Evolutionary Collocation Methods for SMV Skip Trajectory Optimization

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Abstract

This paper presents the simulation results for Space Manoeuvre Vehicles (SMV) skip entry trajectory optimization problem for specified mission. The focus is to investigate methods which are computational efficient and easy to implement. Considering the global optimality and high-accuracy, a two-step optimization approach, the Evolutionary Collocation based on heuristic algorithm and collocation method is presented and discussed. Implementation uses the physical characteristics of the NASA space shuttle in the simulation.

The optimal-control problem is converted into a nonlinear programming problem (NLP) by Legendre-Gauss-Lobatto collocation method. Sequential quadratic programming (SQP) converges faster and the solution accuracy is high when solving NLP, but it is sensitive to initial values. Two heuristic algorithms, Genetic Algorithm (GA) and Particle Swarm Optimization (PSO), are adopted for the descent and exit phases to refine the grids and provide near optimum initial values to circumvent the limitations brought by the classic collocation techniques. The descent and exit are separated in the algorithm to let the user change the parameters at the lowest point. Since the heuristic algorithm does not guarantee the smoothness of generated curves, the control histories (angle of attack, bank angle and thrust) have to be processed to make them more realistic.

Simulation is conducted and the result is compared between two heuristic methods, and the Evolutionary collocation gives a truthful re-entry trajectory satisfying the path constraints and is computation efficient.

1. Introduction

Over the past few decades, the importance of space assets has considerably increased for applications ranging from warfare control to civilian positioning. One of the current objectives is the expansion of Space Manoeuvre Vehicles (SMV): unscrewed, reusable payload carriers with significant manoeuvring capabilities for a dynamic mission profile [1]. These vehicles are expected to achieve a transatmospheric, aeroassisted maneuver such as skip entry. Reference [2] introduced one kind of vehicle, TAVs, which is designed to complete a skip entry maneuver to lower altitude and fulfill the mission of imaging on ground target.

One of the many issues encountered in the development of such vehicles is to predict the dynamics of the aircraft during its re-entry into the Earth's atmosphere and optimize its trajectory accordingly.

Several methods [3-7] have been proposed to optimize the trajectory of a spacecraft entering the atmosphere however most of them relate the optimization to a landing scenario, aiming to improve the control over the range and the touchdown accuracy.

The proposed investigation in this paper will aim to focus on the atmospheric skip, targeting the entry into the atmosphere down to a predetermined point with fixed altitude and the required controls involved in returning to low orbit. Some papers have studied the skip reentry of deep-space spacecraft with high speed over first cosmic velocity, however a high thrust engine would be necessary to return to low orbit [8].

To find an analytic solution to the general trajectory optimization problem is difficult, and many numerical methods are proposed to solve a particular profile. The main difficulty of an transatmospheric entry is the rapid change of atmosphere, the current gradient method used for trajectory optimization will cost a lot of time and memory because of the large gradients which forces the algorithm to refine the grids for a significant increase in computation time [9].

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The aim of this paper will be to continue the current work on trajectory optimization and then improve on two of direct trajectory optimization methods in order to circumvent the limitations brought by the classic gradient techniques. The solution proposed in this paper is to implement a higher level non-gradient optimizer to guide the gradient changes and would help smooth the control over the states discontinuities. Sequential quadratic programming (SQP) methods is considered to solve constrained nonlinear programming problems. And it is generally believed that SQP methods are sensitive to the initial value accuracy. Two heuristic optimizers, GA and PSO are introduced in this paper to optimize initial grids for SQP, which is named as Evolutionary collocation method.

The paper is organized with Section 2 describing the skip entry problem, followed by introduction of three optimization algorithms, optimization objective and constraints. Section 3 presents the simulation results. Section 4 gives a general comparison between two heuristic methods.

2. Problem Definition

General skip reentry problem can be divided into 5 phases: initial roll, down control, up control, Kepler and final entry. Phase 5 (landing) has been thoroughly studied. Considering the mission of SMV, the most challenging phase 2 and 3 will be highlighted in this paper.

