above relationship, the term $p(x_i/I_{i-1}, u_i)$ is the so-called prior p.d.f.; that is the predicted distribution of x_i before acquiring the current measurement set y_i; while $p(y_i/x_i)$ is instead the so-called Likelihood p.d.f; which is used for upgrading the prior into the posterior once measurement y_i is acquired. Finally quantity M_i is simply the normalizing factor allowing the overall right-hand-side exhibiting a unitary integral within the whole space. As it is well known, upgrading the posterior $p(x_i/I_i)$ starting from the previous one $p(x_{i-1}/I_{i-1})$ represents the conceptual framework on the basis of which any state estimation technique (even if of suboptimal type) must be constructed. And to this respect the Chapman-Kolmogorov relationship Eq. (10) clearly show how the process should consequently adhere to the following logic:

$$\begin{array}{ll} p(x_{i-1}/I_{i-1}) \rightarrow p(x_i/I_{i-1}, u_i) \rightarrow & \bullet \rightarrow p(x_i/I_i) \\ & \uparrow \\ & p(y_i/x_i) \end{array} \tag{11}$$

Within the above logic process, and in case of non-linear dynamic, the step in the left - i.e. the evaluation of the prior probability $p(x_i/I_{i-1}, u_i)$ - is generally the most difficult one; which instead reveals very simple to be solved in case of linear dynamics Eq. 6a, Gaussian noises, and also Gaussian previous prior $p(x_{i-1}/I_{i-1})$. More precisely provided that

$$p(x_{i-1}/I_{i-1}) = N\left(x_{i-1}, \widehat{x}_{i-1/i-1}, P_{i-1/i-1}\right) \tag{12}$$

it is well know that with a linear dynamic and white Gaussian noise, as it actually is for our case, we have

$$p(x_i/I_{i-1}, u_i) = N\left(x_i, \widehat{x}_{i/i-1}, P_{i/i-1}\right) \tag{13a}$$

with $\widehat{x}_{i/i-1}$ and $P_{i/i-1}$ just provided by relationships Eq. (7a) and Eq. (7c) respectively; here repeated and renumbered for sake of convenience; and from now on reconsidered with respect to our case:

$$\widehat{x}_{i/i-1} = A\, \widehat{x}_{i-1/i-1} + Bu_i \tag{13b}$$

$$P_{i/i-1} = AP_{i-1/i-1}A^T + BQ_iB^T \tag{13c}$$

(13d)

As regard the evaluation of the Likelihood p.d.f., it is generally much easier than the evaluation of the prior, and in particular for our case it is actually direct; since from Eq. (6b) we immediately have

$$p\left(y_i/x_i\right) = N\left[y_i, C\left({}^{i-2}\theta_{i-1}\right)x_i, S_i\right] \tag{14}$$

thus leading, together with Eq. (13a) and from the general relationship Eq. (10), to the hereafter reported expression for the posterior p.d.f. of our case:

$$p\left(x_i/I_i\right) = \frac{1}{M_i}N\left[y_i, C\left({}^{i-2}\theta_{i-1}\right)x_i, S_i\right]N\left(x_i, \widehat{x}_{i/i-1}, P_{i/i-1}\right) \tag{15}$$

Note however how, despite its first glance appearance the above resulting posterior is however non-Gaussian due to the non-linear dependence of matrix $C\left({}^{i-2}\theta_{i-1}\right)$ from the second component ${}^{i-2}\theta_{i-1}$ of the whole state x_i; thus meaning that any successive propagation of the posterior, in the same form which lead to Eq. (15), can be done only at the expense of suitably approximating the posterior $p\left(x_i/I_i\right)$ resulting at each stage with a Gaussian one, before proceeding with the next stage $i+1$. Such an approximation leads to suboptimal estimation techniques, at least for systems exhibiting non-linearities in the output measurements only (like the one under consideration); these methods, anyway, reveal as quite efficient, even when compared with more sophisticated ones (like for instance the recently introduced Monte Carlo based recursive techniques, generally leading to the Particle Filters (see [6], [7], [8]). Even if such approximations lead to suboptimal estimation techniques, at least for systems exhibiting non-linearities in the output measurements only, like the our, they however reveal as quite efficient, even when compared with more sophisticated ones (like for instance the recently introduced Monte Carlo based recursive techniques; generally leading to the so-called Particle Filtering; which are generally much more cumbersome to be implemented. Particle filtering techniques have been proposed for explicitly dealing with systems which are mainly non linear in their dynamic part, as in Eq. (4a). They efficiently use real time Monte-Carlo methods for propagating the prior p.d.f., since strongly influenced by the non-linearity in the dynamic part (which however is not our case). In the following of the present section we shall briefly illustrate how the mentioned IKF technique allows to propagate Eq. (15) in its Gaussian approximated form, with a reasonable computation effort. To this aim reconsider Eq. (15) and formerly observe how a maximum a posteriori probability (MAP) estimate $\widehat{x}_{i/i}$ can be obtained by maximizing the r.h.s. of Eq. (15) itself with respect to x_i; which is in turn the same of solving the following minimization problem, i.e. minimizing the sum of the exponents in Eq. (15)

$$\widehat{x}_{i/i} = \underset{x_i}{\operatorname{argmin}}\left[\left\|y_i - C\left({}^{i-2}\theta_{i-1}\right)x_i\right\|^2_{S_i} + \left\|x_i - \widehat{x}_{i/i-1}\right\|^2_{P_{i/i-1}}\right] \tag{16}$$

