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Optimal Aircraft Path Planning Considering Wind Uncertainty

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Abstract
The optimization of the aircraft route taking into account wind uncertainty is addressed in this work. The wind uncertainty is obtained from ensemble weather forecasts. A structured airspace is considered, which is defined by a set of waypoints and a set of allowed connections between each pair of waypoints. The analysis is focused on a cruise flight composed of several segments connecting certain waypoints. The optimal route is seen as a path in a graph; therefore, it is obtained by applying a Dijkstra algorithm, following a stochastic approach. Results are presented for a model of B767-300 aircraft, for a given trans-oceanic route, considering a real ensemble weather forecast, and with the objective of minimizing the average total fuel consumption.

1. Introduction

From the operational point of view, trajectory optimization is a subject of great importance in Air Traffic Management (ATM). It aims at defining optimal flight procedures for a given aircraft mission that lead to cost-efficient flights. For commercial transport aircraft, minimizing fuel consumption is of prime importance, both economically and environmentally (because CO2 emissions are directly related to fuel burnt). However, depending on the mission, different performance indices can be considered, such as the direct operating cost (DOC). Aircraft trajectory optimization is an important tool to improve the efficiency of operations and, therefore, it contributes to enhance the efficiency of the ATM system.

Furthermore, in order to maintain high safety standards, any new methodology developed to improve the system performance should integrate uncertainty information. Among the various uncertainty sources that affect the ATM system, weather has perhaps the greatest impact. In particular, weather uncertainty has an important impact on the route planning process. Nilim et al. [1] develop a dynamic routing strategy for the en-route portion of flights subject to adverse weather; they minimize delays modelling the weather processes as stationary Markov chains. Grabbe et al. [2] design a sequential optimization method for traffic flow management, accounting for imperfect weather information, with strategic and tactical control loops; at the tactical level, weather-avoidance rerouting is implemented using a deterministic Dijkstra's algorithm. Sauer et al. [3] analyse the uncertainty related to the displacement and growth of thunderstorm nowcasts to enhance an adverse weather avoidance model for aircraft routing.

In this work, the optimization of the aircraft route taking into account wind uncertainty is addressed; adverse weather phenomena are not considered. The wind uncertainty is obtained from ensemble weather forecasts, and the analysis is focused on the cruise flight. Girardet et al. [4] propose an algorithm for optimal path planning in the presence of a deterministic wind field, and develops an adaptation to spherical coordinates, especially suitable for long flights. An analysis of wind-optimal cruise trajectories using ensemble probabilistic forecasts together with pseudospectral methods is performed in Gonzalez-Arribas et al. [5].

The main objective of this work is to develop a stochastic methodology capable of finding the global optimal path in presence of uncertain winds provided by an ensemble weather forecast.

2. Problem formulation and methodology

In this Section, the optimization of the aircraft route taking into account wind uncertainty is formulated, and the resolution methodology is explained.

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First, ensemble weather forecasting is addressed in Section 2.1, as it is the way used in this work to quantify wind uncertainty. Second, as the analysis is focused on the cruise flight, the airspace structure considered is described in Section 2.2, and the procedure to obtain the flight time and the fuel consumption is developed in Section 2.3. Then, the ensemble trajectory prediction approach is explained in Section 2.4. Finally, the methodology proposed for optimal path planning is described in Section 2.5.

2.1 Ensemble weather forecasting

To model weather for strategic planning horizons, a probabilistic approach is the appropriate one, so that the inherent weather uncertainty can be taken into account. The use of probability forecasts is currently encouraged by meteorologists. For instance, the American Meteorological Society recommends to substantially increase the use of probability forecasts, because they enable users to make decisions based on quantified weather uncertainty, what would lead to socio-economic benefits [6].

Today's trend is to use Ensemble Prediction Systems (EPS), provided by the EWF, which attempt to characterize and quantify the inherent prediction uncertainty based on ensemble modelling. Ensemble forecasting is a prediction technique that consists in running an ensemble of weather forecasts by slightly altering the initial conditions and/or the parameters that model the atmospheric physical processes, and/or by considering time-lagged or multi-model approaches (Arribas et al. [7]; Lu et al. [8]). Thus, this technique generates a representative sample of the possible (deterministic) realizations of the potential weather outcome, as indicated by Steiner et al. [9].