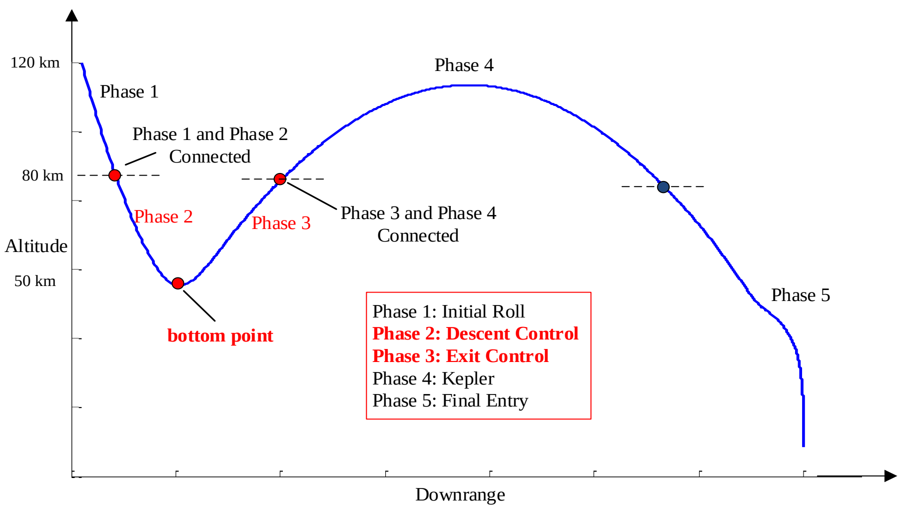

Fig. 1 Whole Phases description of skip reentry

In this section we present the reentry problem. We detail the nonlinear dynamics, the constraints and the cost function.

2.1 Atmospheric modeling

For improved accuracy on the guidance before the initial entry and after the exit, the atmosphere is modeled up to 1000km from the ESDU 77021 documentation.

2.2 Dynamic model

Our initial implementation uses the physical characteristics of the NASA space shuttle found in the literature. The original scenario was set up in British units so most of the literature followed the trend. For this work, international standard units were used except the heating. These parameters can easily be changed to fit a different vehicle.

The equations of motion of the space shuttle are:

$$
\begin{aligned}
&\dot{r} = v\sin\gamma \\
&\dot{\phi} = \frac{\left(v\cos\gamma\cos\psi\right)}{r\cos\theta} \\
&\dot{\theta} = \frac{(v\cos\gamma\sin\psi)}{r} \\
&\dot{v} = \frac{T\cos\alpha - D}{m} - g\sin\gamma \\
&\dot{\gamma} = \frac{L\cos\sigma + T\sin\alpha}{mv} - \frac{g\cos\gamma}{v} + \frac{v\cos\gamma}{r} \\
&\dot{\psi} = \frac{L\sin\sigma}{mv\cos\gamma} + \frac{v\cos\gamma\sin\psi\tan\phi}{r} \\
&\dot{m} = -\frac{T}{I_{sp}g}, \qquad \dot{T} = \frac{K_T\left(T_C - T\right)\frac{\pi}{2}}{T_{max}}
\end{aligned} \tag{1}
$$

where r, ϕ, θ, v, γ, ψ, m, T are state variables, representing radial position, latitude (measured along the local meridian from the equatorial plane, positive northward), longitude (measured along the equator, positive eastward), velocity, flight path angle, heading angle, mass and thrust respectively. Here, angle of attack α, bank angle σ (angle between the lift vector and orbit plane) and thrust T_C are control variables. L and D are the lift acceleration and drag acceleration respectively, which can be defined as:

$$L = \rho V^2 S C_L / (2m) \tag{2}$$

$$D = \rho V^2 S C_D / (2m) \tag{3}$$

Where C_L is the lift coefficient, C_D is the drag coefficient and S is reference surface area. ρ is the density which can be achieved by ESDU 77021 toolbox. The aerodynamic parameters can refer to Reference 6.

2.3 Solutions for Optimization

The optimization problem can be regarded as a nonlinear programming problem. The direct collocation method discretizes all of the variables, equations of motion, and constraint condition equations, and accordingly transforms the SMV reentry trajectory optimization problem into a nonlinear programming problem. The optimal solution of the nonlinear programming problem is obtained using an appropriate method.
Each phase is subdivided into n segments by nodes with corresponding state and control variables. Though the trajectory is divided into 2 phases, it is more efficient to have a number of nodes in each phase proportional to the time it takes to accomplish.