Since the argument of the minimization in Eq. (16) is actually non totally quadratic (still due to the non-linear dependence of matrix $C\left({}^{i-2}\theta_{i-1}\right)$ from the second component ${}^{i-2}\theta_{i-1}$ of the whole state x_i) then, like any non-quadratic minimization problem, a recursive numerical algorithm of gradient type, Newton type, or Newton-Rapson type, can be used for achieving the minimum point $\widehat{x}_{i/i}$. Then, once the minimum point $\widehat{x}_{i/i}$ has been achieved, an approximate evaluation $P_{i/i}$ of the estimation error covariance can be obtained as the inverse Hessian matrix of the argument of Eq. (16), evaluated in correspondence of $\widehat{x}_{i/i}$. In case of employment of a gradient or Newton type procedure, the analytical expression of the Hessian matrix can be a-priory deduced from the analytical expression of the argument of Eq. (16); then its numerical value at $\widehat{x}_{i/i}$ has to be extracted and then inverted. In case instead of a Newton-Rapson procedure, the numerical value $P_{i/i}(k)$ of the Hessian Inverse at the current iteration point $\widehat{x}_{i/i}(k)$ is provided by the structure of the procedure itself. Then, once $\widehat{x}_{i/i}$ and $P_{i/i}$ have been finally achieved, the Gaussian approximation

$$p\left(x_i/I_i\right) \approx N\left(x_i, \widehat{x}_{i/i}, P_{i/i}\right) \tag{17}$$

is introduced for passing to the next stage, and so on. Within the literature the usage of the Newton-Rapson procedure for minimizing any non-quadratic cost resulting from a system which is non linear in its measurement equation only (as it is in our case) is generally termed as IKF; even if we extend this terminology also to the case of employment of any other numerical minimization technique. Finally, it is worth recalling that, still in case of employment of a Newton-Rapson procedure, but initialized at $\widehat{x}_{i/i-1}$ and also arrested at the first iteration, the so resulting incomplete IKF procedure actually turns out coinciding with the previously seen EKF one. A fact this last that clearly shows how IKF always result better performing that the EKF; thus indicating the IKF as the preferable one, provided sustained by the available computing power on-board the vehicle. We shall now conclude the present section by indicating some investigation directions that should be followed for best adapting the IKF procedure to our case. More precisely, by keeping into account of the very particular structure actually attained by the output matrix $C\left({}^{i-2}\theta_{i-1}\right)$; that is, from Eq. (4b), Eq. (5b)

$$C\left({}^{i-2}\theta_{i-1}\right) = Bdiag\left[C_1, C_2\left({}^{i-2}\theta_{i-1}\right)\right] = Bdiag\left\{\begin{bmatrix}0 & 1\\1 & 1\end{bmatrix}, \begin{bmatrix}0 & 1\\1 & R\left({}^{i-2}\theta_{i-1}\right)\end{bmatrix}\right\} \tag{18}$$

with $R\left({}^{i-2}\theta_{i-1}\right)$ as in Eq. (2b); a first investigation direction could be that of trying to exploit such particularity for possibly best structuring the associate Newton-Rapson algorithm working on the whole state x_i. As a second investigation direction, we could instead consider the minimization problem embedded in Eq. (16) once rewritten as

$$\min_{{}^{i-2}\theta_{i-1}} \left\{ \min_{x_i/{}^{i-2}\theta_{i-1}} \left[\left\| y_i - C\left({}^{i-2}\theta_{i-1}\right) x_i \right\|^2_{S_i} + \left\| x_i - \hat{x}_{i/i-1} \right\|^2_{P_{i/i-1}} \right] \right\} \tag{19}$$

Then, by noting that for given ${}^{i-2}\theta_{i-1}$ the inner minimization turns out to be quadratic, an analytical *linear* expression for the resulting conditioned minimum point $\widehat{x}_{i/i}\left({}^{i-2}\theta_{i-1}\right)$ *must* necessarily be obtained of the form (by also exploiting the particular structure of $C\left({}^{i-2}\theta_{i-1}\right)$)

$$\widehat{x}_{i/i}\left({}^{i-2}\theta_{i-1}\right) = L\left[R\left({}^{i-2}\theta_{i-1}\right), y_i, \hat{x}_{i/i-1}\right] \tag{20}$$

Where L just stays for linear and which is actually valid in correspondence of *any* stage. The linear form in Eq. (20), once substituted into Eq. (19) must necessarily lead to an analytical expression for the resulting value of its inner conditioned optimal cost, say it $J\left({}^{i-2}\theta_{i-1}, y_i, \hat{x}_{i/i-1}\right)$, within Eq. (19) itself; thus reducing the residual minimization to the following

$$\min_{x_i/{}^{i-2}\theta_{i-1}} J\left({}^{i-2}\theta_{i-1}, y_i, \hat{x}_{i/i-1}\right) \tag{21}$$

also valid in correspondence of *any* stage; whose numerical minimization now results very much simplified by the fact that it is performed along a one dimensional set. Note however that, as regard the estimated error covariance, this should be instead evaluated at $\widehat{x}_{i/i}$ via the Hessian matrix analytical expression a-priori deduced from the argument of Eq. (16), by possibly exploiting again the particularity Eq. (18). Finally, as a third investigation alternative, we could also keep into account the possibility (however very unlikely) that the analytical expression of $J\left({}^{i-2}\theta_{i-1}, y_i, \hat{x}_{i/i-1}\right)$ might also result with an "easy-to-be-zeroed" partial derivative with respect to its first argument ${}^{i-2}\theta_{i-1}$. A fact this last that, whenever true for our case, would actually reduce the associated quasi optimal IKF state estimation procedure, to the simple and very fast application of some invariant analytical formulas at each stage; thus with an enormous advantage in terms of required computational power.

3.3 Remarks

The State Estimators (STE) of IKF type, or its reduced version EKF, are both *recursive filters* that produce the current stage state estimation by updating their prediction, in turn obtained from the previous estimations. Hence, since the error occurred at any stage will not change in the future and since the absolute vehicle position ad orientation has to be evaluated still via Eq. (2), it is consequently clear how linearly increasing errors have still to be expected for increasing number of stages (with the only difference that such increase is now mitigated by the reduced variances of the relative errors, due to the presence of state estimator). Obviously enough, things might be different in case the incoming acquisitions for increasing stages could be used for producing, not only the current state estimate, but also for bettering, at the same time, all the past state estimations. As a matter of fact this objective can be achieved via the use of the so called Sequence Estimation (SQE) techniques, as discussed in section 4.