An ensemble forecast is a collection of typically 10 to 50 weather forecasts (referred to as members). Cheung et al. [10] review various EPSs: PEARP (form Météo France), consisting of 35 members; MOGREPS (form the UK Met Office), with 12 members; the European ECMWF, with 51 members; and a multi-model ensemble (SUPER) constructed by combining the previous three forming a 98-member ensemble. Some examples of EPS from the US are MEPS (form the Air Force Weather Agency) with 10 members, and SREF (form the National Centers for Environmental Prediction) comprised of 21 members.

Ensemble forecasting has proved to be an effective way to quantify weather prediction uncertainty. The uncertainty information is on the spread of the solutions in the ensemble, and the hope is that this spread bracket the true weather outcome [9]. It is important to notice that for strategic planning the analysis of all the individual ensemble members must be included (rather than an ensemble mean) [11]. Sloughter et al. [12] (and references therein) address the problem of the statistical postprocessing of the ensembles, considering issues like calibration, multimodality and underdispersion.

2.2 Airspace structure

A structured airspace is defined by a set of waypoints and a set of allowed connections between each pair of waypoints, referred to as airways. ICAO Nat Doc 007 [13] defines a route network for aircraft operating within NAT HLA airspace. In particular, it is stated that, for flights operating at or south of 70°N in a predominantly East-West direction, the planned routes are defined by waypoints at the intersection of parallels spaced at intervals of half degree of latitude with meridians spaced at intervals of ten degrees of longitude, from the Greenwich meridian to longitude 70°W.

In this work, a similar route network is considered; however, the regular waypoints grid is not restricted to be within NAT HLA airspace, but it has been enlarged to be directly connected to the departure and arrival airports. This assumption simplifies the problem, because the continental route network close to the departure and arrival airports does not follow such a regular pattern that defines NAT HLA airspace.

All in all, on one hand, the waypoints constitute a regular grid from $\phi_{\min}$ to $\phi_{\max}$ with a step of 0.5º in latitude, and from $\lambda_{\min}$ to $\lambda_{\max}$ with a step of 10º in longitude; and, on the other hand, the grid is bounded by the longitudes of origin and destination, which can be mathematically stated as $\lambda_{\min} - 10^{\circ} < \min\left\{\lambda_{dep}, \lambda_{arr}\right\} < \lambda_{\min}$, and $\lambda_{\max} < \max\left\{\lambda_{dep}, \lambda_{arr}\right\} < \lambda_{\max} + 10^{\circ}$.

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Furthermore, the allowed connections are defined according to the following rules (see Fig. 1). Let ϕ_0 and λ_0 be the latitude and the longitude, respectively, of a certain waypoint; then,

a) if $\phi_0 \neq \phi_{\max}$, this is connected to the waypoint at ($\phi_0 + 0.5^\circ$, λ_0);

b) if $\phi_0 \neq \phi_{\min}$, this is connected to the waypoint at ($\phi_0 - 0.5^\circ$, λ_0);

c) if $\lambda_0 \neq \lambda_{\min}$, this is connected to every node with longitude $\lambda_0 - 10^\circ$; otherwise, this is connected to the westernmost point between origin and destination;

d) if $\lambda_0 \neq \lambda_{\max}$, this is connected to every node with longitude $\lambda_0 + 10^\circ$; otherwise, this is connected to the easternmost point between origin and destination.