The boundary constraints, initial and final conditions, and path constraints constitute nonlinear constraints on the discretized states x and control variables u in the trajectory optimization problem.

The continuous optimization problem was therefore discretized into an NLP problem containing equality and inequality constraints on the vector.

$$\min J = f\left(x\right) \tag{4}$$

Such that

$$h_i\left(x\right) = 0,\ i = 1, 2, \ldots, p \tag{5}$$

$$g_j\left(x\right) \geq 0,\ j = p+1, \ldots, q \tag{6}$$

Following chart gives a chronological view of the method: 1) Trajectory optimization for the descent. 2) Trajectory optimization for the exit. 3) Join the 2 paths on a new mesh based on Legendre pseudo-spectral collocation method. 4) SQP on the complete trajectory

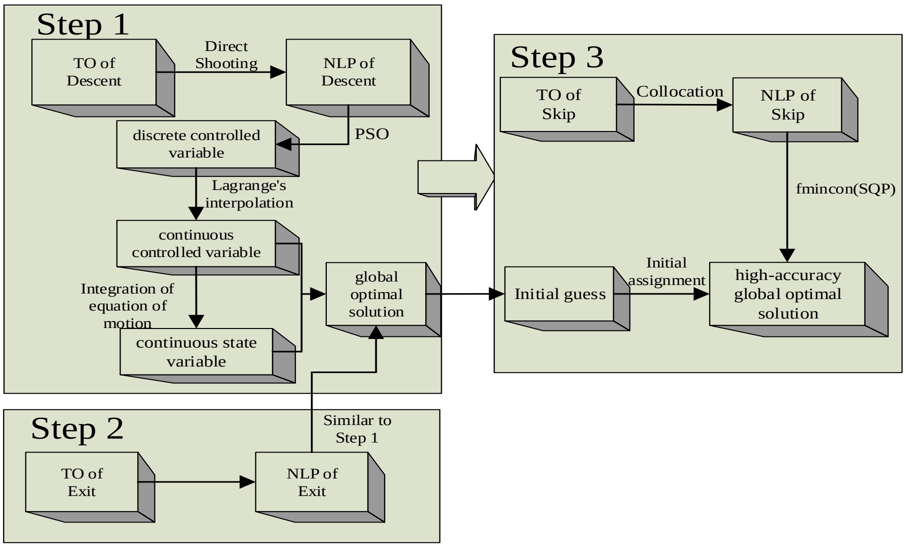

Fig. 2 Flow chart of Trajectory Optimization (TO)

The MATLAB Genetic Algorithm/ Particle Swarm Optimization function also accepts a hybrid function to refine its solution automatically. The NLP can therefore be ran on both segments separately before creating the new mesh and allows the user to skip the NLP on the complete trajectory.

a. Genetic Algorithm

A real-coded GA, where the chromosome for each individual was defined to be the vector of the NLP variables, was used for the multiple-variable trajectory optimization problem in this study [10].

Although the GA also required initial parameters such as the number of collocation points, the population size, convergence criteria, and the temperature cooling coefficient, it was less difficult to determine these parameters based on the results of previous research, and a statistical analysis was often good enough to make an intelligent selection.

b. Particle Swarm Optimization

PSO (Particle Swarm Optimization) algorithm was established by Kennedy and Eberhart [11], based on the model of social psychology. It take inspiration from the social behavior of groups of simple creatures as swarm of bees, colonies of ants, flocks of birds etc, which exhibit some form of collective intelligence based on information exchange. Each agent has two important elements: position and velocity. Every agent of the particle swarm must abide by the success experience of adjacent agents. Agents also have a memory containing the previous particle best position or personal best position and the swarm best position or global best position. All the agents can work together to search a best area of the high dimensional space. Reference [12-13] mentioned some extend study of PSO.

c. Sequential Quadratic Programming

The SQP algorithm proposed by Powell has been considered as one of the most efficient gradient methods for NLP problems [14, 15].

The final result of GA and PSO are treated as the initial point of the SQP algorithm, and then the SQP algorithm is used to find a local optimum near the initial point to achieve a global optimum after remeshing the whole results based on Legendre-Gauss-Lobatto collocation methods.