4. Sequence Estimation

In order to start approaching the argument, let us first introduce the here used notations for denoting the sequences of true relative orientations and positions realized till the $i-th$ stage

$$\theta^{1,i} \doteq \left\{{}^{k-1}\theta_k; k = 1, 2, \ldots, i\right\} \tag{22a}$$

$$r^{1,i} \doteq \left\{{}^{k-1}r_k; k = 1, 2, \ldots, i\right\} \tag{22b}$$

together with the aggregation of the two

$$X^{1,i} \doteq \left\{\theta^{1,i}; r^{1,i}\right\} \tag{22c}$$

and also the following others, for denoting the associated sequences of the collected measurements

$$z^{1,i} \doteq \left\{\left({}^{k-1}\hat{\theta}_k,\ {}^{k-2}\hat{\theta}_k\right); k = 1, 2, \ldots, i\right\} \tag{23a}$$

$$w^{1,i} \doteq \left\{\left({}^{k-1}\hat{r}_k,\ {}^{k-2}\hat{r}_k\right); k = 1, 2, \ldots, i\right\} \tag{23b}$$

they also aggregated into the following:

$$Z^{1,i} \doteq \left\{z^{1,i}; w^{1,i}\right\} \tag{23c}$$

Then, once the $i-th$ stage has been achieved, the problem to be considered here is that of finding the overall sequence estimate

$$\overset{\frown}{X}^{1,i} = \left\{\overset{\frown}{\theta}^{1,i}; \overset{\frown}{r}^{1,i}\right\} = \underset{X^{1,i}}{\operatorname{argmax}}\ p\left(X^{1,i}/Z^{1,i}\right) \tag{24}$$

As it can be realized, provided that the problem is solvable (even in an approximate or suboptimal form), its solution will renew, at each stage i, a batch of interpolated estimate of the *entire* sequence $X^{1,i}$ realized before it, without relying on the estimate of the previous one $X^{1,i-1}$. Due to this, we can argue about the drift errors, generally characterizing IKF and EKF techniques: they should, in this case, result strongly reduced. Still assuming the problem solvability, an obvious drawback is however represented by the apparent increasing dimensionality with the increasing number of stages. The dimensionality increase is actually of linear type and so it is quite acceptable within a reasonably high maximum number of stages, after which the obtained sequence estimate should be "freezed" and the entire procedure then restarted along the successive maximum horizon of stages. At this point, in order to manage the problem, we shall however consider the cascade of the following sub-problems

$$\overset{\frown}{\theta}^{1,i} \doteq \underset{\theta^{1,i}}{\operatorname{argmax}}\ p\left(\theta^{1,i}/z^{1,i}\right) \rightarrow \overset{\frown}{r}^{1,i} \doteq \underset{r^{1,i}}{\operatorname{argmax}}\ p\left(r^{1,i}/w^{1,i}, \overset{\frown}{\theta}^{1,i}\right) \tag{25}$$

where the orientation sequence is estimated on the basis of its own measurements only, while the position sequence is evaluated in cascade, on the basis of its measurements and taking $\overset{\frown}{\theta}^{1,i}$ as a given parameter set. The need of structuring the problem in the above suboptimal way, with respect to the original, actually arises from the necessity to maintain a manageable implementative form that otherwise cannot be guaranteed, whenever considered within its generality as in Eq. (24). Hence, considering the first sub-problem, formerly observe how, via standard application of the Bayes rule, for the involved p.d.f., we actually have

$$p\left(\theta^{1,i}/z^{1,i}\right) = \frac{1}{M_{1,i}} p\left(z^{1,i}/\theta^{1,i}\right) = \prod_{k=2}^{i} \exp\left[-\frac{\left|{}^{k-2}\theta_{k-1} - {}^{k-2}\hat{\theta}_{k-1}\right|^2}{{}^{k-2}\Theta_{k-1}} - \frac{\left|\left({}^{k-2}\theta_{k-1} + {}^{k-1}\hat{\theta}_k\right) - {}^{k-2}\hat{\theta}_k\right|^2}{{}^{k-2}\Theta_k} + \frac{\left|{}^{k-1}\theta_k - {}^{k-1}\hat{\theta}_k\right|^2}{{}^{k-1}\Theta_k}\right] \tag{26}$$

where the first equality simply states that the posterior p.d.f. $p\left(\theta^{1,i}/z^{1,i}\right)$ substantially coincides with the maximum likelihood one $p\left(z^{1,i}/\theta^{1,i}\right)$ since the prior $p\left(\theta^{1,i}\right)$ is actually uniform; on the other hand, the second equality follows directly from the assumed independence and Gaussianity of the added measurement noises. Then we note how maximizing Eq. (26) is equivalent to the following minimization problem:

$$\overset{\frown}{\theta}^{1,i} \doteq \underset{\theta^{1,i}}{\operatorname{argmin}} \sum_{k=2}^{i}\left[\frac{\left|{}^{k-2}\theta_{k-1} - {}^{k-2}\hat{\theta}_{k-1}\right|^2}{{}^{k-2}\Theta_{k-1}} + \frac{\left|\left({}^{k-2}\theta_{k-1} + {}^{k-1}\hat{\theta}_k\right) - {}^{k-2}\hat{\theta}_k\right|^2}{{}^{k-2}\Theta_k} + \frac{\left|{}^{k-1}\theta_k - {}^{k-1}\hat{\theta}_k\right|^2}{{}^{k-1}\Theta_k}\right] \doteq \underset{\theta^{1,i}}{\operatorname{argmin}}\ J^{1,i}\left(\theta^{1,i}\right) \tag{27}$$

Moreover, since by isolating the *first stage* Eq. (27) can be equivalently rewritten as

$$\min_{\theta^{2,i}}\left\{\min_{{}^0\theta_1}\left[\frac{\left|{}^0\theta_1 - {}^0\hat{\theta}_1\right|^2}{{}^0\Theta_1} + \frac{\left|\left({}^0\theta_1 + {}^1\theta_2\right) - {}^0\hat{\theta}_2\right|^2}{{}^0\Theta_2}\right]_{/{}^1\theta_2} + \frac{\left|{}^1\theta_2 - {}^1\hat{\theta}_2\right|^2}{{}^1\Theta_2} + J^{2,i}\left(\theta^{2,i}\right)\right\} \tag{28}$$

(being $J^{2,i}\left(\theta^{2,i}\right)$ the same of Eq. (27) even if now starting from $k = 3$) it then follows that within Eq. (28) we can actually proceed as hereafter indicated.