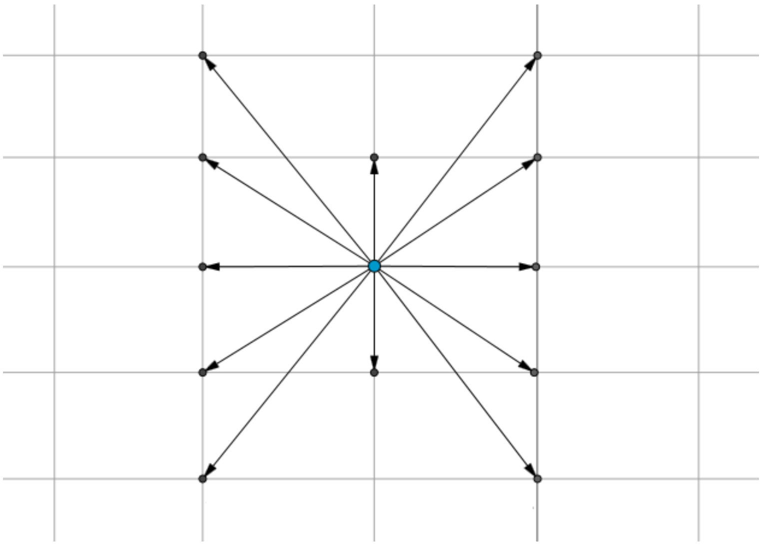

Figure 1: Sketch of connections from an interior waypoint

2.3 Flight time and fuel consumption in cruise flight

As already indicated, in this paper the fuel consumption and the flight time in cruise flight is studied. In accordance with the airspace structure defined in Section 2.2, the cruise is considered to be formed by p cruise segments, each one of them defined by a constant course, and flown at constant Mach number and constant pressure-altitude, as required by Air Traffic Control (ATC) procedures. The Earth is assumed to be spherical, with mean radius $R_E = 6371$ km, and the atmosphere is supposed to be defined by the International Standard Atmosphere (ISA) model plus the winds given by the EPS. The true airspeed is also constant, defined by $V = Ma$, where a is the ISA speed of sound at the given flight pressure altitude.

Let the longitude and latitude of the waypoints that define the cruise segment j be denoted as λ_{j-1}, λ_j, ϕ_{j-1}, and ϕ_j, respectively. Then, the course ψ_j and the segment length $(r_f)_j$ can be computed from the following navigation equations:

$$\tan\psi_j = \frac{\lambda_{j-1} - \lambda_j}{\ln\left[\dfrac{\tan\left(\pi/4 - \phi_{j-1}/2\right)}{\tan\left(\pi/4 - \phi_j/2\right)}\right]} \tag{1}$$

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$$(r_f)_j = \begin{cases} \dfrac{(R_E + h)(\phi_j - \phi_{j-1})}{\cos\psi_j}, & \text{if } \phi_j \neq \phi_{j-1}, \\ (R_E + h)\cos\phi_{j-1}\left|\lambda_j - \lambda_{j-1}\right|, & \text{if } \phi_j = \phi_{j-1}, \end{cases} \tag{2}$$

Sketches of a multi-segment cruise and a generic cruise segment can be found in Fig. 2 and Fig. 3, respectively. In cruise segment j, the flight is subject to along-track winds, $w_{AT_j}(r)$, and crosswinds, $w_{XT_j}(r)$, which vary along the cruise (r represents the distance flown by the aircraft). The effects of the crosswinds are analysed by taking them into account in the kinematic equations, ignoring the lateral dynamics, and translating the crosswind into an equivalent headwind. This leads to a reduced ground speed, which for cruise segment j is given by

$$V_{g_j}(r) = \sqrt{V^2 - w_{XT_j}^2(r)} + w_{AT_j}(r) \tag{3}$$

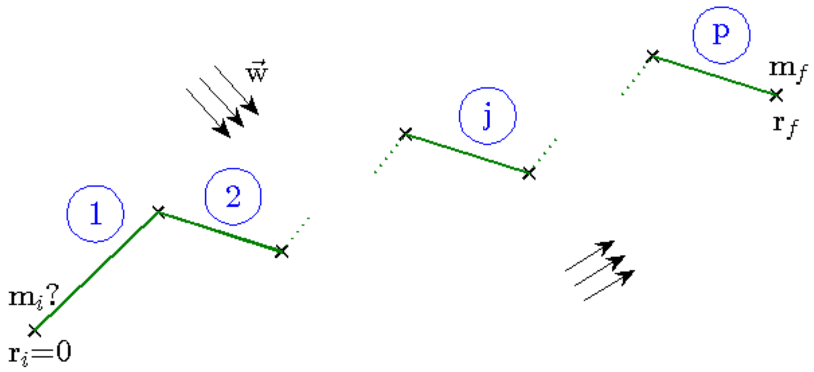

Figure 2: Sketch of a multi-segment cruise.