2.4 Optimization objective and constraints

The descent and exit are separated in the heuristic method to make the parameters at the lowest point adjustable .

Considering the prospective application of SMV, the target state at bottom point is highlighted, other control objective and constraints would be simplified.

Control objective and constraints of two phases will be discussed separately. And the weighting method is adopted for the multi objective optimization.

Control variable, AOA and bank angle both are allowed to vary continuously, so a combination control is used in this paper.

2.4.1 Descent phase for heuristic method

(1) Optimization objective

Multi optimization objectives are considered as follows.

● To minimize the state error at bottom point including the altitude error, flight path error, longitude error, latitude error, heading error with target state.

● To achieve a zero bank angle. Smaller bank angle means to have a level flight and maximize vertical lift for exit.

● To maximize the velocity at the bottom point saves the thrust for exit.

Aim State of control at bottom point is listed below.

Table 1 Simulation parameter setting

Bottom state	value
Height (km)	50
Velocity (km/s)	>3
Flight path angle (deg)	0
Heading (deg)	0
Latitude (deg)	0
Longitude (deg)	0
Bank angle (deg)	0

(2) Constraints

● Angle constraints

Simple angle constraints on the state variable and control variable are considered to demonstrate the effectiveness of two heuristic methods, $[-\frac{\pi}{2},\frac{\pi}{2}]$ rad

● Time constraint

To ensure SMV finish descent in limited time.

● Heating rate constraint

The maximum heating rate usually happens at the bottom point which is closely related with velocity, height and angle of attack.

$$\dot{\mathrm{Q}} = 17700(c_0 + c_1\alpha + c_2\alpha^2 + c_3\alpha^3)\sqrt{\rho}(10^{-4}v)^{3.07} \tag{7}$$

c_0, c_1, c_2, c_3 are coefficients with fixed values [9]. ρ is atmospheric density.

2.4.2 Exit phase for heuristic method

(1) Optimization objective

To minimize the altitude error and flight path angle error at the end of exit.

(2) Constraints

● Time constraint

To ensure SMV finish exit in limited time.

● Heading angle, latitude and longitude

Simple constraints are adopted: $[-\frac{\pi}{2},\frac{\pi}{2}]$ rad.

● Angle of attack

A positive value is adopted: $[0,\frac{\pi}{2}]$ rad.

● Heating rate constraint

Similar with descent phase.

2.4.3 Whole phase for SQP

(1) Control objective:

● To minimize the positional error, flight path angle error and bank angle error at the bottom point.

● To minimize the altitude error and flight path angle error at the exit point.

● To minimize the total thrust applied.

(2) Constraints:

● Velocity at bottom point

To save the energy in exit phase, a larger value is preferable: [5 10] km/s.

● Position constraint at bottom point

Longitude and latitude angle: [-0.2, 0.2] rad

● Other constraints are similar to former settings in 2.4.1 and 2.4.2.

3. Simulation Result

In this part, two test cases are conducted with two heuristic methods. The initial condition and final state are listed in table 2.

Table 2 Simulation parameter setting

	Initial state	**Final state**
Height(km)	80	80
Velocity(km/s)	7.5	Free
Flight path angle(deg)	-1	1
Mass(kg)	90719	Free
Latitude (deg)	80	Free
Longitude (deg)	-45	Free
Heading angle (deg)	10	Free
AOA (deg)	40	Free
Bank angle (deg)	40	0
	Descent phase	Exit phase
Number of nodes	30	30
Thrust (N)	0	$<2*10^6$
Heatflux ($BTU/ft^2/s$, MW/m^2)	400/4.4	
Time (s)	[500 ; 5000]	[50 ; 1000]

It should be noted that in Reference [6], the $Q_{max} = 70$, but it is an orginal value of NASA Space Shuttle nearly forty years ago. Considering the mission requirement of rapid change in attitude at the bottom point, a relatively high value is chosen here.

The simulation can be divided into two scenarios: 1) if the initial position (latitude and longitude) is given a range and the numerical methods can run optimization to find the best boundary values for Phase 1 and 4. 2) If given the fixed value, a better comparison of state at bottom point can be made between two methods.

Interface between optimization routines and high-level modeling packages is well established. All the variables are stored in a single file that the user can modify at will. Models and methods are well seperated and easyly accessible.