1. Minimize the first part for given ${}^{1}\theta_2$: since the part to be minimized is quadratic, the result will be linear in ${}^{1}\theta_2$; thus obtaining the following linear parameterization:

$$ {}^{0}\widehat{\theta}_1 = L_1\left({}^{1}\theta_2\right) \tag{29a} $$

2. Substitute the result of Eq. (29a) into Eq. (28) (the result will necessary be a quadratic function of ${}^{1}\theta_2$) and define:

$$ Q_1\left({}^{1}\theta_2\right) \doteq \min_{{}^{0}\theta_1} \left[\frac{\left|{}^{0}\theta_1 - {}^{0}\hat{\theta}_1\right|^2}{{}^{0}\Theta_1} + \frac{\left|\left({}^{0}\theta_1 + {}^{1}\theta_2\right) - {}^{0}\hat{\theta}_2\right|^2}{{}^{0}\Theta_2} \right]_{/{}^{1}\theta_2} \tag{29b} $$

3. Extract from $J^{2,i}\left(\theta^{2,i}\right)$ its first three terms. Then group the first two with $Q_1\left({}^{1}\theta_2\right)$ (naming it $\bar{Q}_2\left({}^{1}\theta_2, {}^{2}\theta_3\right)$); thus obtaining

$$ \min_{\theta^{2,i}} \left\{ \bar{Q}_2\left({}^{1}\theta_2,\ {}^{2}\theta_3\right) + \frac{\left|{}^{2}\theta_3 - {}^{2}\hat{\theta}_3\right|^2}{{}^{2}\Theta_3} + J^{3,i}\left(\theta^{3,i}\right) \right\} \tag{29c} $$

where $J^{3,i}\left(\theta^{3,i}\right)$ is obviously the same as in Eq. (28) even if now starting from $k = 4$ and where by construction:

$$ \bar{Q}_2\left({}^{1}\theta_2,\ {}^{2}\theta_3\right) = Q_1\left({}^{1}\theta_2\right) + \frac{\left|{}^{1}\theta_2 - {}^{1}\hat{\theta}_2\right|^2}{{}^{1}\Theta_2} + \frac{\left|\left({}^{1}\theta_2 + {}^{2}\theta_3\right) - {}^{1}\hat{\theta}_3\right|^2}{{}^{1}\Theta_3} \tag{29d} $$

4. Finally rewrite Eq. (29c) similarly to Eq. (28) as

$$ \min_{\theta^{3,i}} \left\{ \min_{{}^{1}\theta_2} \bar{Q}_2\left({}^{1}\theta_2,\ {}^{2}\theta_3\right)_{/{}^{1}\theta_2} + \frac{\left|{}^{2}\theta_3 - {}^{2}\hat{\theta}_3\right|^2}{{}^{2}\Theta_3} + J^{3,i}\left(\theta^{3,i}\right) \right\} \tag{29e} $$

and return to the first point, repeating all the steps with respect to Eq. (29e) (and so on for all successive stages).

Note that the linear parameterization ${}^{k-1}\widehat{\theta}_k = L_k\left({}^{k}\theta_{k+1}\right)$ at the current stage k, represents the constraint for the previous stage, according to the back substitution scheme depicted in Fig. 5.

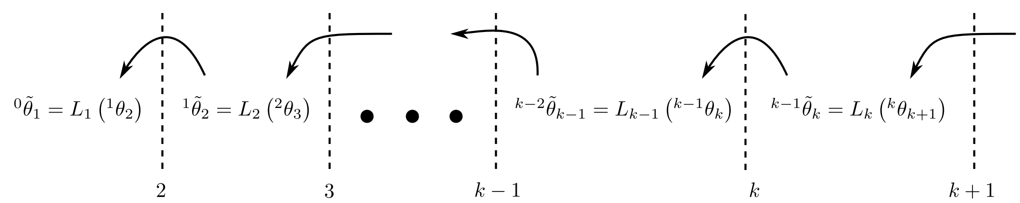

Figure 5: Back substitution scheme

When the $i-th$ stage is finally reached, assuming that no more measurements are acquired (or equivalently assuming that the procedure has to be stopped at such a stage) the last minimization problem (consider that in correspondence of the last stage $J^{i,i}\left(\theta^{i,i}\right) \doteq 0$) can be written as

$$ \min_{\theta^{i-1,i}} \left\{ \bar{Q}_{i-1}\left({}^{i-2}\theta_{i-1},\ {}^{i-1}\theta_i\right) + \frac{\left|{}^{i-1}\theta_i - {}^{i-1}\hat{\theta}_i\right|^2}{{}^{i-1}\Theta_i} \right\} \tag{30} $$

which can be here performed directly with respect to both ${}^{i-2}\theta_{i-1}$, ${}^{i-1}\theta_i$, thus obtaining their estimations ${}^{i-2}\widehat{\theta}_{i-1}$, ${}^{i-1}\widehat{\theta}_i$. Hence, such numerical values can be used to realize the above back substitution scheme; this procedure actually corresponds to the well known Forward Dynamic Programming (FDP) technique, here applied to sequence estimation problems. This method requires to wait until the last $i-th$ stage before providing the estimate of the orientation sequence realized till such stage. Nevertheless, it could also be implemented in such a way that, while the forward phase proceeds, the backward phase is however executed in correspondence of any new incoming stage. In this way, the procedure would totally renew the interpolated sequence in correspondence of each new arrived stage till the last one. Note how the sole backward phase is the one working with a computational effort increasing with the number of stages; thus a limit to the maximum number of processable stages should be substantially established only on this basis. Once the farthest allowed stage is reached, the procedure can be restarted by trivially considering such farthest stage as the new initial one. In such a way drifting errors, occurring within a cascade of long stage batches, would result much more contained than in the case of state filtering techniques. Considering now the sub-problem of estimating the position sequence, just observe that (again via the standard Bayes rule)