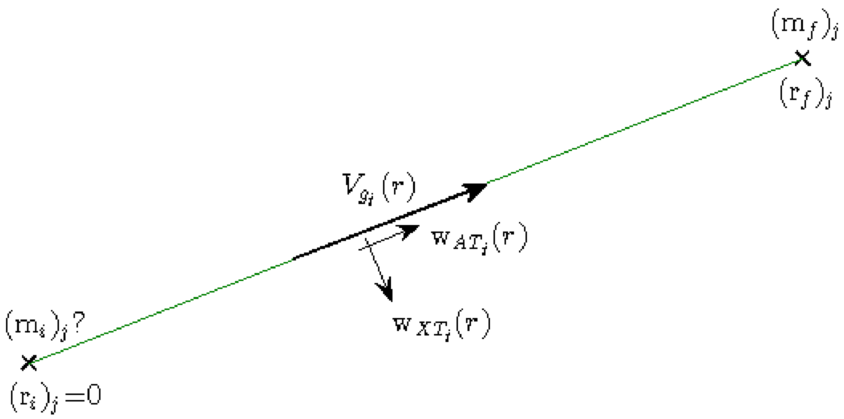

Figure 3: Sketch of a generic cruise segment.

The equations of motion for cruise flight, for segment j, are (see [14]):

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$$\frac{\mathrm{d}r}{\mathrm{d}t} = V_{g_j}(r), \quad \frac{\mathrm{d}m}{\mathrm{d}t} = -c_T T \qquad \textbf{(4)}$$
$$T = D, \quad L = mg$$

where t is the time, T, D, L are the thrust, the aerodynamic drag and the lift, m is the aircraft mass, $g = 9.80665\ \mathrm{m/s}^2$ is the acceleration of gravity, and c_T is the specific fuel consumption.

The drag and the lift can be written as $D = 1/2\,\gamma_a pM^2 SC_D$ and $L = 1/2\,\gamma_a pM^2 SC_L$, where $\gamma_a = 1.4$ is the ratio of specific heats for air, p is the pressure altitude, S the wing surface area, C_D is the drag coefficient, and C_L is the lift coefficient.

The aircraft models considered in this work are defined by Eurocontrol's BADA database ([15]): parabolic drag polar with constant coefficients $C_D = C_{D_0} + C_{D_2} C_L^2$, and specific fuel consumption linear on true airspeed $c_T = C_{f,cr}\, C_{f_1}\left(1 + V[kt]/C_{f_2}\right)$ (note that because V is constant, c_T is constant as well).

The flight time corresponding to cruise segment j is obtained from

$$\left(\Delta t\right)_j = \int_0^{(r_f)_j} \frac{\mathrm{d}r}{V_{g_j}(r)} \qquad \textbf{(5)}$$

Using these definitions and Eqs. (4), the following equation is obtained

$$\frac{\mathrm{d}m}{\mathrm{d}t} = -\left(A + Bm^2\right) \qquad \textbf{(6)}$$

Where the positive constants A and B are given by

$$A = \frac{1}{2} c_T \gamma_a pM^2 SC_{D_0}, \quad B = \frac{2 c_T C_{D_2} g^2}{\gamma_a pM^2 S} \qquad \textbf{(7)}$$

Equation (6) is a non-linear equation that describes the evolution of the aircraft mass as a function of time (this model is adequate to describe the cruise flight of commercial transport aircraft).

In this work, the length of each cruise segment $(r_f)_j$ and the final aircraft mass $(m_f)_p = m_f$ are given. Fixing m_f (instead of the initial aircraft mass) is consistent with having a fixed landing weight. It also allows for a fair comparison for different values of the wind, which lead to different fuel loads and therefore to different values of the initial aircraft mass. Therefore, the whole cruise flight has to be computed backwards, from the destination to the origin, starting from the last segment ($j = p$) and ending at the first one ($j = 1$). For each segment, Eq. (6) is to be solved backwards, from $\left(\Delta t\right)_j$ to $t = 0$, with the boundary condition

$$m\left(\left(\Delta t\right)_j\right) = \left(m_f\right)_j \qquad \textbf{(8)}$$

where $(m_f)_j = (m_i)_{j+1}$, $j < p$ as dictated by mass continuity. This problem (Eqs. 6-8) has the following explicit solution for the final aircraft mass