3.1 Heating flux curve

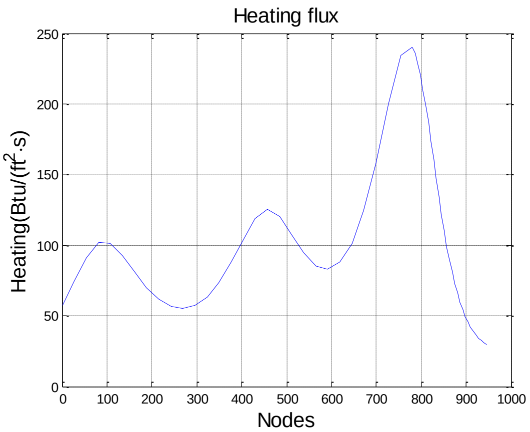

Fig. 3 Heating flux based on GA

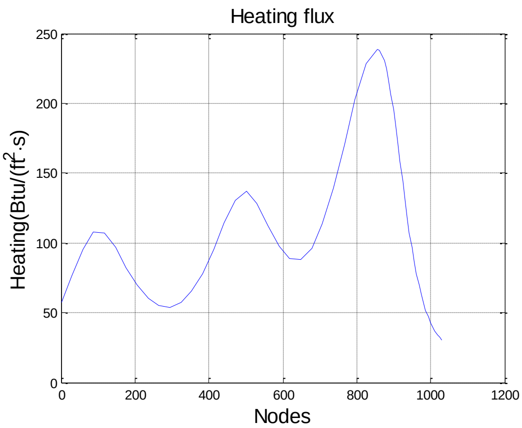

Fig. 4 Heating flux based on PSO

Figure 3 and 4 show the heating flux curve for two methods. Different from the skip re-entry of deep space exploration the maximum heating happens at the bottom point which is similar to the aeroassist maneuver. And the maximum heating value is below the constraint during whole phase.

3.2 Skip reentry trajectory

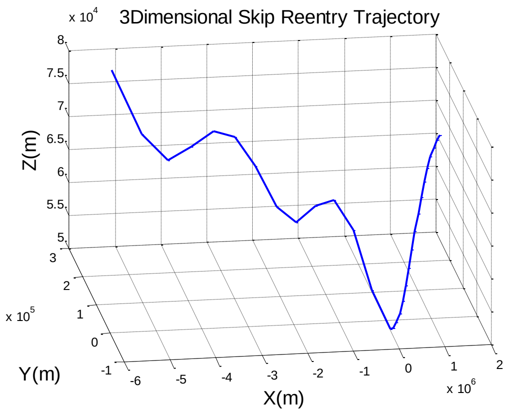

Fig. 5 Skip reentry trajectory based on GA

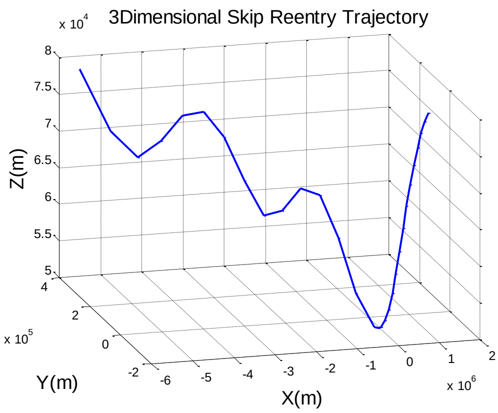

Fig. 6 Skip reentry trajectory based on PSO

Figure 5 and 6 show the trajectory of two evolution collocation methods, 30 nodes are used for collation. The trajectory shows similarity in humps and downs.