$$\begin{aligned} p\left(r^{1,i}/w^{1,i},\overset{\frown}{\theta}^{1,i}\right) = \tfrac{1}{H_{1,i}} p\left(w^{1,i}/r^{1,i},\overset{\frown}{\theta}^{1,i}\right) p\left(r^{1,i}\right) = p\left(w^{1,i}/r^{1,i},\overset{\frown}{\theta}^{1,i}\right) = \prod_{k=2}^{i} \exp\Big[- \left\| {}^{k-2}r_{k-1} - {}^{k-2}\hat{r}_{k-1} \right\|^2_{{}^{k-2}\Sigma^{-1}_{k-1}} + \\ - \left\| \left[{}^{k-2}r_{k-1} + R\left({}^{k-2}\overset{\frown}{\theta}_{k-1}\right) {}^{k-1}r_k \right] - {}^{k-2}\hat{r}_k \right\|^2_{{}^{k-2}\Sigma^{-1}_k} - \left\| {}^{k-1}r_k - {}^{k-1}\hat{r}_k \right\|^2_{{}^{k-1}\Sigma^{-1}_k} \Big] \end{aligned} \tag{31}$$

where, as assumed within the (suboptimal) problem statement in Eq. (25), $\overset{\frown}{\theta}^{1,i}$ has to be taken as a given set of parameters without any probabilistic characterization. Then, under this assumption, the first line of Eq. (31) again states that the posterior p.d.f. substantially coincides with the maximum likelihood one, since the prior p.d.f. is actually uniform. Moreover Eq. (31) holds thanks to the assumed independence and Gaussianity of the added measurement noises. Then, just as before, maximizing Eq. (31) is equivalent to the following minimum problem:

$$\begin{aligned} \overset{\frown}{r}^{1,i} \doteq \underset{r^{1,i}}{\operatorname{argmin}} \sum_{k=2}^{i} \Big[\left\| {}^{k-2}r_{k-1} - {}^{k-2}\hat{r}_{k-1} \right\|^2_{{}^{k-2}\Sigma^{-1}_{k-1}} + \left\| \left[{}^{k-2}r_{k-1} + R\left({}^{k-2}\overset{\frown}{\theta}_{k-1}\right) {}^{k-1}r_k \right] - {}^{k-2}\hat{r}_k \right\|^2_{{}^{k-2}\Sigma^{-1}_k} + \left\| {}^{k-1}r_k - {}^{k-1}\hat{r}_k \right\|^2_{{}^{k-1}\Sigma^{-1}_k} \Big] = \\ \doteq \underset{r^{1,i}}{\operatorname{argmin}} H^{1,i}\left(r^{1,i}\right) \end{aligned} \tag{32}$$

Since for given $\overset{\frown}{\theta}^{1,i}$ the functional to be minimized in Eq. (32) actually exhibits the same structure of its analogous in Eq. (27), all considerations provided for the previous problem in Eq. (27) can be again applied to Eq. (32), thus leading, also for the positions, to a strictly similar FDP based sequence estimation algorithm. This actually follows from the assumption of considering the angular sequence estimates $\overset{\frown}{\theta}^{1,i}$ as a given parameter set: suboptimality is accepted, since otherwise a *joint* estimation (very difficult for the non-linearity) of both sequences $\theta^{1,i}; r^{1,i}$ would be required. Moreover, a drawback of such an implementation is unfortunately exhibited by the sequence position estimation, that is by the need of waiting till the $i-th$ stage for acquiring $\overset{\frown}{\theta}^{1,i}$ and executing the position forward and backward phases. Hence, in order to renew $\overset{\frown}{\theta}^{1,i-1}; \overset{\frown}{r}^{1,i-1}$ into $\overset{\frown}{\theta}^{1,i}; \overset{\frown}{r}^{1,i}$, at each stage the following operations must be achieved:

1. Upgrade the forward phase of the orientation sequence estimator.
2. Perform its entire backward phase, thus obtaining $\overset{\frown}{\theta}^{1,i}$.
3. Perform the entire forward phase of the position sequence estimator, given $\overset{\frown}{\theta}^{1,i}$.
4. Perform its entire backward phase, thus obtaining $\overset{\frown}{r}^{1,i}$.

This until a maximum allowed distance $j-stage$ is achieved, after which a new sequence is restarted. In the following section we shall see how the integration of a state estimator with a sequence one might further bettering the overall estimation performances, while also allowing an efficient management of the available computing power.

5. Exploiting Multiple Sensors

Generally speaking, multi-sensor integration means suitable exploitation of different sensors providing measures of the same set of variables, in order to obtain an augmented sensor characterized by better performances.In this sense the VO technique can be seen as a component sensor that, based on its own internal procedures, at each stage i adds the set of measurements ${}^{i-2}\hat{\theta}_{i-1}, {}^{i-1}\hat{\theta}_i; {}^{i-2}\hat{r}_{i-1}, {}^{i-1}\hat{r}_i$ (plus the auxiliary ones ${}^{i-2}\hat{\theta}_i; {}^{i-2}\hat{r}_i$) of the associated variables, to the collection $\hat{\theta}^{1,i-1}, \hat{r}^{1,i-1}$ of the identical measurements (here also including the auxiliary ones) acquired till the previous stage. Then, accordingly with the previous section, the VO sensor can be integrated with a state estimator (EKF or

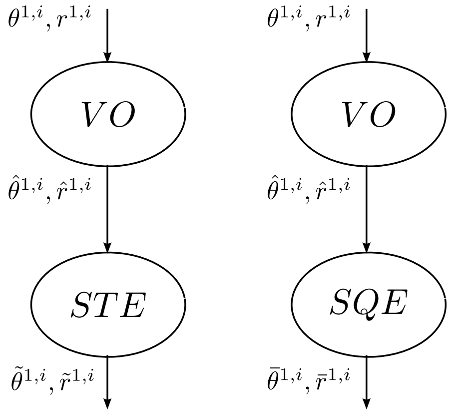

Figure 6: Alternative basic integration schemes

IKF) or, alternatively, with a sequence estimator, in order to obtain better performances, in terms of the variances of the produced estimates (we shall now say $\tilde{\theta}^{1,i-1}, \tilde{r}^{1,i-1}$ for the state estimators and $\bar{\theta}^{1,i-1}, \bar{r}^{1,i-1}$ for the sequence one). Besides the two separate schemes shown in Fig. 6, it could be interesting trying to integrate these two alternatives together, obtaining a tentative scheme for the integration of the three sensor components, as in Fig. 7 (neglect for a while the indicated feedback loop). The rationale for such a scheme follows immediately from the intuitive idea