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$$\arctan\left[\sqrt{\frac{B}{A}}\left(m_i\right)_j\right] = \arctan\left[\sqrt{\frac{B}{A}}\left(m_f\right)_j\right] + \sqrt{AB}\left(\Delta t\right)_j \tag{9}$$

Adding the solutions for all cruise segments, one can easily obtain, on one hand, the total flight time t_f given by

$$t_f = \sum_{j=1}^{p}\left(\Delta t\right)_j \tag{10}$$

And, on the other hand, the initial aircraft mass $(m_i)_1 = m_i$. Then, the cruise fuel consumption follows from $m_F = m_i - m_f$. Hence, one has

$$m_F = \sqrt{\frac{A}{B}}\tan\left\{\arctan\left(\sqrt{\frac{B}{A}}m_f\right) + \sqrt{AB}\; t_f\right\} - m_f \tag{11}$$

2.4 Ensemble trajectory prediction

Ensemble trajectory prediction is one of the main approaches commonly used for trajectory prediction subject to uncertainty provided by EWF, as described in [16, 10]. In this approach, for each member of the ensemble, a deterministic trajectory predictor (TP) is used, leading to an ensemble of trajectories from which probability distributions can be derived.

Suppose that the ensemble has n members. Then, for each member k of the ensemble, the procedure described in Section 2.3 for the computation of the flight time and the fuel consumption can be applied, obtaining t_{f_k} and m_{F_k}. Therefore, for a given cruise path compatible with the airspace structure, the final result is a set of n values of the cruise flight time (t_{f_1}, …, t_{f_n}) and the cruise fuel consumption (m_{F_1}, …, m_{F_n}). Moreover, these sets can be statistically characterized by defining the mean and some measurement of the spread, such as the difference between the maximum and the minimum values.

2.5 Optimal path planning

Because the cruise flight is composed of several segments connecting certain waypoints (including the origin and the destination), the aircraft route can be seen as a path in a graph. Therefore, the optimization of the aircraft route becomes a shortest path problem in the sense of graph theory, and appropriate costs have to be assigned to each segment. In particular, a common objective considered is to minimize the aircraft fuel consumption.

The methodology proposed is based on the application of the well-known Dijkstra's algorithm [17], following a stochastic approach. The Dijkstra Algorithm, developed by Edsger Dijkstra in 1959, is a method to find the shortest paths from an origin node to every other nodes in a graph. It can be restricted to finding the shortest path between an origin node and a destination one, just by stopping its execution once the destination is reached. This algorithm has been widely used in path planning, as it provides an optimal solution if the graph is directed and the connection costs are non-negative.

In the stochastic Dijkstra approach proposed in this work, all the possible wind scenarios, each one of them defined by an ensemble member, are taken into account in the optimization. The problem is stated as selecting the route that minimizes some function of the possible realizations of the fuel consumption. This leads to a unique route to be followed, which is optimal in the sense of the objective function.

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One important case arises when the objective function is the average value (for all members of the ensemble) of the fuel consumption. In this case, several interesting auxiliary solutions can be obtained just by altering the original stochastic problem; these have been extensively addressed in Stochastic Programming (see [18]):

a) Expected Value Solution: In this case, not the whole set of different wind scenarios but only the average wind field is considered, and a deterministic optimization algorithm is applied. This leads to the so called expected value solution, which is common in optimization but can be suboptimal with respect to solving with a stochastic approach. Therefore, the expected value solution sets, in general, an upper bound for the stochastic solution although, in some cases, the optimal path corresponding to the expected value solution may coincide with the stochastic optimal path.

b) Perfect Information Solutions: In this case, for each wind scenario defined by an ensemble member, an optimal route is determined which minimizes the fuel consumption in the presence of that particular realization of the wind field. Again, this is done by applying a deterministic optimization algorithm. Then, one can compute the average value of their corresponding fuel consumptions, which sets a lower bound for the stochastic solution, as indicated in [18]. The differences between them can be seen as a stimulus for forecasts improvement.