3.3 Variable curve of two Evolution Colocation methods

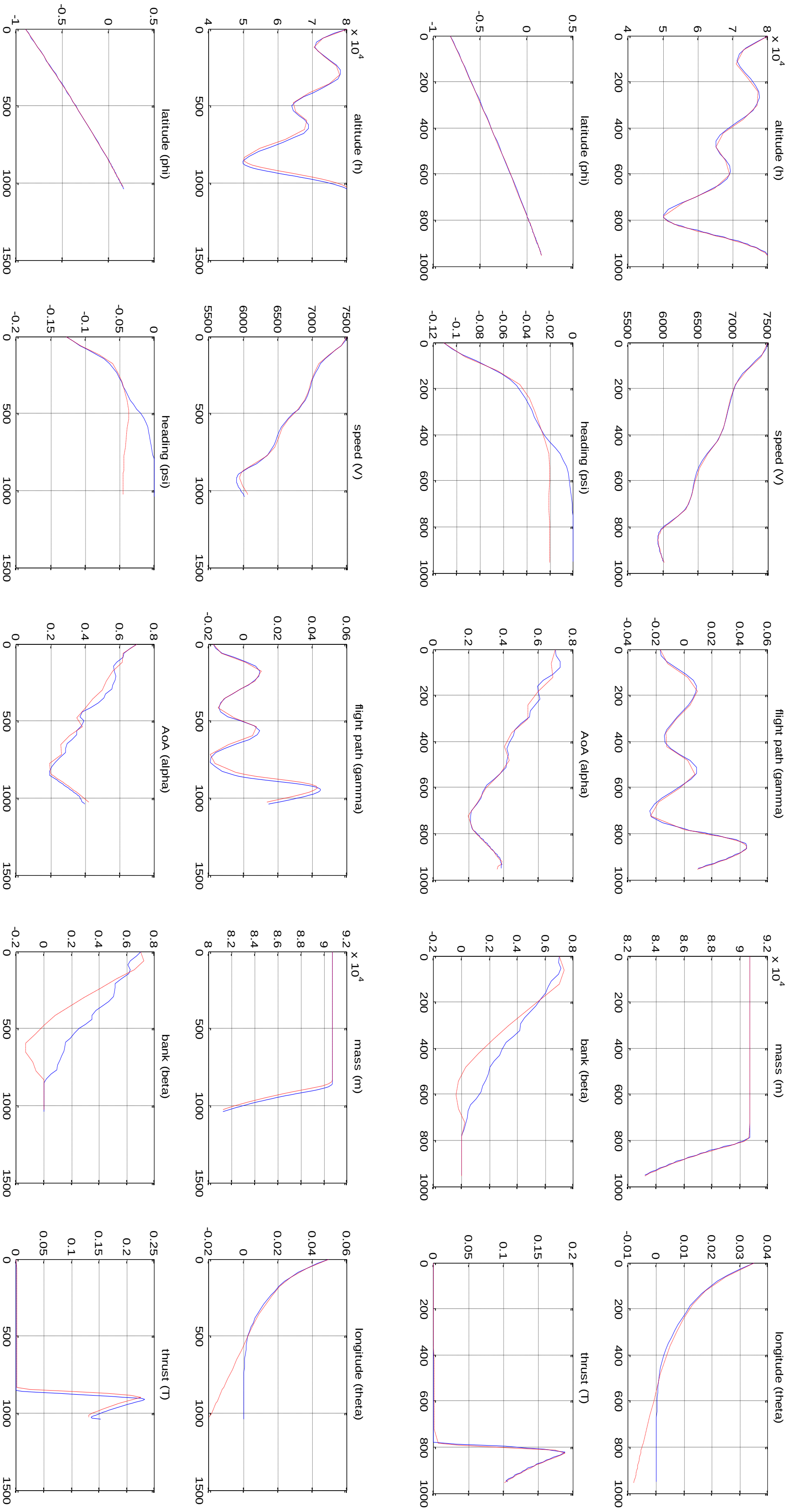

Fig. 7 Curve of state and control variable (a.GA b.PSO)

Figure 7 shows the optimization results given simple constraints on the control variables using numerical methods (blue line). After remeshing the optimization results, SQP gives the final optimization result of NLP problem (red line).

Although there are humps and downs during descent, the velocity remains decreasing with minimum at the bottom followed by a small increase in exit phase to go back to target altitude.

The control variable of AOA decreases gradually to achieve a relatively high velocity and decreases to a value to max L/D at the bottom point. Although in realistic space shuttle control, AOA is maintained at a constant value and decrease linearly, the result in this paper can act as reference.

Bank angle is modulated to control the drag acceleration level and to null range errors. During exit, the bank angle is controlled to zero ensuring minimum thrust. Bank angle is relatively simple to control with rotational thrusters and most vehicles in history tend to use bank angle for trajectory control.