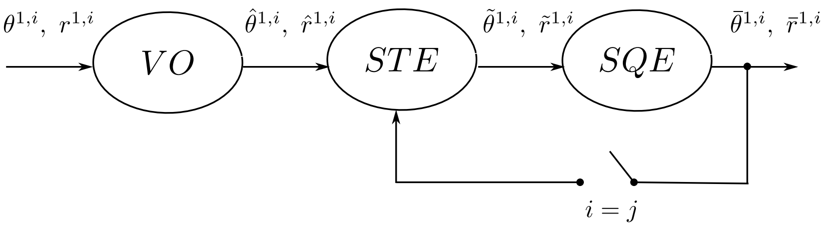

Figure 7: A preliminary tentative integration scheme

that, being the SQE module fed with measurements characterized by smaller variances that those provided by the VO module alone, it should in turn provide sequence estimations with further smaller variances. Moreover, the indicated feedback loop could be used when the maximum allowed distance is achieved, in order to re-initialize the STE module with $\bar{\theta}^{1,j}, \bar{r}^{1,j}$, that is with the hereafter indicated terms

$$\left({}^{j-2}\bar{\theta}_{j-1}, {}^{j-1}\bar{\theta}_j\right); \left({}^{j-2}\bar{r}_{j-1}, {}^{j-1}\bar{r}_j\right) \rightarrow \begin{cases} {}^{j-2}\bar{\theta}_j = {}^{j-2}\bar{\theta}_{j-1} + {}^{j-1}\bar{\theta}_j \\ {}^{j-2}\bar{r}_j = {}^{j-2}\bar{r}_{j-1} + R\left({}^{j-2}\bar{\theta}_{j-1}\right) {}^{j-1}\bar{r}_j \end{cases} \tag{33}$$

in such a way to allow the STE module restarting from stage j with better initial condition and then repeating the entire process for the next batch of stages of maximum length j. Within the above tentative scheme, the idea is to use the STE module in such a way to know (even if less accurately) the vehicle position with respect to the last estimation produced by the SQE.

Unfortunately enough, however, this integrated scheme does not produce any bettering (nor any worsening) with respect to the STE estimation alone. In fact, since the SQE is fed with the STE-provided measurement sequence $\bar{\theta}^{1,j}, \bar{r}^{1,j}$, where in correspondence of each sequential couple of its component we must also evaluate (similarly to Eq. (33) but in correspondence of each intermediate stage i till the $j-th$ one)

$$\left({}^{i-2}\overset{\frown}{\theta}_{i-1}, {}^{i-1}\overset{\frown}{\theta}_i\right); \left({}^{i-2}\overset{\frown}{r}_{i-1}, {}^{i-1}\overset{\frown}{r}_i\right) \rightarrow \begin{cases} {}^{i-2}\overset{\frown}{\theta}_i = {}^{i-2}\overset{\frown}{\theta}_{i-1} + {}^{i-1}\overset{\frown}{\theta}_i \\ {}^{i-2}\overset{\frown}{r}_i = {}^{i-2}\overset{\frown}{r}_{i-1} + R\left({}^{i-2}\overset{\frown}{\theta}_{i-1}\right) {}^{i-1}\overset{\frown}{r}_i \end{cases} \quad ; \quad i = 1, 2, \ldots, j \tag{34}$$

it is easy to see that the corresponding forms of Eq. (28) and Eq. (32), to be minimized inside the SQE, are both zeroed (thus absolutely minimized) just by the same STE-provided sequence: this makes the first suggested scheme *totally useless*. On the other hand, this result is totally coherent with the fact that, once the information has been processed by the STE, the estimation cannot at all be bettered without adding more information (i.e. without acquiring more data). Hence, STE and SQE can be integrated only if they run in parallel, as indicated in Fig. 8.

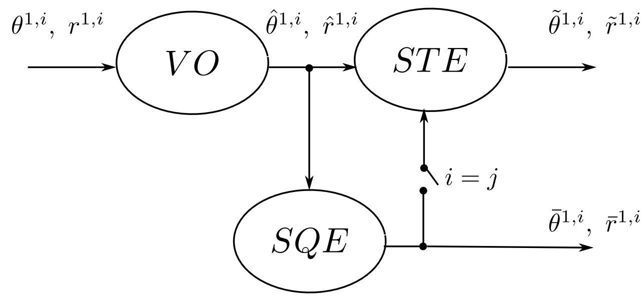

Figure 8: The correct integration scheme

Now that the correct integration scheme between STE end SQE has been established, we can proceed further on by also adding new sensors, like for instance Inertial Management Units (IMU) and/or classical Wheel Odometry (WO) sensor systems (generally present on-board most vehicles), which could improve both the STE and SQE modules.

5.1 State Estimator Improvement

Firstly, consider the case of an IMU system located on-board the vehicle; such a sensor provides (substantially in a continuous-time manner) measurements about the angular velocity vector ω and the linear acceleration vector $\dot{v}$, both projected on the vehicle frame. Since we are for now assuming planar motion only (more general cases will be subjected to further studies), the angular velocity vector will always be about the vertical axis and so its projection on both the vehicle and the absolute frame is the same (we shall indicate it simply as ω). On the other hand, the linear acceleration vector, still due to the planar motion assumption, will always lie on the horizontal plane (we will denote its two-dimensional projection on the horizontal plane of the vehicle frame simply as vector a). Thus the continuous time (typically noisy) measurements provided by an IMU system are typically of the form