A remarkable advantage of the stochastic approach is that it enables performing a trade-off between efficiency and predictability when addressing aircraft path planning. In fact, this approach can also be applied to find the route that minimizes some combination of the average value of the fuel consumption and some measurement of the spread of the trajectories, for instance, the difference between the largest flight time and the smallest one. Note that for this kind of problems it is not possible to define the aforementioned auxiliary deterministic solutions (expected value solution and perfect information solutions).

3. Results

In this section, the application considered in the paper is defined and the results are presented.

In this work, Leonardo da Vinci International Airport (FCO) and John Fitzgerald Kennedy International Airport (JFK) have been selected as origin-destination pair, leading to a trans-oceanic route commonly operated by Air France, Alitalia, and British Airways. Both eastbound and westbound optimal routes are addressed. The coordinates of the airports are included in Table 1.

Table 1: Coordinates of departure and arrival airports

	FCO	**JFK**
Latitude ($\phi_{\min}$)	41º 48’ N	40º 38’ N
Longitude ($\lambda_{\min}$)	12º 14’ E	73º 47’ W

For this origin-destination pair, the waypoints grid is represented in Fig. 4. The limits of the waypoints grid are $\phi_{\min} = 30^{o} N$, $\phi_{\max} = 60^{o} N$, $\lambda_{\min} = 70^{o} W$, and $\lambda_{\max} = 10^{o} E$.

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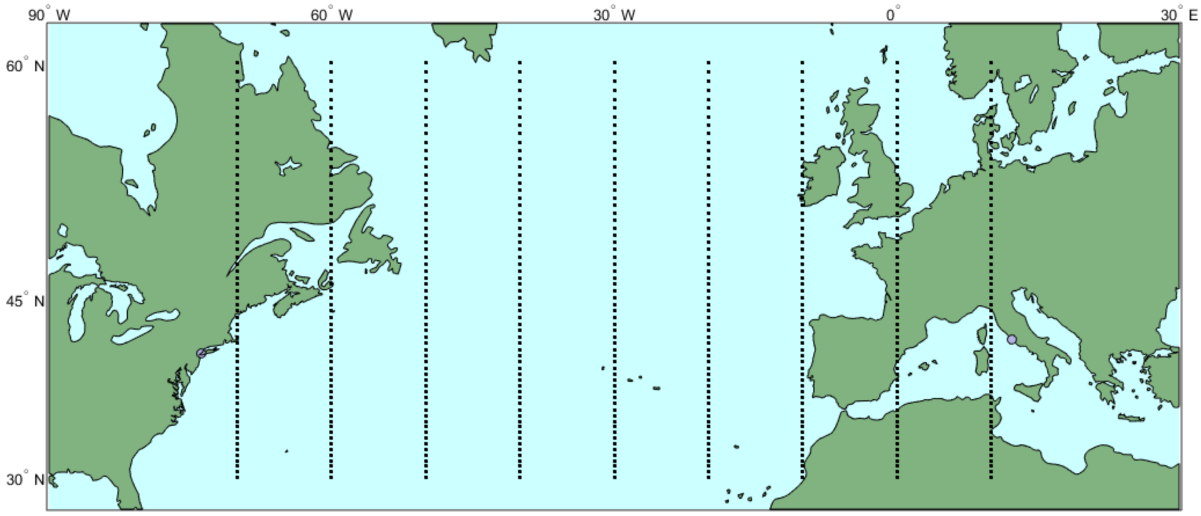

Figure 4: Waypoints grid for JFK-FCO path planning.

The EPS chosen is PEARP, from Météo France. Winds have been retrieved from the ECMWF database, corresponding to 26 June 2016, released at 06:00, with a look-ahead time of 18 hours, and for pressure altitude 200 hPa (this corresponds to an ISA altitude of 11784 m, which is in the stratosphere, where the ISA speed of sound is $a = 295.07$ m/s).

In the following section, results are presented for a model of B767-300. The parameters that define the aircraft model are obtained from BADA; they are given in Table 2. Note that the final aircraft mass m_f is defined as the sum of the minimum mass plus the maximum payload mass (both given by BADA).