For most time of descent, the flight path angle remains between 0 and 2 degree, but in the exit phase, the flight path angle increase to nearly 3 degree because the vehicle's rate of ascent increases.

Although the simulation result shows similarity in control and state variable, the difference between GA and final NLP result is smaller than PSO.

3.4 Error analysis at the bottom point

In former simulation, the initial state is changable within a range, so the atmospheric interface reentry point is also optimized [16]. However, to generally compare the efficiency of two heuristic methods at the bottom point, it is necessary to analyze the trend with fixed state wihch means the same longtitude, latitude and so on in Table 1, the optimization results are showen as follows.

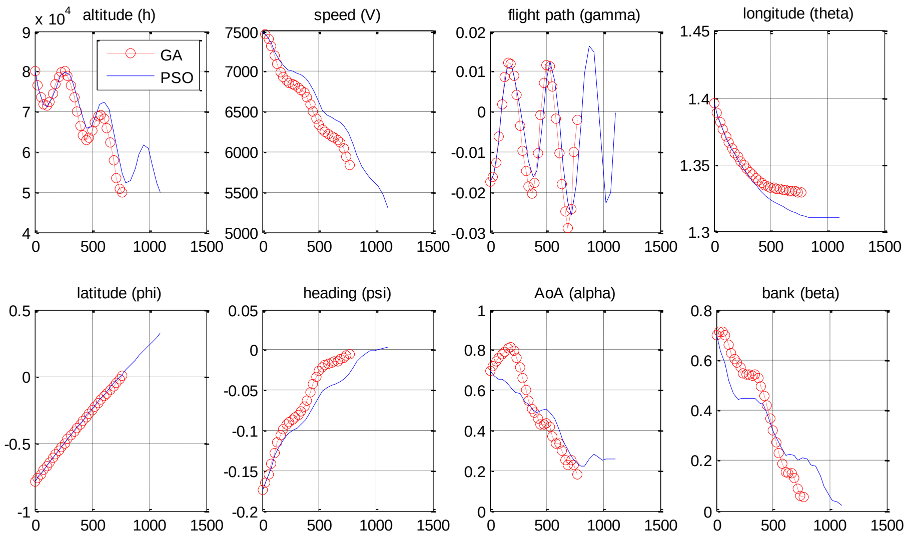

Fig. 8 Simulation result with same initial states

From Figure 8, we can see that both methods can achieve a predetermined altitude at the bottom point, however, GA can achieved a better initial result than PSO because sometimes PSO may be trapped in local minimum fixing state values randomly.

4. Conclusions

An optimal control model is proposed to describe the re-entry problem and the direction collocation method was used to discretize the optimization problem.

This paper describes two heuristic methods which provide initial results for SQP to solve the NLP. The evolutionary collocation method could perform a global search. Two test cases are studied and a general comparison is given in Table 3 and 4.

Table 3 Comparison of simulation time

Time to run GA trajectory	512.27s	Computation time of PSO is only 50 percent of GA.
Time to run SQP	226.22 s	
Time to run PSO trajectory	245.48 s	
Time to run SQP	124.99 s	

Table 4 Comparison between two heuristic methods

Three methods	Computation cost	Optimum	Precision	agents
Genetic Algorithms	expensive computational cost	Global optimum	Lowest precision	individuals are in rivalry
Particle Swarm Optimization	Computationally less expensive, easy to complement	Sometimes Be trapped in local optimum	Medium precision, closer to SQP curve	Swarm intelligence, population-based cooperative behavior of agents

The time to run has a close relation with the generations of GA and particles of PSO, in above simulation, two empirical settings are adopted with generations and particles equal 100 and 40 respectively [17].

So a final conclusion can be reached as follows:

1) The method in this paper minimizes the state error at the bottom point, ensuring the specified mission.

2) With the pre optimized grids provided by heuristic methods, it takes less time for SQP to solve the NLP problem with higher precision.

3) PSO shows advantages in computation efficiency over GA. If the constraint is defined more accurately, the calculation time could be further reduced. However, during simulation process, PSO may be trapped in a local optimum value.

4) From figure 3and 4, we can see that the heating flux hits the maximum value at the bottom point, it would be interesting to investigate how to achieve a trade off between velocity and heating flux. For future work, some other multi-objective optimization methods can be considered, because the current weighting method depends on artificial factor.

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