$$\tilde{\omega}(t) = \omega(t) + e_1(t) \tag{35a}$$

$$\tilde{a}(t) = a(t) + e_2(t) \tag{35b}$$

with $e_1(t), e_2(t)$ assumed continuous-time zero mean white stationary Gaussian noises (for simplicity also uncorrelated between them) with covariance functions $E_1\delta(\tau)$ and $E_2\delta(\tau)$, respectively ($\delta(\tau)$ is the Dirac unit impulse function). In particular note how, from a kinematic point of view, the IMU actually provides first order information for the orientation (the angular velocity measurements $\tilde{\omega}(t)$) and second order ones for the positions (the linear acceleration measurements $\tilde{a}(t)$). Considering now a generic stage i and the associated time instant t_i in correspondence of which the last measurement acquisition by the VO has been performed, the following integral process, starting at the instant t_{i-1}

$$^{i-1}\tilde{\theta}(t) \doteq \int_{t_{i-1}}^{t} \tilde{\omega} dt = {}^{i-1}\theta(t) + {}^{i-1}\mu(t) \tag{36a}$$

with

$$^{i-1}\theta(t) \doteq \int_{t_{i-1}}^{t} \omega(t)\, dt \tag{36b}$$

$$^{i-1}\mu(t) \doteq \int_{t_{i-1}}^{t} e_1(t)\, dt \quad ; \quad {}^{i-1}M(t) = E_1(t - t_{i-1}) \tag{36c}$$

provides a continuous-time measure of the vehicle orientation with respect to frame $< i-1 >$, while moving between time instants t_{i-1} and t_i. Moreover such an estimation becomes an additional measurement ${}^{i-1}\tilde{\theta}_i$ of the angle ${}^{i-1}\theta_i$ at time $t = t_i$, namely:

$$ {}^{i-1}\tilde{\theta}_i = {}^{i-1}\theta_i + {}^{i-1}\mu_i \quad ; \quad {}^{i-1}M_i = E_1\left(t_i - t_{i-1}\right) \doteq E_1 T_i \tag{37} $$

We can note in Eq. (36c) how the superimposed equivalent noise ${}^{i-1}\mu\left(t\right)$ obviously results into a zero mean Wiener process (i-e. a so-called random walk with independent increments), exhibiting the linearly time increasing covariance matrix indicated in the second of Eq. (36c) (in turn becoming the one in Eq. (37) at time t_i).Further note how the equivalent noise sequence ${}^{i-1}\mu_i$ in Eq. (37) results into a zero mean white Gaussian one -with covariance as in Eq. (37)- since it is composed by a sequence if disjoint integrals of the continuous time zero mean white Gaussian noise $e_1\left(t\right)$ (as a matter of fact it would have been sufficient to have the noise $e_1\left(t\right)$ uncorrelated for time delays greater than the integration interval T_i). Similarly, for the continuous time measurement $\tilde{a}$, recalling relationship of Eq. (3a), observe that, kinematically

$$ {}^{i-2}r_i = {}^{i-2}r_{i-1} + R\left({}^{i-2}\theta_{i-1}\right) {}^{i-1}r_i = {}^{i-2}r_{i-1} + T_i\, {}^{i-2}v_{i-1} + R\left({}^{i-2}\theta_{i-1}\right) {}^{i-1}h_i^2 \tag{38a} $$

where ${}^{i-2}v_{i-1}$ is the vehicle velocity at time t_{i-1} projected on frame $< i-2 >$; moreover Eq. (38b) shows the double integral of the true vehicle acceleration projected on frame $< i-1 >$ during the time interval t_{i-1}, t_i, just corresponding to the contribution to the position with respect to $< i-1 >$ provided by the acceleration itself.

$$ {}^{i-1}h_i^2 \doteq \int_{t_{i-1}}^{t_i} d\tau \int_{t_{i-1}}^{\tau} R\left[{}^{i-1}\theta\left(\tau\right)\right] a\left(\tau\right) d\tau \tag{38b} $$

Furthermore, note that for the velocity ${}^{i-2}v_{i-1}$ kinematically we have

$$ {}^{i-2}v_{i-1} = {}^{i-2}v_{i-2} + {}^{i-2}h_{i-1}^1 \tag{39a} $$

with

$$ {}^{i-2}h_{i-1}^1 \doteq \int_{t_{i-2}}^{t_{i-1}} R\left[{}^{i-2}\theta\left(\tau\right)\right] a\left(\tau\right) d\tau \tag{39b} $$

which is the integral of the true vehicle acceleration projected on frame $< i-2 >$, during the time interval t_{i-2}, t_{i-1}, corresponding to the contribution to the velocity provided by the acceleration itself.

Since for ${}^{i-2}r_{i-1}$ kinematically we have

$$ {}^{i-2}r_{i-1} = T_{i-1}\, {}^{i-2}v_{i-2} + {}^{i-2}h_{i-1}^2 \tag{40} $$

then, by deducing ${}^{i-2}v_{i-2}$ from Eq. (40), substituting it into Eq. (39a) and finally substituting the result into the second of Eq. (38a), with very simple algebra we get the linear form:

$$ {}^{i-2}r_i = c_{i-1}^0\, {}^{i-2}r_{i-1} + c_{i-1}^1\, {}^{i-2}h_{i-1}^1 + c_{i-1}^2\, {}^{i-2}h_{i-1}^2 + R\left({}^{i-2}\theta_{i-1}\right) {}^{i-1}h_i^2 \tag{41} $$

Then, still keeping into account the first of Eq. (38a), we can operate the following definitions:

$$ \begin{aligned} y_i &\doteq c_{i-1}^1\, {}^{i-2}h_{i-1}^1 + c_{i-1}^2\, {}^{i-2}h_{i-1}^2 + R\left({}^{i-2}\theta_{i-1}\right) {}^{i-1}h_i^2 = \left(1 - c_{i-1}^0\right) {}^{i-2}r_{i-1} + R\left({}^{i-2}\theta_{i-1}\right) {}^{i-1}r_i = \\ &\doteq \alpha_{i-1}\, {}^{i-2}r_{i-1} + R\left({}^{i-2}\theta_{i-1}\right) {}^{i-1}r_i \end{aligned} \tag{42a} $$