Table 2: Aircraft data from BADA

	M	m_f, kg	S, m^2	C_{D_0}	C_{D_2}	C_{f_1}, kg/(min kN)	C_{f_2}, kt	$C_{f,cr}$
B763	0.8	133800	283.35	0.021112	0.042118	0.74220	2060.5	0.90048

3.1 Minimization of the average total fuel consumption

In the first problem addressed, the objective considered has been to minimize the average value of the total fuel consumption, that is

$$J = \frac{1}{n}\sum_{k=1}^{n} m_{F_k} \tag{12}$$

The optimal east- and west-bound routes are presented in Fig. 5 for the stochastic solution, along with the great-circle route as a reference. For the sake of understanding the results, a representative wind field (the average along the EPS members) is also depicted. For this particular weather forecast, one can see that the route from JFK to FCO is mainly shifted to the North (except at the beginning), in order to take advantage of the predominant tailwinds, whereas the route from FCO to JFK is deviated to the South, to avoid encountering strong headwinds.

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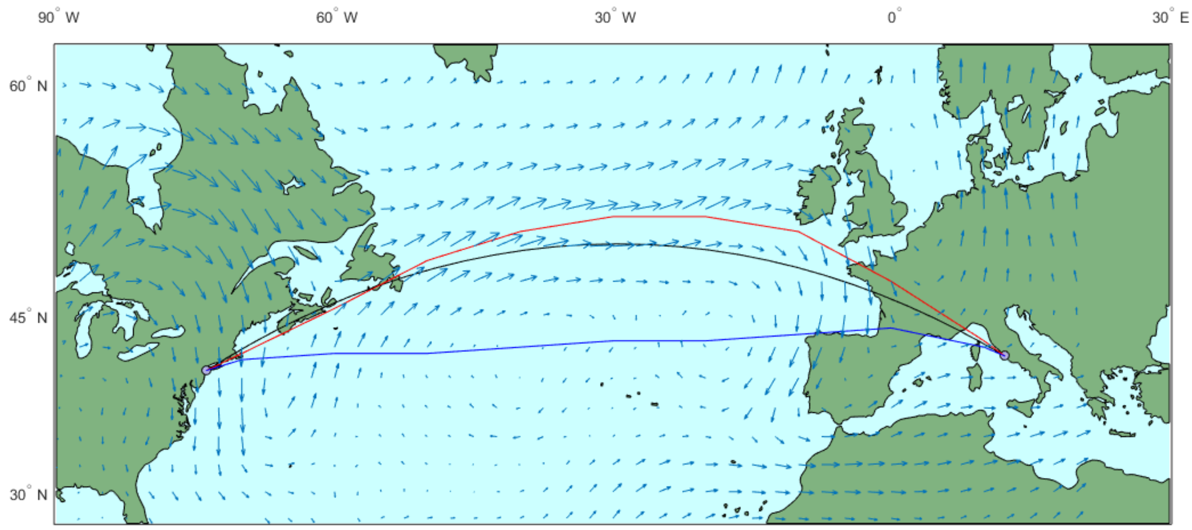

Figure 5: Optimal JFK to FCO routes (red) and optimal FCO to JFK routes (blue) for the stochastic solution.

The values of the optimal total flight time and total fuel consumption corresponding to these trajectories are presented in Table 3. The average $E[\cdot]$ is computed along the EPS members, and the spread $\Delta[\cdot]$ is defined as the difference between the largest value and the smallest one. Note that $E[m_F]$ is the minimum value of J.

Table 3: Average value and spread of the total fuel consumption and the total flight time for the stochastic solution (eastbound and westbound trajectories)

	$E[m_F]$, kg	$\Delta[m_F]$, kg	$E[t_f]$, min	$\Delta[t_f]$, s
Eastbound	36003	762.2	442.0	525.5
Westbound	41467	534.2	504.1	360.8

These results show that for the westbound optimized cruise one has larger values of the mean than for the eastbound optimized cruise (as expected, because in the latter case one can take advantage of the jet stream), but smaller values of the spread.

Note that the values in Table 3 allow for a quantification of the advantages (in a statistical sense) of performing an optimal path planning instead of simply flying the great circle route. For this particular weather forecast, one can save (in average) 1105 kg of fuel when flying from JFK to FCO, and 2537 kg when flying from FCO to JFK; thus, it is especially advisable to perform an optimal path planning for the westbound routes.