As we can see, the l.h.s. y_i of Eq. (42a) actually constitutes a global quantity which can be *noisily* provided by the IMU sensor system. Consequently, provided that we can characterize the so obtained measurement $\tilde{y}_i$ in terms of added noise to the true value y_i itself (that is in the form $\tilde{y}_i = y_i + \sigma_i$ with σ_i possibly white with known covariance), the following relationship can be written:

$$ \tilde{y}_i = \alpha_{i-1}\, {}^{i-2}r_{i-1} + R\left({}^{i-2}\theta_{i-1}\right) {}^{i-1}r_i + \sigma_i \tag{42b} $$

It then becomes clear how Eq. (42b), together with Eq. (37), once repeated for both stages i and $i-1$, results in *three* additional output relationships enriching the set of available measurements. Nevertheless, to this regard, observe how the terms ${}^{i-1}h_i^2, {}^{i-2}h_{i-1}^1, {}^{i-2}h_{i-1}^2$ composing y_i in Eq. (42a), have been represented considering $a(t)$ and $\omega(t)$ as completely unrelated, while this is instead not the case for non-holonomic vehicles (as planetary rovers most probably are); as a matter of fact, non-holonomic constraints might probably introduce simplifications; a fact, this last, that should be better investigated. Finally it should also be remarked that the analytical steps which lead to Eq. (42a) and Eq. (42b) were actually necessary in order to eliminate from the considered kinematic equations all the linear velocity terms (since neither directly measured nor part of the adopted state space model). The integrated overall scheme improving the STE module with the inclusion of the IMU system, obviously results into the one reported in Fig. 9. Note however how the scheme also anticipates the presence of a block relative to the wheel odometry, that can be it also

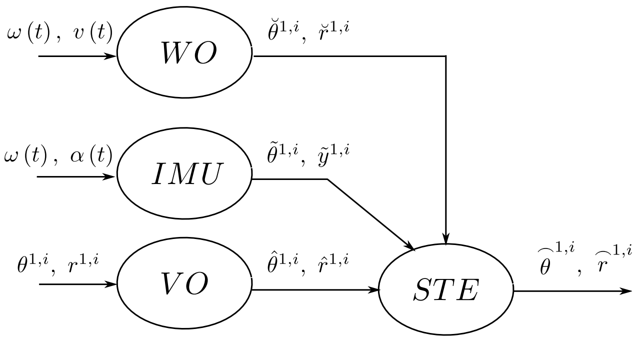

Figure 9: Improving the State Estimator

integrated into the overall system. A wheel odometry system can directly provide, in a substantially continuous-time manner, the measurements $\breve{\omega}$, $\breve{v}$ of the vehicle angular velocity ω and its velocity vector v projected on the vehicle fixed frame. These WO estimates are real-time obtained from the measured rotation velocities of the vehicle wheels and are generally handled within a specific (internal to the WO system itself) least squares algorithm or even a specific state estimator. The WO module continuous time measurements can be expressed as

$$\breve{\omega}(t) = \omega(t) + n_1(t) \tag{43a}$$

$$\breve{v}(t) = v(t) + n_2(t) \tag{43b}$$

where $n_1(t)$ and $n_2(t)$ are continuous-time zero mean white stationary Gaussian noises with covariance functions $N_1\delta(\tau), N_2\delta(\tau)$, respectively. In particular note that, since the WO module actually provides first order measurements for both orientation and position, for the angular velocity measurement in Eq. (43a), similarly to Eq. (37), we can write

$${}^{i-1}\breve{\theta}_i = {}^{i-1}\theta_i + {}^{i-1}\rho_i \quad ; \quad {}^{i-1}\theta_i \doteq \int_{t_{i-1}}^{t_i} \omega(t)\,dt \tag{44a}$$

where the resulting equivalent additional sequence noise (zero mean white Gaussian) expressed as:

$${}^{i-1}\rho_i \doteq \int_{t_{i-1}}^{t_i} n_1(t)\,dt \quad ; \quad {}^{i-1}Z_i = N_1(t_i - t_{i-1}) = N_1 T_i \tag{44b}$$

Moreover, from the first order kinematics we have

$${}^{i-1}r_i = \int_{t_{i-1}}^{t_i} R\left[{}^{i-1}\theta(t)\right] v(t)\,dt \tag{45}$$

simply corresponding to the integral of the velocity projected on $< i-1 >$; since such integral results into a (noisy) quantity that can be measured by the WO system, provided that we can characterize its resulting measure ${}^{i-1}\breve{r}_i$ it also in terms of added noise, that is as

$${}^{i-1}\breve{r}_i = {}^{i-1}r_i + {}^{i-1}\lambda_i \tag{46}$$

it then follows that Eq. (44a) and Eq. (45), once considered for both stages i and $i-1$, actually result into further *six* additional measurements to be used for improving the estimation.

5.2 Conclusions and Comments

Within this paper, a technique to improve rover localization has been suggested; this method employs a Visual Odometry algorithm that has been previously developed at GRAAL Lab. Such a technique can result more efficient, above all for planetary scenarios, as, generally, it is less affected by terrain roughness and slopes that can cause slippage than other on-board sensors. Visual Odometry, as explained, can be improved through the action of state or sequence estimators; furthermore, since a rover is generally endowed also with other sensors, such as classical wheel odometry or inertial measurement units, a multi-sensory integration scheme has been proposed. Anyway, some open issues have been pointed out: first of all, this idea is still under consideration and many simulations and experimental tests have to be carried out. Then, as already suggested, the method is to be extended to more wide motion cases, that is foreseeing not only planar rover movements. Moreover, the method probably might be simplified, mostly taking into account the mentioned non-holonomic constraints for the rover. Finally, it should be noted how the devised multi-sensor integration scheme actually qualifies itself as a “Visual Odometry Centric” scheme: this in the sense that the STE and SQE modules, plus the associated additional sensors, have been adapted to the state space model describing the VO module. In case we had instead started from IMU and/or WO sensor systems, the state space model most probably would have changed, becoming a discrete time, kinematic, state space model (possibly including the vehicle velocities as part of the state), instead of the two-step delay one resulting from the VO. Consequently the eventual successive integration of a VO system should have been adapted to such kinematic model, thus possibly leading to an integration scheme with different characteristics, worth it also to be investigated and compared.

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