In Fig. 6, the optimal east- and west-bound routes for the Expected Value Solutions and for the Perfect Information Solutions are presented, again with the great-circle route as reference. Note that, now, the optimal route is different for each EPS member considered; however these routes are somehow close to each other. Moreover, the optimal route for the Expected Value Solution is encompassed by the routes for the Perfect Information Solutions (one per EPS member).

It is interesting to note that, for the weather forecast considered in this work, the Expected Value Solution coincides with the Stochastic Solution. This result suggests that, if only wind is taken into account and predictability is not rewarded, it is adequate just to consider the average wind field for path planning purposes; however, it may not be the case when the objective is to minimize some combination of the average value of the total fuel consumption and the spread of the total flight time.

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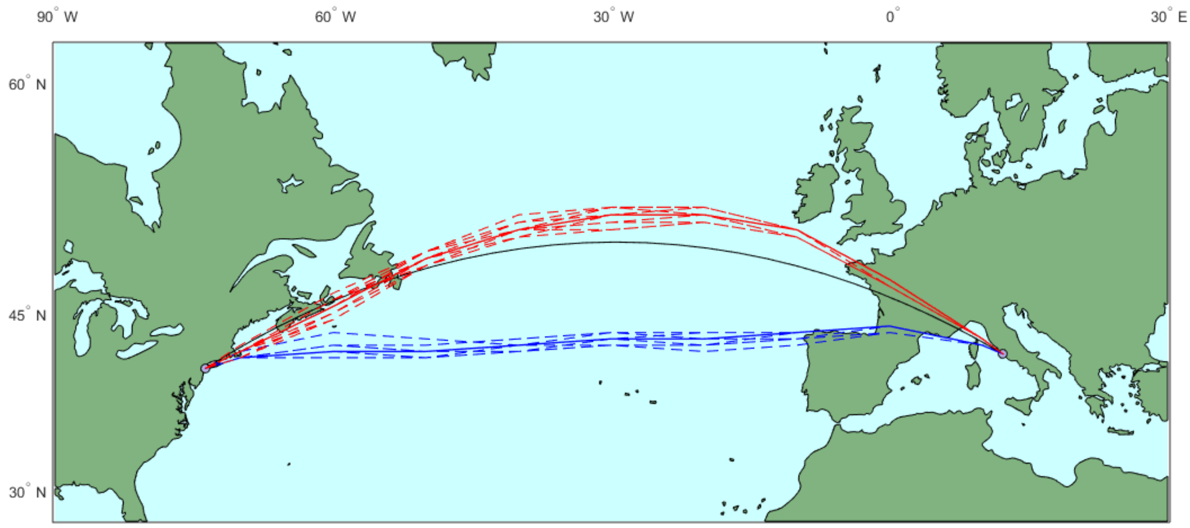

Figure 6: Optimal JFK to FCO routes (red) and optimal FCO to JFK routes (blue) for Perfect Information Solutions (dashed lines) and Expected Value Solution (solid line).

Once the Perfect Information Solutions are obtained, one can compute the average value of their corresponding total fuel consumptions, which gives 35971 kg for the eastbound trajectory, and 41450 for the westbound one. As already expected, the average value of the total fuel consumption for the Perfect Information Solutions is smaller than that for the Stochastic Solution; the difference between them is 32 kg for the eastbound trajectory, and 17 kg for the westbound trajectory.

4. Final remarks

The general framework for this paper is the development of a methodology to manage weather uncertainty suitable to be integrated into the trajectory planning process. In particular, a stochastic methodology has been implemented, which is capable of finding the global optimal aircraft path, considering a structured airspace, in the presence of uncertain winds provided by an EPS. Furthermore, some advantages of applying this methodology have been quantified.

The consideration of temperature uncertainty, also provided by EPS, is left for future work. As cruise segments are usually flown at constant Mach number and constant pressure altitude, the main effect of the temperature distribution is a change in true airspeed (due to the change in the speed of sound), which leads to changes in ground speed and specific fuel consumption.

The trade-off analysis between efficiency and predictability is left for future work.

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